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On the Mechanism of Inferior Mirage Formation

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Abstract

The inferior mirage is an intriguing optical phenomenon observed in nature. A survey of literature reveals that roughly half of the textbooks, scientific dictionaries and encyclopedias mention total internal reflection as the underlying cause of mirage formation while the other half cite refraction of light as being solely responsible for the same. Yet others seem to be uncertain but lean towards simple reflection as the underlying cause. In this paper, the real cause of inferior mirage formation is investigated. This is done from the point of view of both ray optics and wave optics. In the former category, a layer approximation model is frequently used. It was found that objections can be raised against total internal reflection, refraction or reflection in this model. In the wave propagation model, the formation of inverted image could not be realized. We are left with the uncontroversial statement that mirage is formed due to the bending of light as it propagates through a non-uniform medium whose refractive index varies gradually in the vertical direction.

1. INTRODUCTION

A ***mirage*** commonly refers to an optical illusion seen by an observer in the desert who sees the image of a distant object as a pool of water. Mirages are also seen elsewhere on a hot summer day, usually on a roadway. These are referred to as ***inferior mirages*** as opposed to a different kind of mirage called the ***superior mirage*** seen in cold arctic regions. In each kind of mirage, an inverted image of a distant object is produced by the bending of light. In an inferior mirage, the air above the ground is hotter than the air above and light bends upwards. In a superior mirage, the conditions are reversed and light bends downwards. Like most of the optical phenomena observed in nature, mirage formation can be explained by the ***laws of geometrical optics***. However, there is a great amount of controversy as to the precise mechanism of mirage formation in general and that of the inferior mirage in particular. An extensive survey of literature reveals that opinions are evenly divided between two distinct camps. Roughly half of the textbooks, scientific dictionaries and encyclopedias mention ***total internal reflection*** as the underlying cause of mirage formation [1-12] while the other half cite ***refraction of light*** as being solely responsible for the same [13-26]. Curiously, there is yet a third camp of authors who are less certain about the exact cause of mirage formation, but point towards ***reflection*** as the plausible mechanism [27-34]. Phrases like “as if reflected” [29, 30, 32, 33], “appears reflected” [27] and “appears to reflect” [28] have been used. In this paper, we shall critically examine each of the three mechanisms of mirage formation (total internal reflection, refraction and reflection) and offer an alternative uncontroversial statement regarding the actual mechanism of mirage formation. We shall restrict our discussion to the inferior mirage formation, even though it will be applicable to the superior mirage also.

II. LAWS OF GEOMETRICAL OPTICS

Geometrical Optics (also called ***ray optics***) comprises the subject matter of propagation of light in vacuum and through material media. It is governed by a few laws which are at once simple, elegant and understandable. They are recapitulated as follows.

Rectilinear propagation of light. In a uniform isotropic medium, light ***travels in a straight line*** with a velocity $v = c/n$, where c is the velocity of light in vacuum and n is the ***refractive index of the medium***. Since the straight line is the shortest distance between two points, light ***travels along the shortest path*** between two points. Further, if s is the distance between the two points, the time of travel $t = s/v$ is also minimum. In other words, light also ***travels along the fastest path*** between two points within a homogeneous isotropic medium.

Law of reflection. If light is reflected from a surface, the incident and reflected rays make equal angles with the normal to the surface at the point of incidence. Stated alternatively, the angles of incident i and reflection r are equal: $i = r$.

Law of refraction. When light enters from one medium of refractive index n_1 into a second medium of refractive index n_2, it follows ***Snell’s law***:

$$n_1 sin\theta_1 = n_2 sin\theta_2 \tag{1}$$

where θ_1 is the angle of incidence and θ_2 is the angle of refraction. It should be noted that the refracted ray is always accompanied by a reflected ray, which follows the law of reflection (Fig. 1).

