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Soliton generation in a kerr medium: mathematical analysis using Maxwell's equations

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Abstract

In this paper set of equations are derived for kerr medium using Maxwell's equations leading to nonlinear Helmholtz equation. A high intensity beam passes from a non-linear medium its refractive index and susceptibility gets changed. The medium has a kerr coefficient matrix which has been taken as 3x3 in our simulations.Earlier work shows that by applying high intensity electric field the nonlinear material become birefringent and the polarization of field changes in three directions because of its refractive index change. These waves are assumed to be electric field components Ex Ey and Ez.Again perturbation theory is applied while finding the solution of field travelling through the medium. The correlations between the fundamental field and perturbed fields are simulated using Matlab. Kerr effect is proportional to the square of the Electric field similarly it has been assumed that if a nonlinear material has a property of changing its permeability by applying high intensity field (as in Farady and Pockel's effect) then magnetic kerr effect also changes as square of magnetic field H. Substituting this and solving Maxwell's equations we get an EM wave having both scalar and vector components in it. The correlation between the fundamental wave and scalar wave shows a soliton. Correlating again the scalar component with the vector component the EM field we get two peaks in their power spectra.Using these simulations the solitons are proved for 2 to 200THz input EM wave.

AMS subject classification:
Keywords: kerr Nonlinearity, perturbation, soliton.

1. Introduction

Assuming an electric field from a Laser source within a conducting cavity is given by E. If the medium is anisotropic, such as a crystal or a waveguide, the polarization field is not necessarily aligned with the electric field E. In kerr effect refractive index of the material changes depending on the electromagnetic field strength. This also causes light birefringence in the medium [17]. E is a field with amplitude E0 and a polarization direction e. $E = E_0 \exp(-j\omega t)$ field is transverse to the direction of propagation and the polarization vector resolves into two orthogonal components according to birefringence. If susceptibility of the medium χ is a tensor of rank 2 or 3. ϵ is known as the relative permittivity tensor or dielectric tensor. Consequently refractive index must be a tensor. If a wave is polarized in x direction then its refractive index is given by and $n_{xx} = (\epsilon_{xx})^{1/2} = (1+\chi_{xx})^{1/2}$. If a wave is polarized in y direction refractive index is $n_{yy} = (\epsilon_{yy})^{1/2} = (1 + \chi_{yy})^{1/2}$. Thus these waves will see different refractive indices and travel at different speeds. This is due to birefringence. Birefringence occurs in some common crystals such as Calcite and quartz[2]. The quantity k is the amplitude of the wave vector and is given by $K = 2 * \pi/\lambda$. Here in a nonlinear medium we have taken the polarization of electric fields in different directions and also calculated the refractive index for them.

Also it is taken into consideration that the polarization of a dielectric medium in a normal situation is proportional to the electric field strength, but if the strength of the light beam is sufficiently high as in lasers, the P Polarization depends on higher powers of electric field.

We can write polarization as given in [7]

$$P = \epsilon_0(\chi^1 E^1 + \chi^2 E^2 + \chi^3 E^3 + - - - - - - - -)$$

where ϵ_0 is vaccume permittivity $\chi^j (j = 1, 2, 3, \dots)$ is jth order susceptibility tensor. χ^1 contains dominant contribution to P. Its effects are included through the refractive index n. Neglecting higher orders of E polarization is given by

$$P = \chi^1 E + \chi^2 E E$$

and reractive index

$$n^2 = 1 + 4\pi\chi_{eff} = 1 + 4\pi(\chi^1 + \chi^2 E(0)^2)$$

where χ_{ijk} is 3×3 matrix.

2. Formulations and Equations

Electric field travelling in medium is given by-$E = E_0 exp(j(\omega * t - k.x))$. Where E_0 is the amplitude, x is the distance, ω is frequency, K is propagation constant. Taking amplitude as a Normal distribution function [4].

