International Journal of Physics and Applications.
ISSN 0974-3103 Volume 3, Number 3 (2011), pp. 229-241

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Theoretical Design for Double Chirped Mirrors in Femtosecond Pulse Lasers

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Abstract

This paper is concerned with theoretical study on optoelectronics physics to design dielectric double-chirped mirror (DCM) to achieve high reflectivity and dispersion compensation over a broad bandwidth. Analytic expressions for reflectivity, group delay and group delay dispersion are used. Where, most recent research deals with Nanotechnology (10^{-9} seconds). Our research deals with Femtotechnology (10^{-15} seconds), which is very advanced in laser and fiber optics technology.

Their theoretical analysis generally relies on the well-known scattering matrix formalism derived from the Maxwell equations. Laser performance strongly depends on the quality of optical coatings: reflectance of high reflectors should approach the ideal 100% value at the operation wavelengths in order to minimize laser intracavity losses and output coupling has to be set to specific values to ensure optimal operation. The aim is to push the limits of pulse duration to reach femtosecond scale.

The idea of chirped mirrors is based on the quarter-wave stack concept as a building block of dielectric high reflector.The mirror consists of 49 layers: 35 layers from dielectric materials SiO_2 / TiO_2 arranged, as periodic stacks have been used to design chirped mirrors using the Fusedsilica as a substrate and 14 layers from tow dielectric materials SiO_2 / Ta_2O_5 have been used to design anti reflection coating AR with controlled reflectivity and dispersion in the wavelength range $650-900nm$, it exhibits a reflectivity of >99.99%.

Index Terms: Dispersion, double chirped mirror, group delay, group delay dispersion, reflectivity.

Introduction

A central building block for generating femtosecond light pulses are lasers. Within only two decades of the invention of the laser the duration of the shortest pulse shrunk by six orders of magnitude from the nanosecond regime to the femtosecond regime [1]. In a mode-locked laser for femtosecond pulse generation, the dispersion introduced by the gain medium and other optical components in the laser cavity is often not desirable, because it tends to broaden the generated pulses. While the naturally occurring dispersion is usually positive (i.e., normal dispersion), the desired dispersion may either be close to zero or even anomalous. Such dispersion values can be achieved by introducing optical components with anomalous dispersion. Such components are usually either special dispersive dielectric mirrors (e.g. in the form of monolithic Gires-Tournois interferometers or chirped mirrors CM), or prism pairs [1,2]. Dispersion compensation essentially means canceling the chromatic dispersion of some optical elements. However, the term is often used in a more general sense of dispersion management, meaning the control the overall chromatic dispersion of some system. The goal can be, e.g., to avoid excessive temporal broadening of ultrashort pulses and/or the distortion of signals. Dispersion compensation is applied mainly in mode-locked lasers and in telecommunication systems, but also sometimes in optical fibers transporting light e.g. to or from some fiber-optic sensor. It is the control of the overall chromatic dispersion of a system by adding optical elements with a suitable amount of dispersion [1].

Dispersion compensation plays a key role in generation, amplification and propagation of femtosecond pulses. In the dispersive medium, the pulse can be broaden or compressed depending on the sign of chirp and dispersion. To obtain the ultrashort pulses, the pulse group delay GD should have about frequency independence after the dispersion compensation. Especially to compress a pulse to near the transform limit one should not only compensate the GD but also eliminate the high order dispersion term [2,3].

Double chirped mirrors DCM are useful and a compact device for ultrashort pulse generation in sub 10fs regime. The simulation and design presented this mirror offering high reflectivity and controlled group delay dispersion GDD [3]. The proposal stacks design considered in this paper has alternating layers of $SiO_2(n = 1.4716)$, $TiO_2(n = 2.2505)$ and $Ta_2O_5(n = 2.0976)$ at 800nm. The substrate material for the design stacks is $FusedSilica(n = 1.4520)$. The design operates with normal incidence and s-mode of polarization and the behavior studied at design wavelength $\lambda_0 = 800nm$.

