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Magnetohydrodynamic Boundary Layer Flow of Williamson Nanofluid over a Moving Surface in the presence of Gyrotactic Microorganism

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Abstract

The two-dimensional steady incompressible fluid flow of MHD Williamson nanofluid with gyrotactic microorganisms over a moving surface is considered. The four nonlinear governing partial differential equations are reduced to ordinary differential equations, by using a similarity transformation. Homotopy Analysis Method (HAM) is applied to solve the reduced equations and the effect of importance of fluid parameters are explained through graphs. Numerical solution is also obtained and shown that HAM solution matches exactly. The radius of convergence of the solution is determined through Domb-Sykes plot.

Keywords: Williamson nanofluid, Brownian motion, Gyrotactic Microorganisms, MHD, Domb-Sykes plot.

1 INTRODUCTION

Nanofluids are made up of nanoparticles (diameter of less than 100 nanometers) suspended in a base fluid such as water, oil, or ethylene glycol. Nanofluids offer a large surface area for heat transmission between particles and fluids. Nanofluids are characterized by their thermal conductivity. Nanofluids offer significant benefits in a variety of biomedical applications, including cancer therapy, safer surgery by cooling, high-powered lasers, X-ray generators, and magnetic cell separation.

Bioconvection is an occurrence that is characterized by the movement of upward swimming microorganisms such as Bacillus subtilis, Chlamydomonas nivalis, Oxyatic bacteria etc. The nanofluids microorganisms such as oxytactic gyrotactic microorganisms. Here we considered the gyrotactic microorganisms which are very tiny, single celled, self propelled and locally occurring. Due the small scale of microorganisms inertial effect can be neglected and the stability of the nanoparticle suspension may increase. Such type of gyrotactic microorganisms exist in fluids such as seas, lakes, streams and water storages.

Nanoparticles containing microorganisms can improve the thermal efficiency of several systems, including bacteria-powered micro-mixers, microfluidics devices like micro-volumes and enzyme biosensors, microbial fuel cells and bio-microsystems like chip-shaped microdevices. Mobile microbes can enhance the stability of nanoparticle suspensions, microscale mixing, enhanced mass transfer and microvolume use of the suspensions [1, 2, 3, 4]. The development of patterns in layered suspensions of negatively geotactic microbes are discussed by Childress et al. [5].

Talha et al. [6] discussed the two-dimensional steady magnetohydrodynamic boundary layer flow of Williamson nanofluid containing gyrotactic microorganism in the presence of exponential internal heat generation over a stretching sheet and compared the results with and without exponential internal heat generation using Spectral Relaxation Technique (SRT) for numerical solution. Recently Alharbi et al. [7] investigated the two-dimensional stable electrically conducting hybrid nanofluid laminar mixed convection incompressible flow with viscous and gyrotactic microorganism near the stagnation point. A theoretical analysis of migrating gyrotactic microorganism swimming through an isotropically narrowing artery in a non-Newtonian blood base nanofluid is studied by Bhatti et al. [8].

Al-Khaled et al. [9] analyzed the unsteady incompressible fluid flow of tangent hyperbolic nanofluid over an accelerated moving surface and used HAM to find the analytical solution. Lv et al. [10] studied the steady flow of hybrid nanofluid over a spinning disk and used the Parametric Continuation Method (PCM) to solve the transformed equations. The two-dimensional MHD heat transfer and boundary layer flow of Williamson fluid on the exponentially vertical shrinking sheet with variable thickness and thermal conductivity are analyzed by Lund et al. [11]. Atif et al. [12] studied the MHD stratified micropolar bioconvective flow of micropolar nanofluid with gyrotactic microorganisms and converted system of PDE’s are solved by Shooting technique. Yusuf et al. [13] studied the bioconvective flow of Williamson nanofluid under the impact of heat radiation and gyrotactic microorganism in a porus of an aligned semi-infinite plate.

Shahid et al. [14] investigated the effect of the magnetic field, thermal radiation and chemical reaction of nanofluid flow using Successive Taylor’s Series Linearization Method (STSLM). The flow of nanofluid over an isothermal vertical porous Wedge containing gyrotactic microorganisms using the Keller Box method is studied by Mahdy et al. [15]. Non-linear radiation and variable viscosity in two different cases such as oblique and free stream flow are considered by Jayachandra Babu et al. [16] and analyzed the bio conductive stagnation point flow of Williamson nanofluid across a stretching sheet. Khan et al. [17] analyzed the combined impacts of radiative heat and variable viscosity on the oblique hydromagnetic flow with mass and heat transfer of nanofluid over a stretching boundary layer. The bioconvection of a nanofluid with the effects of magnetic field and convective boundary state is examined by Chakrabarty et al. [18]. Dual solutions for the flow of Casson fluid over a vertical rotating cone in a porous medium is investigated by Raju et al. [19].