Using four Maxwell's equations and susceptibility and permeability of the medium changes with the applied electric fields and magnetic fields:

The electric field E is passing through that medium. E_x E_y E_z and magnetic fields H_x H_y H_z are the polarized field travelling with different propagation constants [17]. assuming no bounded charge and magnetization of the material
$\nabla.D = 0;$
$\nabla.B = 0;$
$curl\vec{E} = -\partial(\mu\vec{H})/\partial t;$
$curl\vec{H} = \partial D/\partial t$; where $D = \epsilon\vec{E}$
$\chi(E)$ is proportional to the square of Electric fields which are present inside the medium. Substituting for $\epsilon = \epsilon_0(1 + \chi(\vec{E})$; and $\chi_e = \chi(E)$;
we take

$$\chi(\vec{E}) = (1/2)\sum_{x,y=1}^{3} \alpha_{xy}E_xE_y \tag{1}$$

$$\chi(\vec{H}) = (1/2)\sum_{x,y=1}^{3} \alpha^m_{xy}H_xH_y \tag{2}$$

where α_{xy} and $\alpha^m_{xy}$are kerr coefficients expanding the terms gives following equations

$$\begin{aligned} \chi(E) &= (1/2)(\alpha_{xx}E_x^2 + \alpha_{yy}E_y^2 + \alpha_{zz}E_z^2 + 2\alpha_{xy}E_xE_y \\ &\quad +2\alpha_{xy}E_yE_z + 2\alpha_{xz}E_xE_z) \\ &\quad +O(E^3); \end{aligned} \tag{3}$$

$$div(\epsilon\vec{E}) = 0 \tag{4}$$

$$(\nabla\epsilon, \vec{E}) + \epsilon div\vec{E} = 0 \tag{5}$$

$$\begin{aligned} div(\vec{E}) &= -(\nabla log\epsilon, \vec{E}) = \\ &\quad -(\nabla\vec{\chi}, \vec{E}); \end{aligned} \tag{6}$$

where $\nabla\chi$ is defined as

$$\begin{aligned} \nabla\chi &= \partial\chi/\partial E_x\nabla E_x \\ &\quad +\partial\chi/\partial E_y\nabla E_y \\ &\quad +\partial\chi/\partial E_z\nabla E_z \end{aligned} \tag{7}$$

where

$$\nabla = \hat{x}\partial/\partial_x + \hat{y}\partial/\partial_y + \hat{z}\partial/\partial_z$$

$$\begin{aligned} \nabla\chi &= (\alpha_{xx}E_x + \alpha_{xy}E_y + \alpha_{xz}E_z)\nabla E_x \\ &\quad +(\alpha_{yx}E_x + \alpha_{yy}E_y + \alpha_{yz}E_z)\nabla E_y \\ &\quad +(\alpha_{zz}E_z + \alpha_{zy}E_y + \alpha_{zx}E_x)\nabla E_z; \end{aligned} \tag{8}$$

where $\nabla E_y = E_{yx}\hat{x} + E_{yy}\hat{y} + E_{yz}\hat{z}$
where $\nabla E_z = E_{zx}\hat{x} + E_{zy}\hat{y} + E_{zz}\hat{z}$
if $E_{yx} = \partial E_y/\partial x$ so $\nabla\chi(\vec{E})$; can be written as

$$\nabla\chi = \sum_{x,y=1}^{3} E_x(\partial\chi/\partial E_y)E_{yx}\hat{e_x}; \tag{9}$$

from Maxwell's equations

$$\nabla \times (\nabla \times \vec{E}) = -\nabla \times \partial(\mu\vec{H})/\partial t$$

$$\begin{aligned} &\nabla div\vec{E} - \nabla^2 E \\ &\quad = -\nabla\partial(\mu \times \vec{H})/\partial t \\ &\quad -\nabla\mu \times \partial\vec{H}/\partial t \\ &\quad -\partial \quad \mu(\nabla \times \vec{H})/\partial t - \mu\partial(\nabla \times \vec{H})/\partial t; \end{aligned} \tag{10}$$

from equation (6) and (9) substituting in eq (11)

$$\begin{aligned} (\nabla\chi, \vec{E}) &= div\vec{E} \\ &= -\Sigma_{y,x,z=1}^{3}(\alpha_{yx}\vec{E}_y\vec{E_{yx}})\hat{e_x}.\vec{E}_\rho; \end{aligned} \tag{11}$$