Principle of Operation

The principle of operation can be understood as follows:
Each interface between the two materials contributes a Fresnel reflection [4]:

$$r_F = (n_h - n_l)/(n_h + n_l) \tag{1}$$

Where r_F Fresnel reflectivity, n_h high refractive index and n_l low refractive index.

For the design wavelength, the optical path length difference between reflections from subsequent interfaces is half the wavelength; in addition, the reflection coefficients for the interfaces have alternating signs. Therefore, all reflected components from the interfaces interfere constructively, which results in a strong reflection. The reflectivity achieved is determined by the number of layer pairs and by the refractive index contrast between the layer materials. The reflection bandwidth is determined mainly by the index contrast [4].

A bandwidth of a CM and its GDD spectral shape are defined by, among other parameters, impedance mismatch between the ambient medium and mirror stack. The impedance mismatch can be overcome by using glass as the medium of incidence. Unfortunately, this solution limits the highest value of GDD, because the mirror stack should compensate an additional thin non-parallel substrate or glass wedge. Additionally, in this case, the aperture cannot be high and the beam stability problem may arise. There are several other approaches to reducing GDD ripples: double-chirped mirrors (Fig.1), Brewster-angled CMs and complementary CM pairs. Alternatively, time-domain optimization deals directly with pulse compression and can therefore lead to the shortest pulses even in spite of large GDD oscillations [5].

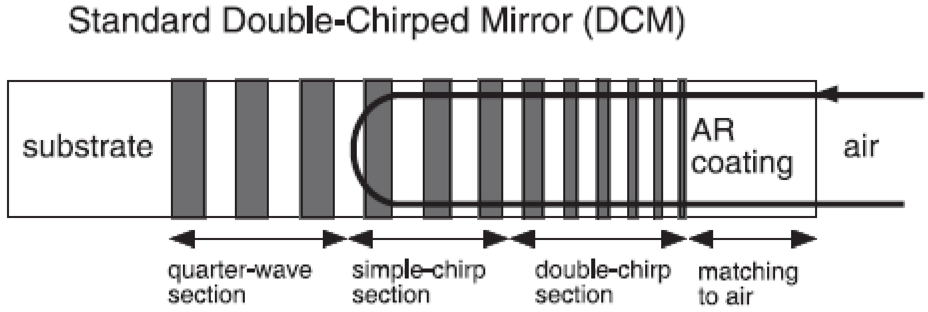


Figure 1: Standard Double dielectric chirped mirror [5,6].

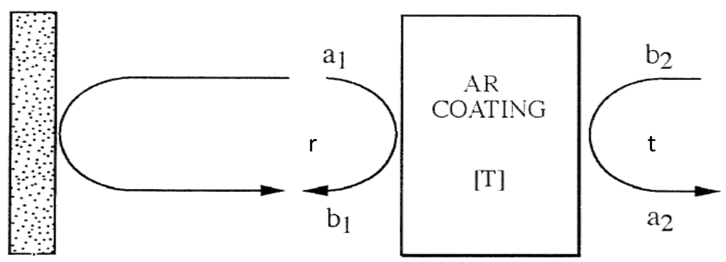


Figure 2: is a schematic illustrating reflectivity of the double-chirped mirror of Fig.1 [5].

In the schematic above, AR coating is represented as a two-port with two incoming waves a_1, b_1 and two outgoing waves b_2, a_2. Assuming the multi-layer AR coating is loss-less (no absorption), the connection between the waves at the left port and the right port is described by a transfer matrix [T]. Here, r and t are the reflectivity and transmission, respectively, at port1 assuming reflection-free termination of port2.The black rectangle represents the back mirror, which includes all of double-chirped mirror except AR coating. That is, back mirror includes the SiO_2 substrate and all the alternating high and low refractive index layers and including double-chirped portion. Back mirror is assumed to be perfectly matched, and has full reflection over the total bandwidth under consideration, such that its complex reflectivity in the wavelength range of interest [5,8].