Bees et al. [20] proposed the nonlinear structure of deep, stochastic, gyrotactic bioconvection. A linear analysis is reviewed and a weakly non-linear analysis is used to show the supercritical nature of the bifurcation. Akbar [21] proposed a peristaltic flow of a nanofluid in an asymmetric channel including gyrotactic microorganism and provided numerical results using long wavelength and low Reynold's number approximation. A dilute suspension of gyrotactic and oxytactic microorganisms in water is proposed by Kuznetsov [22, 23]. He concluded that gyrotactic microorganisms always have a destabilizing effect and the Galerkin method is applied to get the results. Ahmed et al. [24, 25] explained the flow of nanofluids across a porous media and obtained numerical solution by using the Successive over Relaxation (SOR) parameter method.

Ramzan et al. [26] concluded that the effect of increase in bioconvection Lewis number and Peclet number of microorganisms decreases the motile density. The effects of solar radiation on hydromagnetic bioconvection of water-based nanofluid flow via a permeable surface in the residence of gyrotactic microorganisms are examined by Acharya et al. [27]. Achala et al. [37, 38, 39, 40] applied HAM to solve ordinary and partial differential equations, obtained region of convergence by Domb-Sykes plot and applied the Pade approximation to identify the singularities. Sathyanarayana et al. [41, 42] applied HAM to obtain exact analytical solution to the two dimensional laminar compressible boundary layer flow with pressure gradient.

In this paper we are analysing Williamson nanofluid model in boundary layer over a moving surface by considering presence of gyrotactic microorganisms. We are extending the work of Talha et al. [6] for moving surface. The distribution of the paper is first section contains Introduction, second section involves governing equations of the problem, third section is Homotopy Analysis solution of the problem, fourth section is result and discussion and fifth section consists of graphs and tables.

2 MATHEMATICAL FORMULATION

The two dimensional steady incompressible fluid flow of MHD Williamson nanofluid with gyrotactic microorganism over a moving surface [6] is considered. u and v are velocity components along x and y directions respectively. Uniform magnetic field B_0 is considered. The fundamental equations for this model are as follows.

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2} + \sqrt{2}\nu\Gamma\frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial y^2} - \frac{\sigma B_0^2 u}{\rho}, \tag{2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2}+\frac{\rho_p c_p}{\rho c}\left(D_B\frac{\partial C}{\partial y}\frac{\partial T}{\partial y}+\frac{D_T}{T_\infty}\left(\frac{\partial T}{\partial y}\right)^2\right), \tag{3}$$

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=D_B\frac{\partial^2 C}{\partial y^2}+\frac{D_T}{T_\infty}\frac{\partial^2 T}{\partial y^2}, \tag{4}$$

$$u\frac{\partial n}{\partial x}+v\frac{\partial n}{\partial y}=D_n\frac{\partial^2 n}{\partial y^2}-\frac{dW_c}{C_w-C_\infty}\frac{\partial}{\partial y}\left(n\frac{\partial C}{\partial y}\right), \tag{5}$$

where u and v are velocity components in x and y direction, ν is the kinematic viscosity, α is the thermal diffusivity of nanofluid, ρ is density of the fluid, B_0 is the magnetic field, σ is the electrical conductivity, τ is heat capacity ratio, T is the temperature of the fluid, C is the concentration of the fluid, D_T is thermophoresis coefficient, D_B is Brownian diffusion coefficient, n is the density of the motile microorganism, D_n is diffusivity of microorganism, W_c is maximum swimming speed, λ is power exponent parameter. The corresponding boundary conditions are given by

$$u=\lambda_1 U, v=0, T=T_\omega=T_\infty+Ax^\lambda, C=C_\omega=C_\infty+Bx^\lambda, n=n_\omega = n_\infty+Ex^\lambda \text{ at } y=0;$$

$$u\to U, T\to T_\infty, C\to C_\infty, n\to n_\infty \text{ as } y\to\infty. \tag{6}$$

Introducing the following similarity transformations to reduce the governing equations into a system of ordinary differential equations.

