Performance Analysis of MIMO with Amplify and Forward Relay Network for Mobile to Mobile Fading Channel Model

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Abstract

Wireless relaying is essential to provide reliable transmission, high throughput and broad coverage for wireless networks in a variety of applications. In a cellular environment a relay can be used to overcome shadowing effect due to obstacles. In wireless communication, multiple-antenna systems provide higher rate data transmission than a single-antenna system in a scattered environment. This paper analyses capacity of MIMO with two hop relay systems for mobile to mobile fading channel model (M2M), in which NLOS (Non-line-sight) propagation conditions are assumed from the source mobile station to mobile relay and also from mobile relay to destination station. A non regenerative amplify and forward (AF) relay is used to optimize the capacity between the source and destination, and optimal relaying scheme is compared with few other alternative relaying schemes in terms of capacity for realistic channel model. It is found that the capacity offered by optimal relaying scheme is better than other relaying systems for ideal and realistic channel model.

Keywords: Multiple antennas, ergodic capacity, relay network, Amplify and forward relay, Mobile to Mobile Fading channel

Introduction

The demand for spectral efficiency in wireless communication is ever increasing. The use of multiple antennas at both ends of a wireless link, known as multiple-input multiple-output (MIMO) wireless yields significant improvements in spectral

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efficiency and link reliability through spatial multiplexing [1], [4],[9],[11] &[12] and space-time coding [1],[3],[7] & [13] respectively. A Wireless relay networks is based on the principle of cooperative schemes and viewed as a generalization of the typical multihop approach where the relaying terminal retransmits symbols received from base station or central controller. The main advantage of the cooperative schemes with respect to classical relaying strategies, to create a "virtual" multiple-input multiple-output (MIMO) system, which might offers significant capacity gains in shadowing propagation conditions. For mobile ad hoc networks, relaying is essential not only to overcome shadowing due to obstacles but also to reduce unnecessary transmission power from source and hence radio frequency interference to neighboring nodes. There are two different approaches for cooperative transmission, according to the role played by the relaying terminal: AF scheme and decode and forward scheme (DF) [14]. The simplest approach is the AF which is a nonregenerative where the relay amplifies and retransmits the signal received from the source. In practical relay systems, AF method shows the advantage of simple implementation and low computational complexity compared to DF regenerative systems, since the relay node linearly processes only the received baseband signal without decoding the information [15].

Materials and Methods

MIMO with Relay System Model

A three-terminal orthogonal MIMO relay channel as shown in Fig. 1. All terminals in the relay model are equipped with multiple antennas. The source transmits signals to the relay and destination through channel 1, and relay transmits to the destination in channel 2. Channels 1 and 2 are orthogonal to each other. Based on the assumption, MIMO channel responses between the source and relay, relay and destination, and source and destination, are represented by constant (as opposed to polynomial) matrices H_0 , H_1 and H_2 respectively. The transfer function of a non-regenerative relay is equivalent to a memoryless weighting matrix F that transforms the waveform received at relay to waveform transmitted from relay. Furthermore, assumed that during the transmission of each packet of data H_0 , H_1 , H_2 and F remain constant is given in [7]. The numbers of antennas equipped at the source, destination and relay are denoted as N_t , N_d and N_r respectively. There is little need to consider a non-square F since all N_r antennas at the relay also used for both receiving and transmitting.

Direct Link without Relay:

The signal received at the destination without relay is

$$r = H_0 s + n_0 \tag{1}$$

where *s* is assumed to be a $M \times 1$ zero mean circularly symmetric complex Gaussian signal transmitted by the source terminal. Also assumed that the source works in spatial multiplexing mode, i.e., source transmits independent data streams from

different antennas and over different sub-carriers. The instantaneous capacity between the source and destination is given by [10]

$$C_{I,A} = \log_2 \left| I_M + \rho_0 H_0^+ H_0 \right|$$
⁽²⁾

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Relay without Direct Link

The signal received at the destination is

$$r = H_2 F H_1 s + H_2 F n_1 + n_2 \tag{3}$$

where $n_1 \sim CN(0, \sigma_1^2 I_L)$ and $n_2 \sim CN(0, \sigma_2^2 I_N)$.