Total internal reflection. When light enters from a denser medium of refractive index n_1 to a rarer medium of refractive index n_2 ($n_1 > n_2$), the angle of refraction θ_2 is greater than the angle of incidence θ_1. In accordance with Snell’s law, θ_2 becomes 90^o, when

$$\theta_1 = sin^{-1}\left(\frac{n_2}{n_1}\right) \tag{2}$$

which is called the ***critical angle*** for refraction from the denser medium into the rarer medium. For angles of incidence greater than the critical angle, there is no refraction of light and the incident light is ***totally reflected*** back into the denser medium.

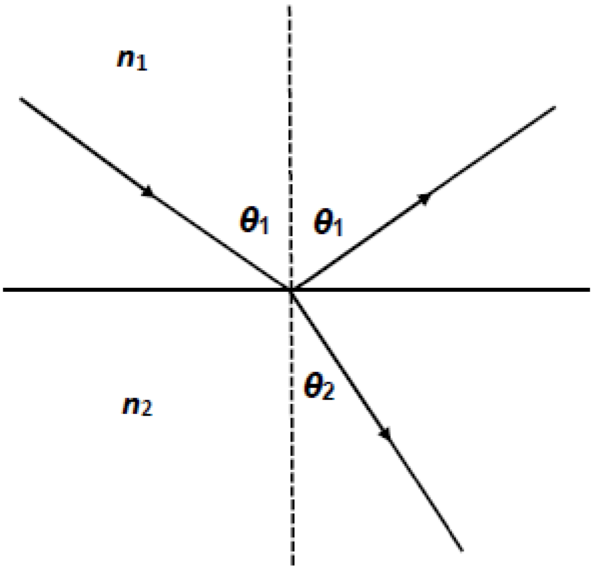

Fig. 1

Fermat's principle. The ***optical path*** *L* is defined as the product of the refractive index *n* and the actual path length *s*: $L = ns$. ***Fermat's principle*** states that the ***optical path*** taken by light in propagating between two points, either directly, or via reflection or refraction, ***is a minimum***. Since by definition, $L = ct$, Fermat's principle equivalently states that the ***time taken*** by light in propagating between two points, either directly, or via reflection or refraction, ***is also a minimum***. It can be shown that the laws of rectilinear propagation, reflection and refraction of light all follow from Fermat's principle (cf. [35]). In other words, Fermat's principle represents a ***unification of the laws of geometrical optics***. A violation of Fermat's principle has been reported in the literature [36]. However, it was pointed out that Fermat's principle is always upheld locally, since any reflecting or refracting surface is considered locally flat [37].

The general form of Fermat's principle for the passage of light between two points 1 and 2 is written as:

$$\delta \int_1^2 n ds = 0 \tag{3}$$

This is analogous to ***Hamilton's principle*** of classical mechanics. By applying methods analogous to dynamical problems, the equation of the path of a ray can be obtained in a medium of varying refractive index [38]. In the case of an inferior mirage, the refractive index of air can be assumed to increase linearly with height *y*: $n = n_0 + \alpha y$, where n_0 is the refractive index of air at ground level and α is the gradient of increase. The equation of the path of the ray is found to be [38]:

$$y = h + \frac{n_h}{\alpha}\left[cosh\left(\frac{\alpha x}{n_h}\right) - 1\right] \tag{4}$$

where n_h is the refractive index at height *h* from the ground level (Fig. 2). Equation (4) is that of a ***caterary***, which is the shape of a uniform chain hanging under gravity.

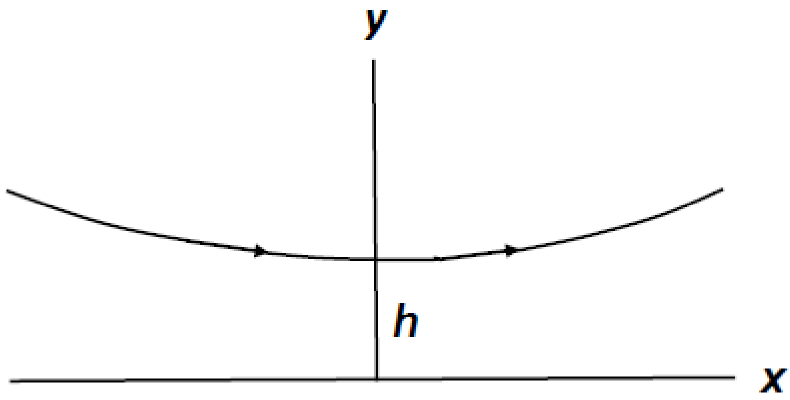

Fig. 2

III. MECHANISM OF INFERIOR MIRAGE FORMATION

It is relevant to point out that the vast majority of authors [1-34] opine about the underlying cause of the mirage formation, whether they refer to it as total internal reflection, refraction alone, or simply reflection, but do not elaborate the actual mechanism involved. Following are some of the descriptions aimed at the latter.