$$\Psi=\sqrt{(2U\nu x)}f(\eta), \theta(\eta)=\frac{T-T_\infty}{T_\omega-T_\infty}, \phi(\eta)=\frac{C-C_\infty}{C_\omega-C_\infty}, \xi(\eta)=\frac{n-n_\infty}{n_\omega-n_\infty}, \eta=y\sqrt{\frac{U}{2\nu x}}. \tag{7}$$

Where Ψ is stream function. Substituting equation (7) in (2) - (6) we get

$$f'''(1+\gamma f'')+ff''-Mf'=0, \tag{8}$$

$$\theta''+Pr(f+Nb\phi')\theta'-2Pr\lambda f'\theta+PrNt(\theta')^2=0, \tag{9}$$

$$\phi''+Lef\phi'-2Le\lambda f'\phi+\frac{Nt}{Nb}\theta''=0, \tag{10}$$

$$\xi''+(PrLbf-Pe\phi')\xi'-(2PrLb\lambda f'+Pe\phi'')\xi-Pe\sigma\phi''=0. \tag{11}$$

$$f=0, f'=\lambda_1, \theta=1, \phi=1, \xi=1 \;\; at \;\; \eta=0,$$

$$f'\to 1, \theta\to 0, \phi\to 0, \xi\to 0 \;\; as \;\; \eta\to\infty. \tag{12}$$

Where

$M = \left(\frac{2\sigma B_0^2 x}{\rho U}\right)$; Magnetic field parameter,

$\gamma = \Gamma \frac{U^{\frac{3}{2}}}{\sqrt{\nu x}}$; Non Newtonian Williamson parameter,

$Pr = \frac{\nu}{\alpha}$; Prandtl number,

$Pe = \left(\frac{dW_c}{D_n}\right)$; Peclet number,

$Le = \frac{\nu}{D_B}$; Lewis number,

$Nb = \left(\frac{(\rho c_p) D_B (C_w - C_\infty)}{\nu(\rho c_f)}\right)$; Brownian motion parameter,

$Nt = \left(\frac{(\rho c_p) D_T (T_w - T_\infty)}{\nu T_\infty (\rho c_f)}\right)$; Thermophoresis parameter,

$Lb = \frac{\alpha}{D_n}$; Bioconvection Lewis number,

$\sigma = \frac{n_\infty}{n_w - n_\infty}$; Bioconvection parameter.

3 HOMOTOPY ANALYSIS SOLUTION

Shijun Liao (1992) [29, 30, 31, 32, 33, 34, 35] explained Homotopy Analysis Method (HAM) to solve non-linear differential equations analytically. Using this method [36] we solve coupled nonlinear equations of this problem. The steps of the method are,

$$N[f(\eta)] = f'''(1 + \gamma f'') + ff'' - Mf', \tag{13}$$

$$N[\theta(\eta)] = \theta'' + Pr(f + Nb\phi')\theta' - 2Pr\lambda f'\theta + PrNt(\theta')^2, \tag{14}$$

$$N[\phi(\eta)] = \phi'' + Lef\phi' - 2Le\lambda f'\phi + \frac{Nt}{Nb}\theta'', \tag{15}$$

$$N[\xi(\eta)] = \xi'' + (PrLbf - Pe\phi')\xi' - (2PrLb\lambda f' + Pe\phi'')\xi - Pe\sigma\phi''. \tag{16}$$

Linear operators considered are as follows,

$$L_f(f) = \frac{\partial^3 f}{\partial \eta^3} + \frac{\partial^2 f}{\partial \eta^2}, \tag{17}$$

$$L_\theta(\theta) = \frac{\partial^2 \theta}{\partial \eta^2} + \frac{\partial \theta}{\partial \eta}, \tag{18}$$

$$L_\phi(\phi) = \frac{\partial^2 \phi}{\partial \eta^2} + \frac{\partial \phi}{\partial \eta}, \tag{19}$$

$$L_\xi(\xi) = \frac{\partial^2 \xi}{\partial \eta^2} + \frac{\partial \xi}{\partial \eta}, \tag{20}$$

which gives initial approximations as,

$$f_0 = (\lambda_1 - 1) + \eta + (1 - \lambda_1)e^{-\eta}, \tag{21}$$

$$\theta_0 = e^{-\eta}, \tag{22}$$

$$\phi_0 = e^{-\eta}, \tag{23}$$

$$\xi_0 = e^{-\eta}. \tag{24}$$

The nonlinear equations for approximate solutions are,

$$(1-p)L_f[f(\eta,p) - f_0(\eta)] = hp\left[\frac{\partial^3 f}{\partial \eta^3}\left(1 + \gamma\frac{\partial^2 f}{\partial \eta^2}\right) + f\frac{\partial^2 f}{\partial \eta^2} - M\frac{\partial f}{\partial \eta}\right], \tag{25}$$