Figure 1: A three-terminal orthogonal MIMO relay channel model

The instantaneous capacity between the source and destination is given by

$$C_{I,B} = \log_2 \left| I_L + \rho_1 H_1 H_1^+ - \rho_1 H_1 H_1^+ S^{-1} \right|$$
(4)

with $\rho_1 = \frac{P_1}{M\sigma_1^2}$ which is the (normalized) SNR at the relay destination. Similarly the signal to noise ratio at the receiver given in [7] is

$$S = I_{L} + \frac{\sigma_{1}^{2}}{\sigma_{2}^{2}} F^{+} H_{2}^{+} H_{2} F$$
(5)

Since the transmitted signal from the relay is $FH_1s + Fn_1$, the power constraint on the relay leads to the following constraint on

$$F: \sigma_1^2 tr \left\{ F(I_L + \rho_1 H_1 H_1^+) F^+ \right\} \le P_2$$
(6)

where $tr{\cdot}$ represents the trace of a matrix. For convenience,

$$G = \frac{\sigma_1}{\sigma_2} F$$
(7)

Using (7) in (4), (5) and (6), the optimization problem for the MIMO relaying system working in mode B is given by

$$\max C_{I,B} = \log_2 \left| I_L + \rho_1 H_1 H_1^+ - \rho_1 H_1 H_1^+ S^{-1} \right|$$
(8)
s.t. $tr \left\{ G(I_L + \rho_1 H_1 H_1^+ G^+) \le \rho_2 L \right\}$

where $S = I_L + G^+ H_2^+ H_2 G$ and $\rho_2 = \frac{P_2}{L\sigma_2^2}$ is (normalized) SNR at the destination in channel 2. If the relay does not know the CSI (i.e., H_1 and H_2), the relay matrix F (or G) might be chosen by maximizing the ergodic capacity $C_{e,B} = \xi_{H_1H_2} \langle C_{I,B} \rangle$ with the average power constraint $\varepsilon_{H_1} \langle tr \langle G(I_L + \rho_1 H_1 H_1^+) G^+ \rangle \rangle \leq \rho_2 L$. It is showed that the maximal ergodic capacity with unknown CSI and the relay could be achieved by using a diagonal weighting matrix.

Naive Amplify-and-Forward (NAF) Scheme

This scheme simply normalizes the received signal to meet the power constraint and then forward the signal to the destination. In this, the weighting matrix at the relay is

$$F_{naf} = \eta_1 I_L \tag{9}$$

The power constraint is given by (6) and hence

$$\eta_{1} = \frac{\sigma_{2}}{\sigma_{1}} \sqrt{\frac{\rho_{2}L}{tr\{I_{L} + \rho_{1}H_{1}H_{1}^{+}\}}}$$
(10)

Pseudo Match-and-Forward (PMF) Scheme

Another simple choice of the weighting matrix at the relay as used in [5] is

$$F_{pmf} = \eta_2 H_2^+ H_1^+ \tag{11}$$

To meet the power constraint, $\eta 2$ is given by

$$\eta_2 = \frac{\sigma_2}{\sigma_1} \sqrt{\frac{\rho_2 L}{tr\{H_1^+ H_1(I_L + \rho_1 H_1 H_1^+) H_2 H_2^+\}}}$$
(12)

This scheme is asymptotically optimal when the numbers of relay nodes in the MIMO parallel relay channel approaches infinity [5]. When the number of relays is sufficiently large, the signals received by destination combined coherently, which yields a nice scaling law for the two-hop relay networks. However, the performance of this scheme was not shown in [5] when there exists a single relay, and thus it is compared.

Three Ring Mobile to Mobile (M2M) Fading Channel Model

To include the channel characteristics, the three ring scattering model for narrowband MIMO M2M fading channels in AF relay type cooperative networks channel model

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proposed [2] adopted in the simulation. It is a widely used model for outdoor environment. It has multiple transmit antennas (N_t) at source mobile station, multiple antennas (N_r) at mobile relay and multiple receiver antennas (N_d) at the destination mobile stations. There are M local scatterers around the source mobile station, denoted by $S_s^{(m)}$ (m = 1, 2, 3, ..., M), are positioned on a ring of radius R_s , whereas N local scatterers $S_D^{(n)}$ (n = 1, 2, 3, ..., N) lie around the destination mobile station on a separate ring of radius R_D . Furthermore, the local scatterers $S_R^{(k)}$ (k = 1, 2, 3, ..., K) and $S_R^{(l)}$ (l = 1, 2, 3, ..., L) are located on a third ring of radius R_R around the mobile relay. The number of local scatterers around the mobile relay is K = L and $S_R^{(k)} R = S_R^{(l)}$ The subscripts S, R, and D represents source mobile station, mobile relay, and destination mobile station. Considering a $2 \times 2 \times 2$ antenna configuration, the source mobile station, mobile relay, and destination mobile stations are equipped with two antennas each and assumed that the NLOS propagation conditions in all the transmission links. The complete system separated into 2×2 MIMO subsystems. One of the MIMO subsystems (source mobile station and mobile relay) is denoted by the S-R MIMO subsystem. While the other MIMO subsystem (mobile relay and destination mobile station) is termed as RD MIMO subsystem. The input-output relation for the S-R MIMO subsystem expressed as