In this paper, we shall present the transfer matrix method allowing solving Maxwell equations in multilayer dielectric structures. We shall consider an example of a periodical structure and derive general equations in planar structures. In the beginning, we consider propagation of light in the normal to layer planes direction. We shall generalize the transfer matrix approach for TE and TM linear polarizations of light. By definition, TE-polarized (also referred to as s-polarized) light has the electric field vector parallel to the layer planes, TM-polarized light (also referred to as p-polarized) has the magnetic field vector parallel to the planes (see Fig.3).

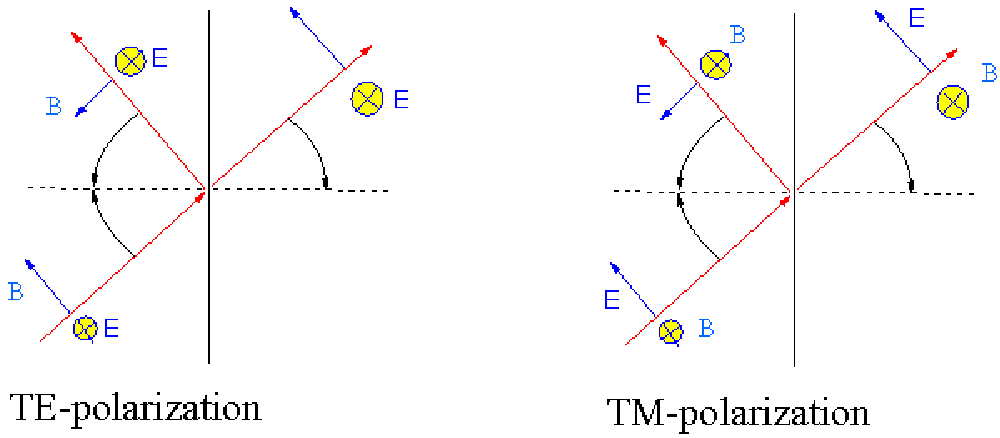


Figure 3: Orientation of electric and magnetic fields in TE- and TM-polarized incident on a planar boundary [9].

What happens to the electromagnetic field at the planar interface between two dielectric media with different refractive indices? The answer can be found by resolving the system of Maxwell equations independently in the two media and then matching of the solutions for electric and magnetic fields by the Maxwell boundary conditions at the interface. These conditions require continuity of the tangential components of both fields. They can be microscopically justified for any abrupt interface in the absence of free charges and free currents.

Consider a transverse light-wave propagating along the z-direction in a medium characterized by a refractive index n that is homogeneous in the xy plane but possibly z-dependent. The wave equation in this case becomes [1,8]:

$$\frac{\partial^2 E}{\partial z^2} = -k_0^2 n^2 E \tag{2}$$

where k_0 is the wave-vector of light in a vacuum. The general form of the solution of Eq. (1) writes:

$$E = A^+ \exp(ikz) + A^- \exp(-ikz) \tag{3}$$

where $k = k_0 n$, A^+, A^- are coefficients. Using the Maxwell equation one can easily obtain the general form of the magnetic field amplitude

$$B = A^+ n \exp(ikz) - A^- n \exp(-ikz) \tag{4}$$

If we consider reflection of light incident from the left side to the boundary $(z = 0)$ between two semi-infinite media characterized by refractive indices n_1 (left) and n_2 (right), the matching of the tangential components of electric and magnetic fields would give:

$$A_1^+ + A_1^- = A_2^+ \tag{5}$$

$$(A_1^+ - A_1^-)n_1 = A_2^+ n_2 \tag{6}$$

where A_1^+, A_1^- and A_2^+ are the amplitudes of incident, reflected and transmitted light, respectively. One can easily obtain the amplitude reflection coefficient

$$r \equiv \frac{A_1^-}{A_1^+} = \frac{n_1 - n_2}{n_1 + n_2} \tag{7}$$

and the amplitude transmission coefficient

$$t \equiv \frac{A_2^+}{A_1^+} = \frac{2n_1}{n_1 + n_2} \tag{8}$$

The ratio of reflected to incident energy flux (reflectivity) is given by

$$R = |r|^2 \tag{9}$$

and the ratio of transmitted to incident energy flux (transmittance) is

$$T = \frac{n_2}{n_1} |t|^2 \tag{10}$$

In the last formula, the factor $\frac{n_2}{n_1}$ comes from the ratio of light velocities in two media [9].