$$(1-p)L_\theta[\theta(\eta,p) - \theta_0(\eta)] = hp\left[\frac{\partial^2 \theta}{\partial \eta^2} + Pr\left(f + Nb\frac{\partial \phi}{\partial \eta}\right)\frac{\partial \theta}{\partial \eta} - 2Pr\lambda\frac{\partial f}{\partial \eta}\theta + PrNt\left(\frac{\partial \theta}{\partial \eta}\right)^2\right], \tag{26}$$

$$(1-p)L_\phi[\phi(\eta,p) - \phi_0(\eta)] = hp\left[\frac{\partial^2 \phi}{\partial \eta^2} + Lef\frac{\partial \phi}{\partial \eta} - 2Le\lambda\frac{\partial f}{\partial \eta}\phi + \frac{Nt}{Nb}\frac{\partial^2 \theta}{\partial \eta^2}\right], \tag{27}$$

$$(1-p)L_\xi[\xi(\eta,p) - \xi_0(\eta)] = hp\left[\frac{\partial^2 \xi}{\partial \eta^2} + \left(PrLbf - Pe\frac{\partial \phi}{\partial \eta}\right)\frac{\partial \xi}{\partial \eta} - \left(2PrLb\lambda\frac{\partial f}{\partial \eta} + Pe\frac{\partial^2 \phi}{\partial \eta^2}\right)\xi - Pe\sigma\frac{\partial^2 \phi}{\partial \eta^2}\right], \tag{28}$$

with following boundary conditions,

$$f(0,p) = 0, f_\eta(0,p) = \lambda_1, f_\eta(\infty,p) = 1, \tag{29}$$

$$\theta(0,p) = 1, \theta(\infty,p) = 0, \tag{30}$$

$$\phi(0,p) = 1, \phi(\infty,p) = 0, \tag{31}$$

$$\xi(0,p) = 1, \xi(\infty,p) = 0. \tag{32}$$

Varying the values of p from 0 to 1 we get the solution from first approximation to required solution. Defining

$$f_0(\eta) = f(\eta,0) = \zeta_0(\eta), \tag{33}$$

$$\theta_0(\eta) = \theta(\eta,0) = \mu_0(\eta), \tag{34}$$

$$\phi_0(\eta) = \phi(\eta, 0) = \psi_0(\eta), \tag{35}$$

$$\xi_0(\eta) = \xi(\eta, 0) = \tau_0(\eta). \tag{36}$$

Using Maclaurin's series expansion and applying Leibnitz theorem we get the series solution. The convergence of the series solution is derived by calculating the convergence parameter h.

$$L_f[\zeta_k - \chi_k \zeta_{k-1}] = h r_k(\eta), \tag{37}$$

$$L_\theta[\mu_k - \chi_k \mu_{k-1}] = h s_k(\eta), \tag{38}$$

$$L_\phi[\psi_k - \chi_k \psi_{k-1}] = h t_k(\eta), \tag{39}$$

$$L_\xi[\tau_k - \chi_k \tau_{k-1}] = h w_k(\eta). \tag{40}$$

$$\text{where} \quad \chi_k = \begin{cases} 0, & \text{when } k \le 1 \\ 1, & \text{when } k > 1 \end{cases} \quad \text{and} \tag{41}$$

$$r_k(\eta) = \zeta'''_{k-1}(\eta) + \gamma \sum_{m=0}^{k-1} \zeta''_{k-1-m}(\eta)\zeta'''_m(\eta) + \sum_{m=0}^{k-1} \zeta_{k-1-m}(\eta)\zeta''_m(\eta) - M\zeta'_{k-1}(\eta), \tag{42}$$

$$s_k(\eta) = \mu''_{k-1}(\eta) + Pr\left(\sum_{m=0}^{k-1} \zeta_{k-1-m}(\eta) + Nb \sum_{m=0}^{k-1} \psi'_{k-1-m}(\eta)\right)\mu'_m(\eta) - 2Pr\lambda \sum_{m=0}^{k-1} \zeta'_{k-1-m}(\eta)\mu_m(\eta) + PrNt \sum_{m=0}^{k-1} \mu'_{k-1-m}(\eta)\mu'_m(\eta), \tag{43}$$