$$\nabla^2\vec{E} + \Sigma_{y,x,z=1}^{3}\nabla((\alpha_{yx}\vec{E}_y\vec{E_{yx}})\hat{e_y}\vec{E}_z) \tag{12}$$

let

$$((\alpha_{yx}\vec{E}_y\vec{E_{yx}})\vec{E}_z) = D'$$

$$\nabla D' = \Sigma_{\rho=1}^{3}(\Sigma_{xy=1}^{3}\alpha_{xy}\vec{E_{yx}}\vec{E}_x\vec{E}_z)_z\hat{e_z}; \tag{13}$$

where $X_z = \partial X/\partial z$. RHS of the eq (11) is presented in terms of T1, T2, T3, T4 and add them

$$\begin{aligned} &-\nabla\partial/\partial t(\mu \times \vec{H}) - \nabla\mu \times \partial\vec{H}/\partial t \\ &- \quad \partial\mu(\nabla \times \vec{H})/\partial t \\ &- \quad \mu\partial(\nabla \times \vec{H})/\partial t; \end{aligned} \tag{14}$$

$$\nabla\mu \times \partial\vec{H}/\partial t \tag{15}$$

$$\mu = \mu_0(1+\chi_m) \tag{16}$$

$$\begin{aligned} \nabla\mu &= \mu_0 \nabla\alpha_m(\vec{H}) \qquad (17) \\ &= \mu_0 \sum_{y'x'=1}^{3} (\alpha^m_{y'x'} \vec{H}_{y'} \vec{H_{y'x'}}) \hat{e_{x'}} \end{aligned}$$

(18)

similar to eq (9)

$$\vec{H} = H_{x'}\hat{e_{y'}} + H_{x'}\hat{e_{y'}} + H_{z'}\hat{e_{z'}} \tag{19}$$

taking

$$\begin{aligned} &\partial\vec{H}/\partial t \qquad (20) \\ &= \sum_{x'} \vec{H_{x't}} \hat{e_{x'}} \end{aligned}$$

let

$$\begin{aligned} T1 &= \nabla\mu \times \partial\vec{H}/\partial t \qquad (21) \\ &= \mu_0 \sum_{x',y',z'=1}^{3} \alpha^m_{y',x'} \vec{H}_{y'} (\varepsilon_{z',x',y'}) \vec{H_{y'x'}} \hat{e_{x'}} \\ &\quad \times \sum_{x'=1}^{3} \vec{H_{x't}} \hat{e_{x'}}; \end{aligned}$$

where $\varepsilon_{z',y',x'}$ = product of $\alpha_{y'x'} H_{y'} H_{y'x'}$; where x'not equal x it is assumed that kerr nonlinearity also affects permeability. Taking magnetic susceptibility as $\chi_m(\vec{H})$

$$\mu = \mu_0(1+\chi_m(\vec{H})); \tag{22}$$

$\nabla\mu = \mu_0\nabla(\chi_m)$; $\chi_m = \chi(H)$

$$\begin{aligned} &= \mu_0(\partial\chi/\partial x')\nabla H_{x'} + (\partial\chi/\partial y')\nabla H_{y'} \qquad (23) \\ &\quad +(\partial\chi/\partial z')\nabla H_{z'}; \end{aligned}$$

$$\begin{aligned} \nabla\chi_m &= (\sum_{y',x'=1}^{3})\alpha^m_{y'x'} \qquad (24) \\ &\quad \times \vec{H}_{y'} \vec{H_{y'x'}} \hat{e_{x'}} \end{aligned}$$

writing magnetic field $\overrightarrow{H}$ as

$$\vec{H} = \sum_{x'=1}^{3} \vec{H_{x'}} \hat{e_{x'}} \tag{25}$$

let

$$\begin{aligned} T2 &= \partial/\partial t \nabla\mu \times \vec{H} \hat{e_{x'}} \qquad (26) \\ &= \delta\mu_0 \partial(\sum_{y'x'=1}^{3} \alpha^{m}_{y'x'} \vec{H_{y'}} \vec{H_{y'x'}})/\partial t \times \sum_{x'=1}^{3} \vec{H_{x'}} \hat{e_{x'}}; \end{aligned}$$