In multilayer structures, direct application of Maxwell boundary conditions at each interface leads to the necessity to resolve a substantial number of algebraic equations (two per interface). A convenient method allowing reducing the number of equations to be resolved to a strict minimum (four in general case) is the transfer matrix method, which we are going to describe briefly here.

Let us introduce the vector

$$\vec{\Phi}(z) = \begin{bmatrix} E(z) \\ B(z) \end{bmatrix} = \begin{bmatrix} E(z) \\ -\dfrac{i}{k_0}\dfrac{\partial E(z)}{\partial z} \end{bmatrix} \tag{11}$$

where E(z), B(z) are the amplitudes of the electric and magnetic field of any light wave propagating in the z direction in the structure under study. Note that $\vec{\Phi}(z)$ is continuous at any point in the structure due to the Maxwell's boundary conditions. In particular, it is continuous at all interfaces where n changes abruptly.

By our definition, the transfer matrix $\hat{T}_a$ across the layer of width a is such a 2×2 matrix that:

$$\hat{T}_a \vec{\Phi}\Big|_{z=0} = \vec{\Phi}\Big|_{z=a} \tag{12}$$

It is easy to verify by substitution into Eq.(11) of the electric and magnetic amplitudes (2), (3) that if n is homogeneous across the layer,

$$\hat{T}_a = \begin{bmatrix} \cos ka & \dfrac{i}{n}\sin ka \\ in\sin ka & \cos ka \end{bmatrix} \tag{13}$$

The transfer matrix across a structure composed of m layers can be found as

$$\hat{T} = \prod_{i=m}^{i=1} \hat{T}_i \tag{14}$$

Where $\hat{T}_i$ is the transfer matrix across i-th layer. The order of multiplication in Eq. (13) is essential. The amplitude reflection and transmission coefficients (r_s and t_s) of a structure containing m layers, and sandwiched between two semi-infinite media with refractive indices n_{left}, n_{right} before and after the structure, respectively, can be found from the relation

$$\hat{T}\begin{bmatrix} 1 + r_s \\ n_{left} - n_{left} r_s \end{bmatrix} = \begin{bmatrix} t_s \\ n_{right} t_s \end{bmatrix}. \tag{15}$$

One can easily obtain

$$r_s = \frac{n_{right}t_{11} + n_{left}n_{right}t_{12} - t_{21} - n_{left}t_{22}}{t_{21} - n_{left}t_{22} - n_{right}t_{11} + n_{left}n_{right}t_{12}} \quad (16)$$

$$t_s = 2n_{left}\frac{t_{12}t_{21} - t_{11}t_{22}}{t_{21} - n_{left}t_{22} - n_{right}t_{11} + n_{left}n_{right}t_{12}} \quad (17)$$

The intensities of reflected and transmitted light normalized by the intensity of the incident light are given by

$$R = |r_s|^2\,,\; T = |t_s|^2 \frac{n_{right}}{n_{left}} \quad (18)$$

Respectively, In its turn, the transfer matrix across a layer can be expressed via reflection and transmission coefficients of this layer. If the reflection and transmission coefficients for light incident from the right-hand side and left-hand side of the layer are the same, and $n_{left} = n_{right} \equiv n$ (the symmetric case realized, in particular, in a quantum well embedded in a cavity), the Maxwell boundary conditions for light incident from the left and right sides of the structure yield:

$$\begin{aligned} &\hat{T}\begin{bmatrix} 1+r_s \\ n - nr_s \end{bmatrix} = \begin{bmatrix} t_s \\ nt_s \end{bmatrix}, \\ &\hat{T}\begin{bmatrix} t_s \\ -nt_s \end{bmatrix} = \begin{bmatrix} 1+r_s \\ -n+nr_s \end{bmatrix}. \end{aligned} \quad (19)$$