$$t_k(\eta) = \psi''_{k-1}(\eta) + Le \sum_{m=0}^{k-1} \zeta_{k-1-m}(\eta)\psi'_m(\eta) - 2Le\lambda \sum_{m=0}^{k-1} \zeta'_{k-1-m}(\eta)\psi_m(\eta) + \frac{Nt}{Nb}\mu''_{k-1}(\eta), \tag{44}$$

$$w_k(\eta) = \tau''_{k-1}(\eta) + \left(PrLb \sum_{m=0}^{k-1} \zeta_{k-1-m}(\eta) - Pe \sum_{m=0}^{k-1} \psi'_{k-1-m}\right)\tau'_m(\eta) - \left(2PrLb\lambda \sum_{m=0}^{k-1} \zeta'_{k-1-m}(\eta) + Pe \sum_{m=0}^{k-1} \psi''_{k-1-m}\right)\tau_m(\eta) - Pe\sigma\psi''_{k-1}(\eta), \tag{45}$$

with boundary conditions,

$$\zeta_k(0) = 0, \zeta'_k(0) = 0, \zeta'_k(\infty) = 0, \tag{46}$$

$$\mu_k(0) = 0, \mu_k(\infty) = 0, \tag{47}$$

$$\psi_k(0) = 0, \psi_k(\infty) = 0, \tag{48}$$

$$\tau_k(0) = 0, \tau_k(\infty) = 0. \tag{49}$$

The required solution is

$$f = \zeta_0 + \zeta_1 + \zeta_2 + \zeta_3 + \ldots, \tag{50}$$

$$\theta = \mu_0 + \mu_1 + \mu_2 + \mu_3 + \ldots, \tag{51}$$

$$\phi = \psi_0 + \psi_1 + \psi_2 + \psi_3 + \ldots, \tag{52}$$

$$\xi = \tau_0 + \tau_1 + \tau_2 + \tau_3 + \ldots. \tag{53}$$

Soving equtions (37)-(40) by using MATHEMATICA we get

$$\zeta_1 = \frac{1}{4}(-h - 4hM + h\gamma - 2h\lambda 1 + 4hM\lambda 1 - 2h\gamma\lambda 1 + 3h\lambda 1^2 + h\gamma\lambda 1^2) + \frac{1}{4}e^{-2\eta}\Big(2e^{\eta}(3h - 2hM - h\gamma - 6h\lambda 1 + 2hM\lambda 1 + 2h\gamma\lambda 1 + 3h\lambda 1^2 - h\gamma\lambda 1^2) + h\big(-2e^{2\eta}M\eta^2 + (-1+\gamma)(-1+\lambda 1)^2 - 2e^{\eta}(-1+\lambda 1)(-2+\eta^2+2M(2+\eta) + 4\lambda 1 + 2\eta\lambda 1)\big)\Big), \ldots \tag{54}$$

$$\mu_1 = \frac{1}{2}e^{-2\eta}\Big(h\mathrm{Pr}(-1 + \mathrm{Nb} + \mathrm{Nt} + 2\lambda + \lambda 1 - 2\lambda\lambda 1) + e^{\eta}\Big(2h + h\mathrm{Pr} - h\mathrm{NbPr} - h\mathrm{NtPr} - 6h\mathrm{Pr}\lambda - 3h\mathrm{Pr}\lambda 1 + 2h\mathrm{Pr}\lambda\lambda 1 + h\big(-2 + \mathrm{Pr}\eta^2 + 4\mathrm{Pr}\lambda + 2\mathrm{Pr}\lambda 1 + 2\eta(-1 + 2\mathrm{Pr}\lambda + \mathrm{Pr}\lambda 1)\big)\Big)\Big), \ldots \tag{55}$$

$$\psi_1 = \frac{1}{2\mathrm{Nb}}(2\mathrm{Nb}e^{-2\eta}(h\mathrm{LeNb}(-1 - 2\lambda(-1+\lambda 1) + \lambda 1) + e^{\eta}(2h\mathrm{Nb} + h\mathrm{LeNb} + 2h\mathrm{Nt} - 6h\mathrm{LeNb}\lambda - 3h\mathrm{LeNb}\lambda 1 + 2h\mathrm{LeNb}\lambda\lambda 1 + h(-2\mathrm{Nt}(1+\eta) + \mathrm{Nb}(-2 + \mathrm{Le}\eta^2 + 4\mathrm{Le}\lambda + 2\mathrm{Le}\lambda 1 + 2\eta(-1 + 2\mathrm{Le}\lambda + \mathrm{Le}\lambda 1)))))), \ldots \tag{56}$$