$$X(t) = H_{S-R}(t)S(t) + N_R(t)$$
(13)

where X(t) is a 2 × 1 received signal vector at the mobile relay, S(t) is a 2 × 1 signal vector transmitted by the source mobile station, and $N_R(t)$ is a 2×1 additive white Gaussian noise (AWGN) vector. In (13), $H_{S-R}(t)$ is a 2×2 channel matrix, which models the M2M fading channel between the source mobile station and mobile relay $H_{S-R}(t)$ expressed as

$$H_{S-R}(t) = \begin{pmatrix} h_{S-R}^{11}(t) & h_{S-R}^{12}(t) \\ h_{S-R}^{21}(t) & h_{S-R}^{22}(t) \end{pmatrix}$$
(14)

Considering the geometrical three-ring scattering model (Fig 2), the m^{th} homogeneous plane wave emitted from $A_S^{(q)}$, first encounters the local scatterers $S_S^{(m)}$ round the source mobile station. Moreover, before impinging on $A_R^{(i)}$, the plane wave is captured by local scatterers $S_R^{(k)}$ around the mobile relay, however the reference model is based on the assumption that number of local scatterers, *M* and *K*, around the source mobile station and mobile relay is infinite. Following [8], the diffuse component $h_{S-R}^{11}(t)$ of the transmission link from $A_S^{(1)}$ to $A_R^{(1)}$ approximated as

$$h_{S-R}^{11}(t) = \frac{1}{\sqrt{MK}} \sum_{m=1}^{M} \sum_{k=1}^{K} g_{S-R}^{(mk)} \exp(j[2\pi(f_{S}^{m} + f_{S-R}^{(k)}(t) + (\theta_{S-R}^{(mk)} + \theta_{S-R})]$$
(15)

with joint gains $\frac{1}{\sqrt{MK}}$ and joint phases $\theta_{S-R}^{(mk)}$ caused by the interaction of the local scatterers $S_{S}^{(m)}$ and $S_{R}^{(k)}$. The joint phases, $\theta_{S-R}^{(mk)}$, S-R are considered to be independent

and identically distributed (i.i.d.) random variables, each having a uniform distribution over the interval $[0, 2\pi]$.

$$g_{S-R}^{(mk)} = a_{S}^{(m)} b_{R}^{(k)} c_{S-R}^{(mk)}$$

$$a_{S}^{(m)} = e^{j(\pi/\lambda)\delta_{S}} \cos(\varphi_{S}^{(m)} - \beta_{S})$$

$$b_{R}^{(k)} = e^{j(\pi/\lambda)\delta_{R}} \cos(\varphi_{S-R}^{(m)} - \beta_{R})$$

$$c_{S-R}^{(mk)} = e^{j(2\pi/\lambda)} \left[R_{S} \cos(\varphi_{S}^{(m)} - \gamma_{S}) R_{R} \cos(\varphi_{S-R}^{(k)} - \gamma_{S}) \right]$$

$$\theta_{S-R} = -\frac{2\pi}{\lambda} \left(R_{S} + D_{S-R} + R_{R} \right)$$

$$f_{S}^{m} = f_{Smax} \cos(\varphi_{S}^{(m)} - \alpha_{S}) \text{ and } f_{S-R}^{(k)} = f_{Rmax} \cos(\varphi_{S-R}^{(k)} - \alpha_{R})$$
(16)

Where ${}^{f_{Smax}}$ maximum Doppler frequency caused by the motion of source mobile station (mobile relay) and λ denotes the carrier wavelength. The diffuse component ${}^{h_{S-R}^{22}(t)}$ obtained by replacing ${}^{a_{s}^{(m)}}$ and ${}^{b_{R}^{(k)}}$ by their complex conjugates ${}^{a_{s}^{(m)*}}$ and ${}^{b_{R}^{(k)*}}$, components ${}^{h_{S-R}^{12}(t)}$ and ${}^{h_{S-R}^{21}(t)}$, realized likewise by substituting ${}^{a_{s}^{(m)}}$ and ${}^{b_{R}^{(k)*}}$, by their complex conjugates ${}^{a_{s}^{(m)}}$ and ${}^{b_{R}^{(k)*}}$ [8].