This allows the matrix $\hat{T}$ to be expressed as:

$$\hat{T} = \frac{1}{2t}\begin{bmatrix} t_s^{\,2} - r_s^{\,2} + 1 & -\dfrac{(1+r_s)^2 - t_s^{\,2}}{n} \\ n\left((r_s - 1)^2 - t_s^{\,2}\right) & t_s^{\,2} - r_s^{\,2} + 1 \end{bmatrix} \quad (20)$$

For a quantum well, $t_s = 1 + r_s$, and Eq. (19) becomes

$$\hat{T}_{QW} = \begin{bmatrix} 1 & 0 \\ -2n\dfrac{r_s}{t_s} & 1 \end{bmatrix} \quad (21)$$

In the oblique incidence case, in the TE-polarization, one can use the basis $\begin{bmatrix} E_\tau(z) \\ B_\tau(z) \end{bmatrix}$, where E_τ, B_τ are the tangential (in-plane) components of the electric and magnetic fields of the light wave. In this case, the transfer matrix (4) keeps its form provided that the following substitutions are made:

$$k_z = k\cos\varphi,\ n \to n\cos\varphi \tag{22}$$

where φ is the propagation angle in the corresponding medium ($\varphi = 0$ at normal incidence).

In the TM-polarization, following Born and Wolf [1] we use the basis $\begin{bmatrix} B_\tau(z) \\ E_\tau(z) \end{bmatrix}$ which still allows the transfer matrix (12) to be used provided that the substitutions are done:

$$k_z = k\cos\varphi,\ n \to \frac{\cos\varphi}{n} \tag{23}$$

Note that the transfer matrices across the interfaces are still identity matrices, and Eq. (13) for the transfer matrix across the entire structure is valid.

In the formulas for reflection and transmission coefficients (15-18) one should replace, in the TE-polarization

$$n_{left} \to n_{left}\cos\varphi_{left},\ n_{right} \to n_{right}\cos\varphi_{right} \tag{24}$$

And in the TM-polarization

$$n_{left} \to \frac{\cos\varphi_{left}}{n_{left}},\ n_{right} \to \frac{\cos\varphi_{right}}{n_{right}} \tag{25}$$

where φ_{left}, φ_{right} are the propagation angles in the first and last media, respectively. The same transformations would be applied to the transfer matrices (19), (20). Note that any two propagation angles φ_i, φ_j in the layers with refractive indices n_i, n_j are linked by the Snell-Descartes law:

$$n_i \sin\varphi_i = n_j \sin\varphi_j \tag{26}$$

which is also valid in the case of complex refractive indices, when the propagation angles formally become complex as well.

The group delay is defined as the negative of the derivative of the phase response with respect to frequency [10→14], GD, also known as "Envelope Delay" [3]. In physics and in particular in optics, the study of waves and digital signal processing, the term group delay has the following meaning:

The rate of change of the total phase shift with respect to angular frequency [11,12]:

$$GD = -\frac{d\phi}{d\omega} \tag{27}$$

Through a device or transmission medium, where ϕ is the total phase shift in radians, and ω is the angular frequency in radians per unit time, equal to $2\pi f$, where f is the frequency (hertz if group delay is measured in seconds).