$$\tau_1 = \frac{1}{2}e^{-2\eta}\Big(h\big(-2\mathrm{Pe} + \mathrm{LbPr}(-1 + 2\lambda + \lambda 1 - 2\lambda\lambda 1)\big) + e^{\eta}\Big(-e^{-\eta}h(-2e^{\eta} - 2\mathrm{Pe} - \mathrm{LbPr} - 2e^{\eta}\eta + e^{\eta}\mathrm{LbPr}\eta^2 + 2\mathrm{LbPr}\lambda + 4e^{\eta}\mathrm{LbPr}\lambda + 4e^{\eta}\mathrm{LbPr}\eta\lambda + \mathrm{LbPr}\lambda 1 + 2e^{\eta}\mathrm{LbPr}\lambda 1 + 2e^{\eta}\mathrm{LbPr}\eta\lambda 1 - 2\mathrm{LbPr}\lambda\lambda 1 + 2e^{\eta}\mathrm{Pe}\sigma + 2e^{\eta}\mathrm{Pe}\eta\sigma) + h\big(\mathrm{LbPr}\eta^2 + 2(-1 + \mathrm{LbPr}(2\lambda + \lambda 1) + \mathrm{Pe}\sigma) + 2\eta(-1 + \mathrm{LbPr}(2\lambda + \lambda 1) + \mathrm{Pe}\sigma)\big)\Big)\Big), \ldots \tag{57}$$

4 RESULTS AND DISCUSSION

The equations (9) - (12) with boundary conditions (13) are solved analytically by employing Homotopy Analysis Method and numerically by employing Runge-Kutta method. The obtained convergent solution depends on auxiliary linear operator L, auxiliary parameter h and the initial solution. For all variables h-curve is plotted to find the value of h and is used to get the convergent solution.

The significant parameters of this problem are Williamson parameter γ, Magnetic field parameter M, Power exponent parameter λ, Brownian motion parameter Nb, Thermophoresis parameter Nt, Prandtl number Pr, Lewis number Le, Peclet number Pe, Bioconvection Lewis number Lb and Bioconvection parameter σ. The effect of these parameters on different profiles are as follows. In figure 2 we observed that velocity profile decreases with increase in γ. From figures 3 and 7 it is observed that increase in γ and Nt increases the temperature. From figures 4, 5, 6 and 8 we have observed that temperature profile decreases with increase in M, λ, Nb and Pr.

From figures 9, 10, 12 and 13 it is observed that the increase in values of λ, M, Nb and Le decreases the concentration profile, where as from figure 11 we can observe that increase in Nt increases the concentration profile. It is observed that increase in M, λ, Pr, Pe and Lb decreases the microorganism profile seen from figures 14, 15, 16, 17 and 18, where as increase in σ increases the microorganism profile shown in figure 19. Applied magnetic field will reduce temperature, concentration and gyrotactic microorganism in Williamson fluid.

We have obtained radius of convergence for velocity $R = 5.1342$, temperature $R = 7.0816$, concentration $R = 7.6923$ and gyrotactic microorganism $R = 7.6511$ by drawing Domb-Sykes plot in figures 20, 21, 22 and 23.

Comparison of numerical solutions and HAM solutions for velocity, temperature, concentration and gyrotactic microorganism profiles are shown from figures 24, 25, 26, 27 and in table 1 and observed good agreement. We compare our HAM solution with existing numerical solution of Haile [28] by the method Runge-Kutta integration technique. We have also solved numerically by R K Method and observe a very good matching in every case. We conclude that HAM gives almost exact solution.