The Layer Approximation Model. In this model, the air above the heated surface is approximated by thin horizontal layers of constant refractive indices (Fig. 3). Since the refractive index is inversely proportional to the temperature [39], it increases with height. As a ray from a distant object descends from above, it bends away from the normal (Fig. 3). At some point, the angle of incidence exceeds the critical angle (cf. Eq. 2), and the ray is totally reflected (Fig. 3). It continues to bends upwards thereafter following the path prescribed by Eq. (4). In this ***layer approximation model***, both refraction of light and total internal reflection are involved. But since the latter assumes a more critical role of bending the ray upwards, authors subscribing to the total internal reflection scenario [1-12] must have had the layer approximation model in their minds.

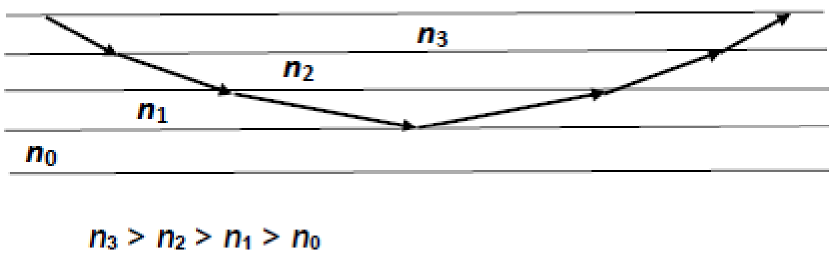

Fig. 3

Some authors [18, 40] have argued against total internal reflection as having a role in mirage formation since “transition between the layers is gradual everywhere” [40] and “refraction alone accounts for the mirage” [18]. However, the same arguments can be used against refraction. By definition, refraction refers to the change in direction of a ray of light entering from one medium into a second. There exists a ***surface of separation*** between the two media, also referred to as an ***interface***. A part of the ray is reflected at the interface following the law of reflection (Fig. 1) and consequently the intensity of the refracted ray is diminished. This is repeated at each interface and the intensity of light should diminish rapidly. Since this apparently does not happen, there must be no refraction at the ‘virtual’ interfaces in the layer approximation model. Thus the validity of refraction at the interfaces is as questionable as is total internal reflection. In summary, layer approximation, refraction and total internal reflection are useful assumption in explaining mirage formation, even though the validity of each assumption is highly questionable.

Huygens’ Wave Propagation Model. ***Huygens’ principle*** deals with the propagation of light as waves. According to this principle, every point on a wave-front acts as the source of spherical wavelets and the forward envelope of the wavelets defines the new wave-front. Huygens’ principle is used not only to analyze ***diffraction*** and ***interference*** of light in ***physical optics*** but also to confirm the laws of ***rectilinear propagation***, ***reflection*** and ***refraction*** in ***geometrical optics***. Some authors (e.g., [41]) have used Huygens’ principle to explain mirage formation. Figure 4 shows the propagation and evolution of a wave-front AB from an object producing a mirage. Since cooler air lies above the warmer air and light travels faster in warmer air, the distances B_1B_2, B_2B_3, B_3B_4, B_4B_5 and B_5B_6 are longer than A_1A_2, A_2A_3, A_3A_4, A_4A_5 and A_5A_6, respectively. Consequently light bends upwards as in the layer approximation mode.

