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Short Communications: Compressible Flows

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PART 1:

EQUILIBRIUM THERMODYNAMICS IN COMPRESSIBLE FLOWS

Abstract: For an ideal gas, we have the equations of state pv = RT, de = C_vdT for a unit mass. We show that for differential of q, the external heat supplied to raise the temperature under arbitrary conditions, is not an exact differential. We show that 1/T plays the role of an integrating factor and that differential q/T is an exact differential for equilibrium thermodynamics.

Keywords: Ideal gas, equations, thermodynamics, equilibrium, exact, differential

We start with the following where k(T) is the thermal conduction.

$$q = -k \bigtriangledown T$$

$$\bigtriangledown q = \bigtriangledown(-k) \bigtriangledown^2 T$$

$$\bigtriangledown^2 q = \bigtriangledown^2(-k) \bigtriangledown^3 T$$

But, the Laplacian squared of (-k) is just a constant. This implies the following:

$$\bigtriangledown^2 q = -k \bigtriangledown^3 T$$

Next, divide both sides by Laplacian squared of T, to get the following:

$$\frac{\bigtriangledown^2 q}{\bigtriangledown^2 T} = -k \frac{\bigtriangledown^3 T}{\bigtriangledown^2 T}$$

$$\triangle(q/T) = -k \bigtriangledown T$$

Therefore, the differential of q/T is an exact differential since q, the heat flux, satisfies the Fourier Law in a perfect or ideal gas.

PART 2:

SPEED OF SOUND UNDER SONIC CONDITIONS IN COMPRESSIBLE FLOWS

Abstract: The speed of sound under sonic conditions c_* is obtained when u = c_* locally; and in an ideal gas c^2 = y (rho / p). We show that the stagnation enthalpy h_s can be written has h_s = ½(y+1/y-1) (c_*)^2. We consider now a stationary, normal shock with flow speed u_1 upstream and u_2 downstream. Using the matching relations across the shock demonstrable u_1 times u_2 = (c_*)^2. This result is called the Prandhl-Meyer equation. By further considering the stagnation enthalpy and its implied relationship between u squared and c squared, we show that if u/c star > 1 then u/c > 1 and that u/c > u/c star.

Keywords: Ideal gas, equations, thermodynamics, equilibrium, exact, differential

$$h_s = h + (1/2)u^2$$

$$C_pT_s = C_pT + (1/2)u^2$$

$$\frac{T_s}{T} = 1 + \frac{u^2}{2C_pT} = 1 + \frac{\gamma - 1}{2}(\frac{u^2}{\gamma RT})$$

If we have the following:

$$C_p = \gamma R/(\gamma - 1)$$

Then, we get this equation:

$$\frac{T_0}{T} = 1 + \frac{\gamma - 1}{2}M^2$$

Now,

$$u_1 \to u, u_2 \to u \Rightarrow u^2 = c_*^2$$

$$M \equiv u/c > 1; c_* = u/M, M \equiv 1 \Rightarrow c_* = u.$$

$$\Rightarrow u/c_* = 1 \Rightarrow M \equiv u/c > u/c_* \geq 1.$$

PART 3:

STATIONARY OBLIQUE SHOCKS & SHOCK FRONTS IN A COMPRESSIBLE FLOW

Abstract: Consider the flow with velocity v_1 incident on an oblique shock front that is stationary in the flow, making an included angle beta to the shock. The exit flow speed is v_2 and is deflected by angle theta. We obtain the appropriate balances across the shock, and establish that the velocity components tangential to the shock are continuous across the shock. In terms of M_1 = v_1/c_1 and Beta, we show what the shock relations are.

Keywords: Ideal gas, equations, thermodynamics, equilibrium, exact, differential

We start with the following flows:

$$v_1 = \sqrt{u_1^2 + v^2}, \beta = tan^{-1}(u_1/v)$$

$$v_2 = \sqrt{u_2^2 + v^2}, \beta - \theta = tan^{-1}(u_2/v)$$

We get the following shock relations:

$$M_{n1} = u_1/c_1 = M_1 sin\beta > 1$$

$$M_{n2} = u_2/c_2 = M_2 sin(\beta - \theta) < 1$$

$$P_2/P_1, \rho_2/\rho_1, T_2/T_1, (S_2 - S_1)/C_v$$

For normal shock and replacing Maqam with M_1 sin Beta, we get the following equations:

$$\frac{P_2}{P_1} = 1 + \frac{2\gamma}{\gamma + 1}(M_1^2 sin\beta - 1)$$

$$\frac{\rho_2}{\rho_1} = \frac{(\gamma + 1)M_1^2 sin^2\beta}{(\gamma - 1)M_1^2 sin^2\beta + 2} = \frac{u_1}{u_2} = \frac{tan\beta}{tan(\beta - \theta)}$$

This is the upstream/downstream Mach relationship.

$$M_2^2 sin^2(\beta - \theta) = \frac{(\gamma - 1)M_1^2 sin^2\beta + 2}{2\gamma M_1^2 sin^2\beta + 1 - \gamma}..$$

PART 4:

A SUPERSONIC COMPRESSIBLE FLOW WHERE SHOCK FORMS ON A CORNER & A WEDGE

Abstract: A supersonic flow is incident on a corner where the corner makes an angle alpha. A shock forms at the corner. We use the results of Part 3 to determine the angle of the shock to the incoming flow and specify the outflow conditions. Similarly, an oblique shock forms where a supersonic flow is incident on a wedge. We show what the angle is of the shock to the wedge and where there are the outflow conditions. We demonstrate that if alpha is large, roughly alpha greater than 24 degrees say, then such a configuration cannot be maintained. Under these conditions, a detached shock forms upstream of the wedge.

Keywords: Ideal gas, equations, thermodynamics, equilibrium, exact, differential

We look at mass conservation in equations of motion on a disk/plate surface. u/v, u/r, w are functions of z where (u, v, w) are velocity components of r, theta and z or (r, theta, z) in cylindrical coordinate system with u/r = 0.

Beta is shock wave at corner; alpha is a deflection at corner. Therefore, we have the following:

$$tan(\alpha) = 2cot(\beta)(\frac{M_1^2 sin^2\beta - 1}{M_1^2(\gamma + cos(2\beta) + 2)})$$

Using the above and tan (- alpha) instead where beta is a shock wave at a wedge and alpha is a deflection from a wedge such that we have the following:

$$tan(-\alpha) = 2cot(-\beta)(\frac{M_1^2 sin^2(-\beta) - 1}{M_1^2(\gamma + cos(2(-\beta)) + 2)})$$

If M_1 = 2, then we have a maximum deflection angle. Otherwise, M_1 implies alpha trending towards 0 at Beta = pi/2 (normal shock) and at Beta = arcsin of (1/M_1).

$$M_1 = 2 \Rightarrow \alpha_{max} \simeq 24^0.$$

$$M_1 = \alpha \to 0, \beta = \pi/2; \beta = sin^{-1}\frac{1}{M_1}.$$

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