5 Graphs and tables

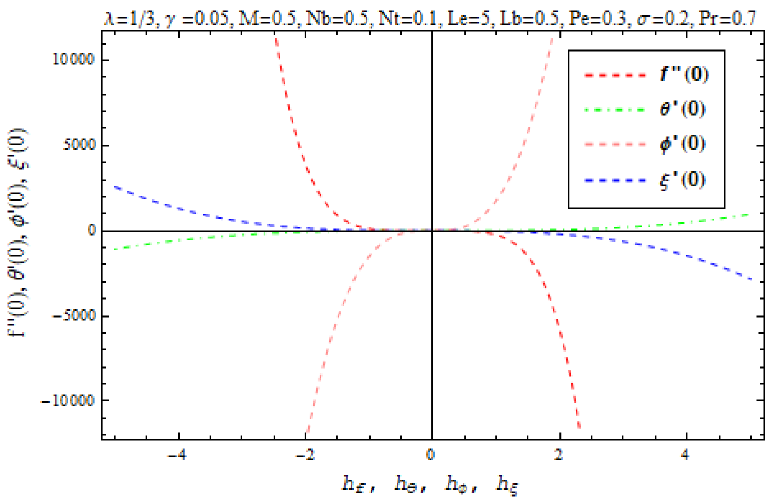

Figure 1: Combine plot for h-curves

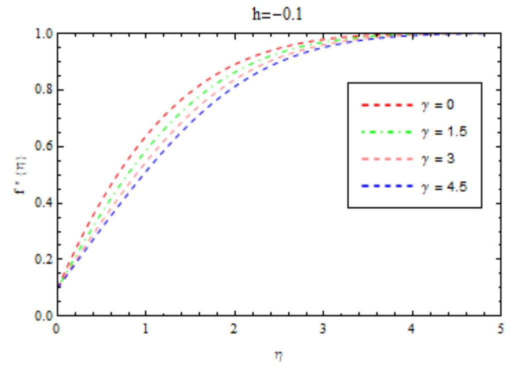

Figure 2: Effect of Williamson parameter γ on velocity profile

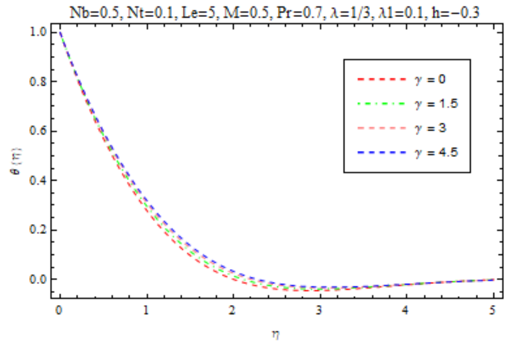

Figure 3: Effect of Williamson parameter γ on temperature profile

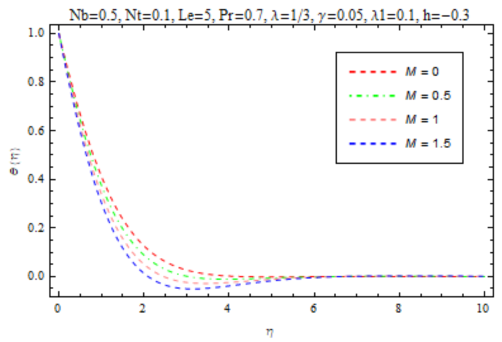

Figure 4: Effect of Magnetic field parameter M on temperature profile

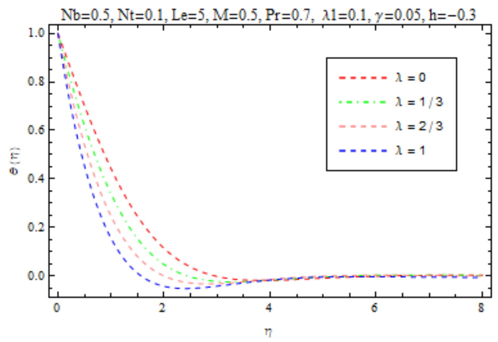

Figure 5: Effect of power exponent parameter λ on temperature profile

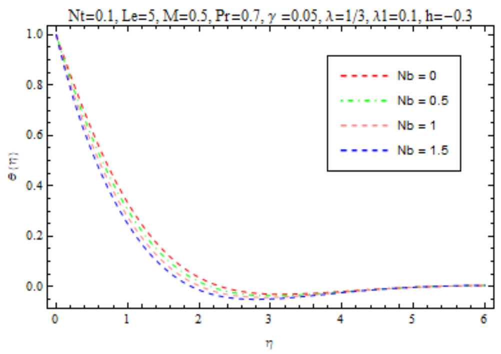

Figure 6: Effect of Brownian motion parameter Nb on temperature profile

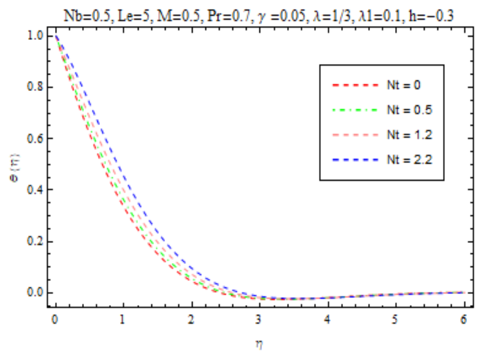