if perturbation δ is introduced in the $\chi_e(\vec{E})$ and $\chi_m(\vec{H})$ adding T1 and T2 we get

$$\begin{aligned} &\delta\mu_0 \partial/\partial t \sum_{y'x'=1}^{3} \alpha^{m}_{y'x'} \vec{H_{y'}} \vec{H_{y'x'}} \hat{e_x} \times \sum_{x'=1}^{3} \vec{H_{x'}} \hat{e_{x'}} \qquad (27) \\ &+\delta\mu_0 \sum_{y',x'=1}^{3} (\alpha^{m}_{y'x'} \vec{H_{y'}} \vec{H_{y'x'}}) \hat{e_{x'}} \times \sum_{x'=1}^{3} \vec{H_{x't}} \hat{e_x}; \end{aligned}$$

let

$$\begin{aligned} T3 &= (-\partial\mu/\partial t)(\nabla \times \vec{H}) = (\partial\mu/\partial t)\partial(\epsilon\vec{E})/\partial t \qquad (28) \\ &= -\mu_0\epsilon_0(\partial\chi_m/\partial t)(\partial(1+\chi_e)\vec{E}/\partial t); \\ &= (-1/c^2)\delta\partial(\chi_m \vec{E}t + O(\delta^2 + -)/\partial t; \qquad (29) \end{aligned}$$

where

$$\vec{E} = \vec{E^0} + \delta\vec{E}^1 + \delta^2\vec{E}^2 + ------$$

$$\text{T3} = -1/c^2 \partial(\chi_m(\vec{E_t}^{\,0})/\partial t$$

(considering only first power of δ) if

$$\begin{aligned} T4 &= -\mu\partial(\nabla \times \vec{H})/\partial t = \mu\partial^2(\epsilon\vec{E})/\partial t^2; \qquad (30) \\ &= \mu_0\epsilon_0\partial^2(1+\delta\chi(E))/\partial t^2; \qquad (31) \end{aligned}$$

here from kerr nonlinearity

$$\chi(\vec{E}) = (1/2)\alpha_e \vec{E}^2;$$

and

$$\mu = \mu_0(1+\delta\chi_m);$$

$$\chi_m(\vec{H}) = (1/2)\alpha_m \vec{H}^2.$$

From Maxwell's equations left hand side of equation is $\nabla Div\vec{E} - \nabla^2\vec{E}$ and combining equations (11), (28), (30), (32) we get a nonlinear Helmholtz equation.

$$\begin{aligned}
\nabla Div\vec{E} + \nabla^2\vec{E} &= -\delta[\mu_0(\partial/\partial t)\sum_{x,y=1}^{3}\chi_{y'x'}\vec{H_{y'}}^0\vec{H_{y'x'}}^0\hat{e_{x'}} \qquad (32)\\
&\times\sum_{x,x'=1}^{3}\vec{H_{x'}}^0\hat{e_{x'}} + \mu_0\sum_{x',y'=1}^{3}(\chi_{y'x'}\vec{H_{y'}}^0\vec{H_{y'x'}}^0)\hat{e_{x'}}\\
&\times\sum_{x'=1}^{3}\vec{H_{x't}}\hat{x'} + (\partial(\sum_{yy'=1}^{3}\alpha_m\\
&\vec{H_{x'}}^0\vec{H_{y'}^0})/\partial t)(1/(2*c^2))(\partial(\vec{E}^0\delta\vec{E}^1)/\partial t)\\
&+(\partial^2(\vec{E}^0 + \delta\vec{E}^1 + ..)/\partial t^2)(1/(2*c^2))\\
&+\sum_{y',x'=1}^{3}\vec{H_{x'}}^0\vec{H_{y'}}^0\partial^2(\vec{E}^0 + \delta\vec{E}^1 + \cdots)/\partial t^2\\
&+(1/2*c^2)\partial^2((\sum_{x,y=1}^{3}\alpha_{xy}\vec{E_x^0}\vec{E_y}^0)/\partial t^2)\vec{E}^0\hat{x}\\
&+(1/c^2)\partial(\alpha_{x,y})\vec{E_x^0}\vec{E_y^0}E^0/\partial t^2];
\end{aligned}$$