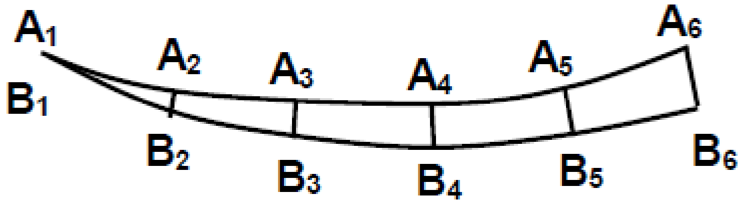

Fig. 4

The wave propagation model also has its own drawbacks. In this case, the light waves will always be divergent and light rays will not intersect each other. Now we know that mirages are ***inverted images*** of distant objects and light rays must intersect to form an inverted image as in Fig. 5 (cf. [39]). Thus, we must revert back to ***ray optics*** to explain mirage formation. Figure 6 (from [42]) is an example of an actual mirage which clearly depicts the inverted images.

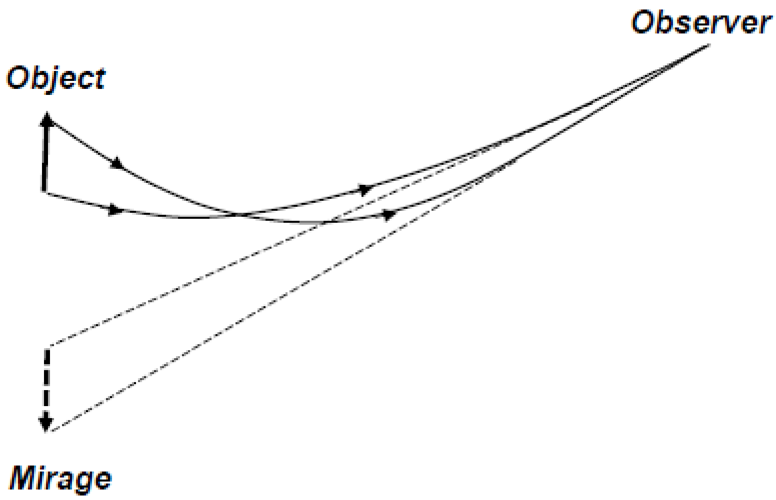

Fig. 5

Fig. 6

IV. CONCLUSION

In summary, we have seen that the intriguing optical phenomenon of mirage formation has evoked great interest and elicited contradicting explanations in the literature. Upon scrutiny, each explanation is found to be questionable in some respect. The lack of actual surface of separation in air renders the concepts of reflection, refraction and total internal reflection into question. Also the wave propagation model is unable to account for the inverted image formation. An uncontroversial statement of the mechanism of mirage formation would be the bending of light as it propagates through a non-uniform medium whose refractive index varies gradually in the vertical direction.

REFERENCES

[1] Sir William Bragg, *The Universe of Light*, Dover (New York, 1959), p. 75.

[2] Albert Edward Caswell, *An Outline of Physics*, MacMillan (New York, 1938), p. 411.

[3] W.H.A. Fincham, *Optics*, The Hatton Press (London, 1965), p.43.

[4] W.H. Furry, E.M. Purcell & J.C. Street, *Physics for Science and Engineering Students*, McGraw-Hill (New York, 1960), p.432.

[5] K.W. Lyon & D.W. Scott, *Light*, Edward Arnold (London, 1960), p. 17.

[6] A.E.E. McKenzie, *General Physics*, Cambridge (Cambridge, 1961), p. 203.

[7] Clarence Joseph Smith, *Intermediate Physics*, Edward Arnold (London, 1949), p. 417.

[8] Alpheus W. Smith & John N. Cooper, *Elements of Physics*, McGraw-Hill (New York, 1964), p. 343.

[9] *Chambers Science and Technology Dictionary*, Peter M.B. Walker ed., Chambers (Cambridge, 1988), p. 577.

[10] *Concise Dictionary of Physics and Related Subjects*, J. Thewlis ed., Pergamon Press (Oxford, 1979), p. 221.

[11] *The Facts on File Dictionary of Physics*, John Daintith ed., Facts on File (New York, 1981), p. 119.

[12] *The International Dictionary of Physics and Electronics*, Walter C. Mitchels ed., Van Nostrand (Princeton, 1961), p. 751.