Group delay dispersion is a ubiquitous, and often irritating, phenomenon in ultrafast laser labs. When ultrashort pulses propagate through dispersive media, their frequency components emerge at different times due to GDD, causing the resulting pulse to be chirped and stretched and reducing the pulse's peak power [13].

$$GDD = -\frac{d^2\phi}{d\omega^2} \tag{28}$$

This effect can be compensated by using a pulse compressor, which can introduce negative GDD [20]. The standard method for computing the GDD is to compute complex reflection coefficients using the transfer matrix technique and then take successive finite difference over frequency [21,22]. Group-delay dispersion of optical elements is a critical parameter for the generation and control of femtosecond laser pulses. GDD can either increase or decrease then pulse duration by modulating the spectral phase of the femtosecond laser pulses. The effect of GDD becomes more significant as the laser pulse duration gets shorter. Ideally, a femtosecond dielectric mirror should not only have high reflectance but also low dispersion over a sufficiently broad spectral bandwidth [24].

Experimental Results and Discussion

In this simulation, we used a quarter waves double-chirped mirror. The structure of double-chirped mirror design consists of 49 layers, with optical thicknesses=200. The Bragg wavelength $\lambda_B = 800nm$ in the rang $650-900nm$.

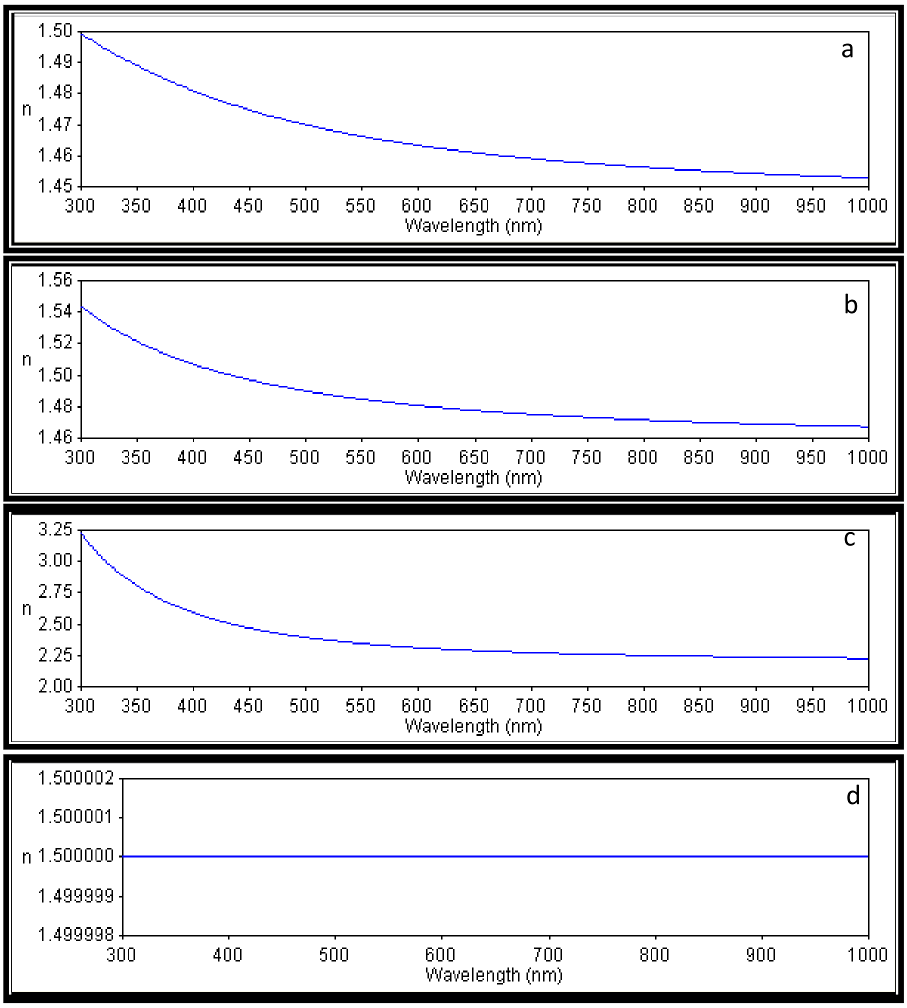
Figs.4 shows the relationship between the reflective index and the wavelength for (the $FusedSilica$ substrate, TiO_2 material, SiO_2 material and Ta_2O_5 material. This determines the dispersion properties of the structure. Fig.5 The relationship between the refractive index and the distance from substrate. Fig.6 The relationship between the phase and the wavelength we see the properties of phase shift in the rang between 650-900nm for DGMs.