where $\vec{H_{\alpha' t}}$ denotes the differentiation of H_α w.r.t t neglecting higher orders of δ and substituting equation (14) and (31) in equation (32) where LHS E is replaced by ($E = E^0 + \delta E^1 + \delta^2 E^2 + \cdots$) Nonlinear Helmholt'z equation 32 gives solution of E^1 in terms of the E^0 by solving first order of δ.

3. Results

The equations for fundamental mode and perturbed modes are simulated using matlab. The frequency is taken from GHz to THz range. The amplitude of the signal is taken as: $A_v \sim N(0, R_v)$ It is taken as Gaussian distribution. When kerr nonlinearity is assumed to depend both on electric and magnetic field components of the wave the output $E1$ is simulated which is having scalar and vector components in the equation. The magnitude plot of scalar components generates a modulated wave a Soliton wave plotted in fig 1. Fig 2 shows different waves propagating in a soliton. Power spectral density plots are shown in the figure 3. Vector component has a psd shown in figure 4 and 5. The correlation between scalar and vector components is plotted in fig 6.

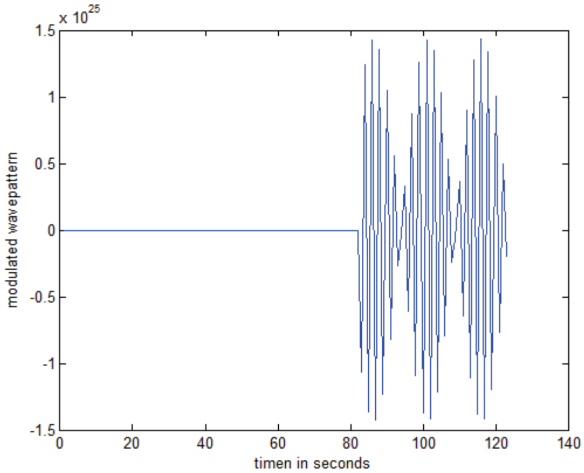

Figure 1: solitons generated by taking autocorrelation of the scalar component of perturbed wave with fundamental wave.

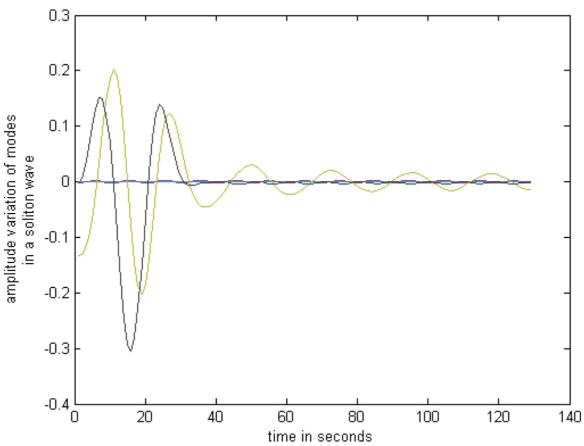

Figure 2: different waves propagating at different phases and amplitudes(generated by scalar part of perturbed wave).

4. Conclusion

Using Maxwell's equations in the kerr medium along with perturbation in the signals produced Solitons. Kerr nonlinearity is used here for Terahertz range. Using the Electromagnetic analysis of kerr nonlinearity in the presence of high frequency light we reach up to conclusion that the perturbations in the electric and magnetic fields in the form of scalar and vector field components are highly correlated which gives solitons in the optical waveguides. It has tremendous scope for the comming technologies in optical communication.