Figure 2: Geometrical three-ring scattering model for a $2 \times 2 \times 2$ MIMO Mobile to Mobile fading channel

In the same way, the input-output relationship of the R-D MIMO subsystem is written as

$$R(t) = H_{R-D}(t)X(t) + N_D(t)$$
(17)

where R(t) is a 2 × 1 received signal vector at the destination mobile station, X(t) is a 2×1 signal vector transmitted by mobile relay, $H_{R-D}(t)$ is a 2 × 2 R-D fading channel matrix, and $N_D(t)$ is a 2 × 1 AWGN vector. The diffuse component $h_{R-D}^{11}(t)$ is obtained by replacing the index S by R, R by D and S-R by R-D in (17) respectively. Correlation properties of the transmission link between source and destination is derived between from diffuse components of $h_{S-R-D}^{11}(t)$ and $h_{S-R-D}^{22}(t)$. Closed-form expression of the source correlation function is given by

$$\rho_{\rm S}(\delta_{\rm S},\tau) = J_0 \left(2\pi \sqrt{\left(\frac{\delta_{\rm S}}{\lambda}\right)^2 + \left(f_{\rm Smax}\tau\right)^2 - 2\left(\frac{\delta_{\rm S}}{\lambda}\right)f_{\rm Smax}\tau\cos(\alpha_{\rm s}-\beta_{\rm s})} \right)$$
(18)

Closed form expression of the relay correlation function is given by

$$\rho_{R}(\delta_{R},\tau) = 2\left\{J_{0}(2\pi f_{Rmax}\tau)\right\}^{2} + J_{0}\left\{\left(2\pi\sqrt{\left(\frac{\delta_{R}}{\lambda}\right)^{2} + \left(f_{Rmax}\tau\right)^{2} - 2\left(\frac{\delta_{R}}{\lambda}\right)f_{Rmax}\tau\cos(\alpha_{R}-\beta_{R})}\right)\right\}^{2} + J_{0}\left\{\left(2\pi\sqrt{\left(\frac{\delta_{R}}{\lambda}\right)^{2} + \left(f_{Rmax}\tau\right)^{2} + 2\left(\frac{\delta_{R}}{\lambda}\right)f_{Rmax}\tau\cos(\alpha_{R}-\beta_{R})}\right)\right\}^{2}\right\}$$

$$(19)$$

Results and Discussion

The comparison of simulation results of optimal relaying scheme with other alternative non-regenerative relaying schemes such as Naïve Amplify-and-Forward (NAF) and Pseudo Match-and-Forward (PMF) for independent identically distributed channel model is shown in Fig. 3. The results revealed that the capacity of optimal relaying scheme is very close when the signal to noise ratio at the destination is high. It is clear that the Naive Amplify-and-Forward is far from the optimum and the Pseudo Match-and Forward method is even worse than the Naive Amplify-and Forward method. In real time channel characteristics, the three ring channel model is used in simulation, which is a widely used model for outdoor environment. Fig 4 and 5 shows the correlation function of source and relay in three ring M2M fading isotropic scattering channel model by using simulation tilt angle, $\beta_s = \beta_R = \pi/2$, $\alpha_s = \pi/4$, $\alpha_R = 0$ $f_{Smax} = f_{Rmax} = 91$ Hz and wavelength $\lambda = 0.15$ m. Fig 6 and 7 shows the results for nonisotropic scattering condition for M = 40 and K = L = 20 and von Mises distribution employed to characterize nonisotropic scattering around the source mobile station (destination mobile station) and mobile relay. In this simulations, the parameters of the von Mises distribution were $\varphi_{S}^{(0)} = \varphi_{S-R}^{(0)} = \varphi_{R-D}^{(0)} = 60^{\circ}$ and $\kappa_{S} = \kappa_{S-R} = \kappa_{R-D} = 40$.



Figure 3: Mean capacity of non-regenerative relaying schemes



Figure 4: Source correlation function of 2*2*2 MIMO system with Mobile to Mobile channel model under isotropic scattering conditions.



Figure 5: Relay correlation function of 2*2*2 MIMO system with Mobile to Mobile channel model under isotropic scattering conditions.



Figure 6: Absolute value of the source Correlation function of 2*2*2 MIMO system under nonisotropic scattering conditions



Figure 7: Absolute value of the relay Correlation function of 2*2*2 MIMO system under nonisotropic scattering conditions



Figure 8: Mean capacity of non-regenerative relaying schemes for realistic channel condition.



Figure 9: Mean capacity of various antenna configurations.

Conclusion

Optimal relaying scheme is compared with few other alternative non-regenerative relaying schemes for ideal channel and realistic mobile to mobile fading channel model. In mobile to mobile fading channel scatters are present around mobile source, relay and destination stations, so analysis is also done with the source and relay station correlation functions. It has been found that the capacity offered by optimal relaying scheme is better than other relaying systems for realistic channel model. Finally MIMO system for M2M fading channel model is compared with MIMO system with and without direct link and single antenna system. MIMO system with direct link provides better capacity results than other systems. Realistic channel capacity of MIMO system is somewhat less than ideal channel model due to correlated channel condition.

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