Results given in Fig.7 reveal almost high reflector at design wavelength ($\lambda_B = 800nm$) when the distribution double-chirped mirror consist of alternating periodic quarter-wavelength stack of low and high refractive index. It exhibits a reflectivity of >99.9995%.

Fig.8 shows the relationship between the reflectance and the thickness. Where the reflectance increase with thickness increase.

Fig.9 shows the characteristics of GD as a function of wavelength, where the group delay is nearly constant over a bandwidth of about 740-860nm. While Fig.10 shows the dependence GDD on the optical wavelength. It is clearly seen that the decrease of the oscillation in GDD in the rang wavelength from 670-900nm. In the case of high reflectors, a combination of materials with the highest refractive-index ratios n_h / n_l is usually preferred since the higher the ratio, the higher the theoretical reflectance and bandwidth of standard quarter wave stacks.

These stacks have a peak in the reflectivity at 800nm. The group delay and group delay dispersion has low oscillation.



Figsure 4: The relationship between the reflective index and the wavelength for: a-(Fusedsilica) substrate. b- TiO2 material. c- SiO2 material. d- Ta2O5 material.

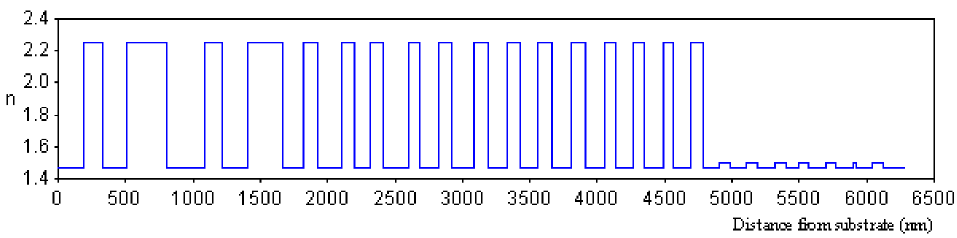


Figure 5: The relationship between the refractive index and the distance from substrate.

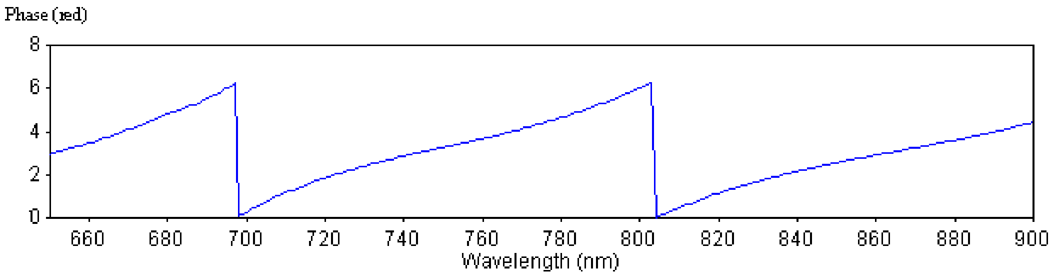


Figure 6: The relationship between the phase and the wavelength.

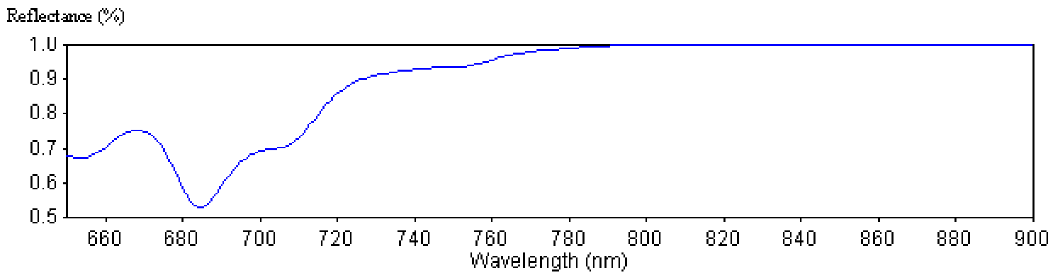


Figure 7: The relationship between the reflectance and the wavelength.