[13] John E. Betts, *Physics for Technology*, Reston Publishing (Reston, 1981), p. 489.

[14] Newton Henry Black, *An Introductory Course in College Physics*, MacMillan (New York, 1954), p. 656.

[15] Leonard Paul Elliott & William F. Wilcox, *Physics: A Modern Approach*, MacMillan (New York, 1957), p. 492.

[16] Erich Hausman & Edgar P. Slack, *Physics*, Van Nostrand (Princeton, 1957), p. 578.

[17] Henry Margenau, William W. Watson & C.G. Montgomery, *Physics: Principles and Applications*, McGraw-Hill (New York, 1953), p. 626.

[18] M.G.J. Minnaert, *Light and Color in the Outdoors* (Springer, 1974), p.64.

[19] James A. Richards, Francis Weston Sears, M. Russell Wehr & Mark W. Zemansky, *Modern College Physics*, Addison-Wesley (Reading, 1962), p. 616.

[20] Francis Weston Sears & Mark W. Zemansky, *College Physics*, Addison-Wesley (Reading, 1960), p. 787.

[21] James T. Shipman, Jerry L. Adams & Jerry D. Wilson, *Introduction to Physical Science*, D.C. Heath (Lexington, 1983), p. 475.

[22] George Shortley & Dudley Williams, *Physics: Fundamental Principles for Students of Science and Engineering*, Prentice-Hall (New York, 1950), p. 634.

[23] Oscar M. Stewart, *Physics: A Textbook for Colleges*, Ginn & Co. (Boston, 1950), p. 620.

[25] Frank L. Verwiebe, Gordon E. Van Hooft & Robert R. Suchy, *A Basic Science*, Van Nostrand (Princeton, 1962), p. 410.

[25] Robert L. Weber, Marsh W. White & Kenneth V. Manning, *College Physics*, McGraw-Hill (New York, 1959), p. 488.

[26] *Concise Encyclopedia of the Sciences*, John-David Yule ed., Facts on File (New York, 1978), p. 383.

[27] David S. Falk, Dieter R. Brill & David G. Stork, *Seeing the Light*, Harper & Row (New York, 1986), p. 59.

[28] Eugene Hecht & Alfred Zajac, *Optics*, Addison-Wesley (Menlo Park, 1979), p. 69.

[29] Arthur L. Kimball, *A College Textbook of Physics*, Henry Holt & Co. (New York, 1937), p. 586.

[30] Jurgen R. Meyer-Arendt, *Introduction to Classical and Modern Optics*, Prentice Hall (Englewood Cliffs, 1972), p. 315.

[31] James A. Richards, Francis W. Sears, M. Russell Wehr & Mark W. Zemansky, *Modern University Physics*, (Addison-Wesly, Reading, 1960), p. 612.

[32] Bruno Rossi, *Optics*, Addison-Wesley (Reading, 1957), p. 54.

[33] Paul A. Tipler, *College Physics*, Worth Publishers (New York, 1987), p. 623.

[34] Robert W. Wood, *Physical Optics*, MacMillan (New York, 1934), p. 87.

[35] https://en.wikibooks.com/wiki/Optics/Fermat%27s_Principle.

[36] Francis A. Jenkins & Harvey E. White, *Fundamentals of Optics*, McGraw-Hill (New York, 2001), p. 17.

[37] A. Tan, A. Ranasinghe & V.M. Edwards, On principle of Fermat in refraction of light , *Lat. Am. Phys. Educ.*, *8* (2014), p. 4304-1.

[38] James Evans & Mark Rosenquist, "*F=ma*" optics, *Am. J. Phys.*, *54* (1986), pp.876-883.

[39] David K. Lynch & William Livingston, *Color and Light in Nature* (Cambridge, 1995), pp. 52-54.

[40] Robert T. Bush & Robert S. Robinson, A note on explaining the mirage, *Am. J. Phys.*, *42* (1974), p. 774.

[41] Paul A. Tipler, *College Physics*, Worth Publishers (New York, 1987), p. 623.

[42] www.weathercast.co.uk/weather-news/article/what_causes_mirages.html.