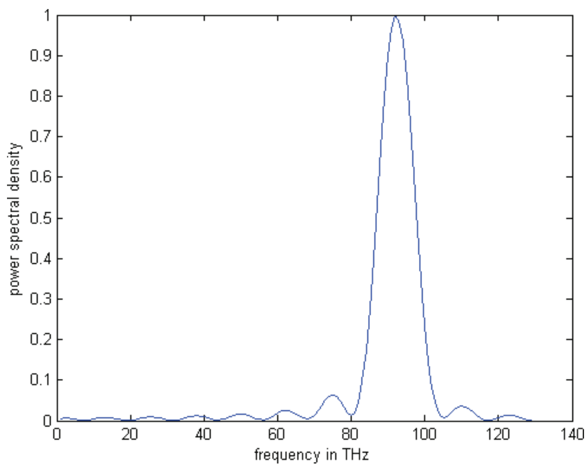

Figure 3: power spectral density of a soliton at 2THz frequency.

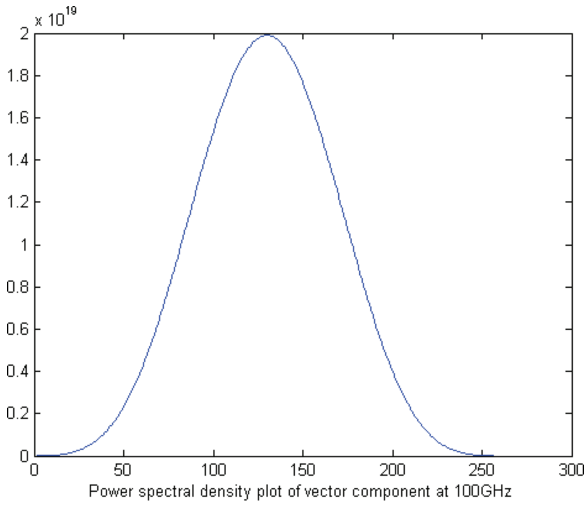

Figure 4: Power spectral density plot of vector component of perturbed wave.

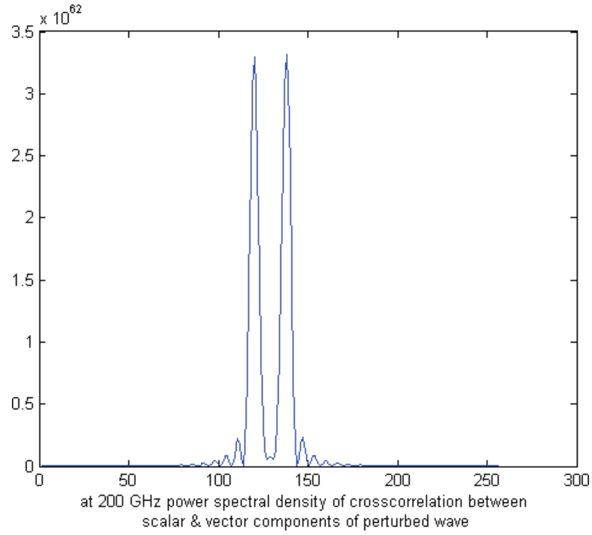

Figure 5: Psd plot of correlation of vetor & scalar component of perturbed wave at 200 GHz.

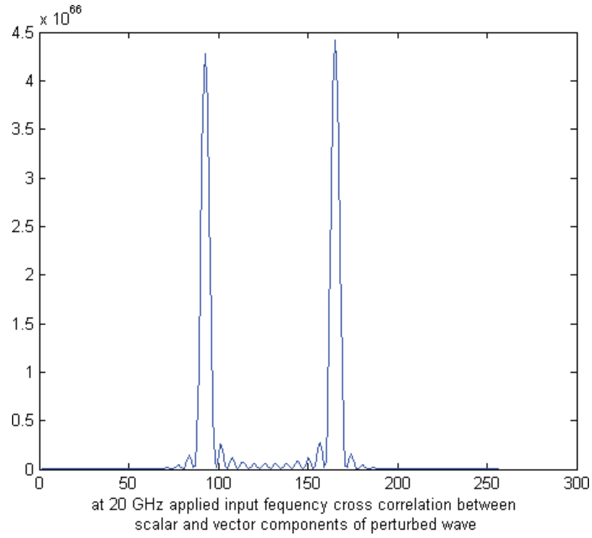

Figure 6: Psd plot of correlation of vetor & scalar component of perturbed wave at 20 GHz.

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