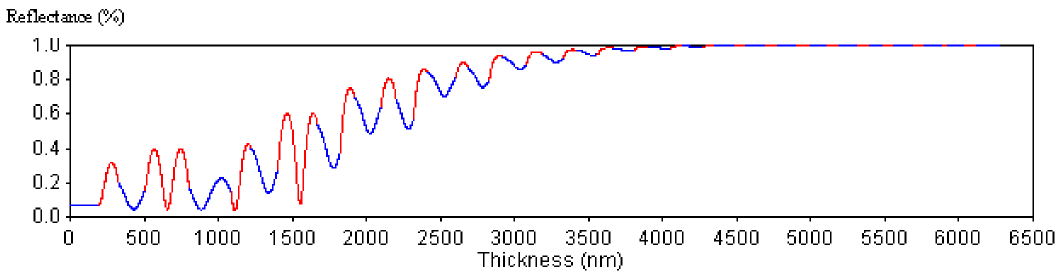


Figure 8: The relationship between the reflectance and the thickness.

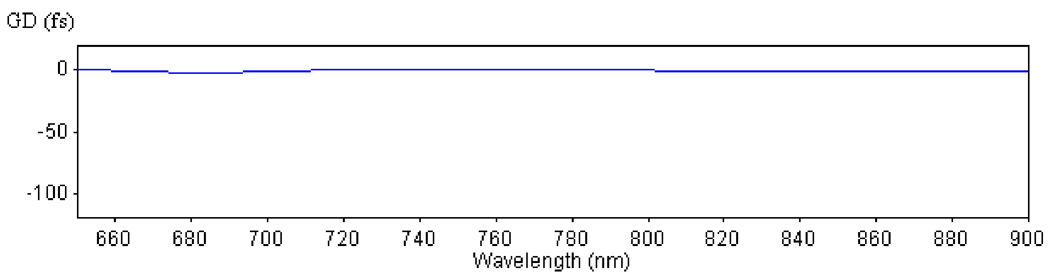


Figure 9: The relationship between the group delay and the wavelength where the GD has a minimum oscillations in the range from 650-900nm.

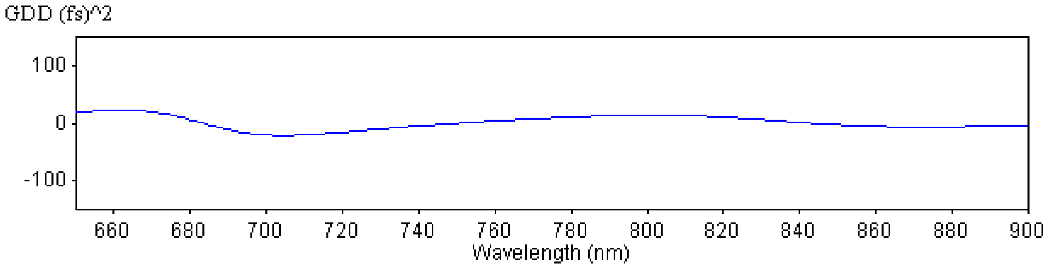


Figure 10: The relationship between the group delay dispersion and the wavelength. It is clearly seen that the decrease of the oscillations in the GDD in the rang wavelength from 650-900nm.

Conclusion

In view of the results presented in this study, the main contributions of this research can be summarized below:

- Tow materials SiO_2 / Ta_2O_5 have been used to design chirped mirrors with a flat GDD over bandwidth of 650-900nm and the reflectivity was found to be greater than 99.995%. Therefore, chirped mirror is a good device for generating pulse duration down to $10\, fs^2$.

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