Figure 7: Effect of thermophoresis parameter Nt on temperature profile

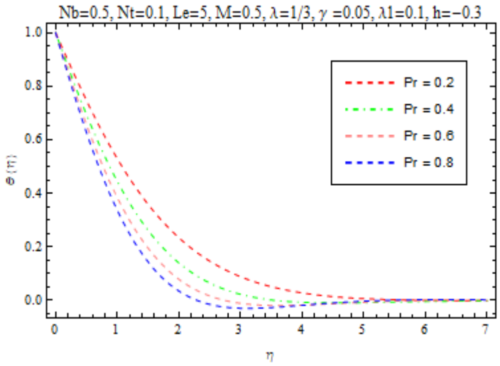

Figure 8: Effect of Prandtl number Pr on temperature profile

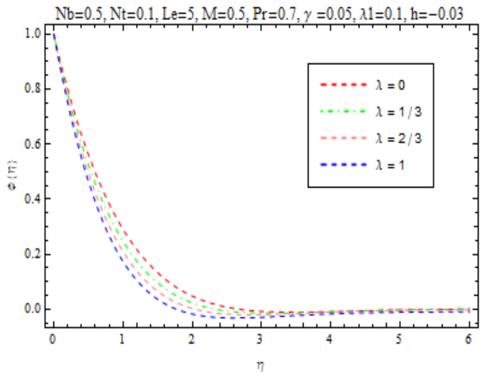

Figure 9: Effect of power exponent parameter λ on concentration profile

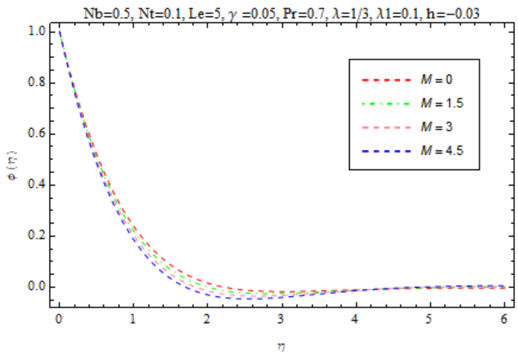

Figure 10: Effect of Magnetic field parameter M on concentration profile

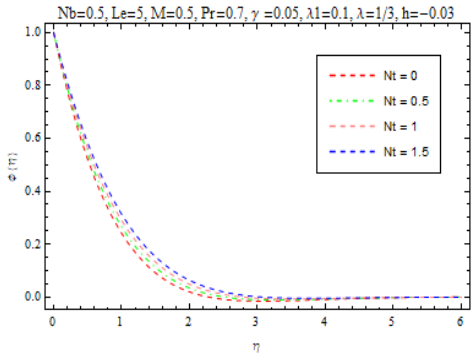

Figure 11: Effect of thermophoresis parameter Nt on concentration profile

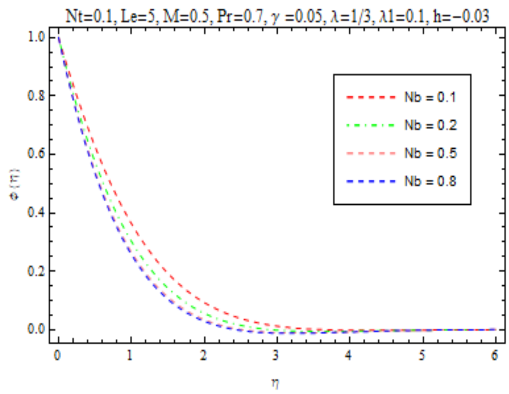

Figure 12: Effect of Brownian motion parameter Nb on concentration profile

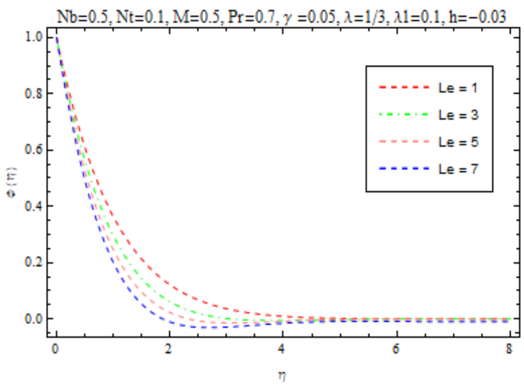

Figure 13: Effect of Lewis number Le on concentration profile

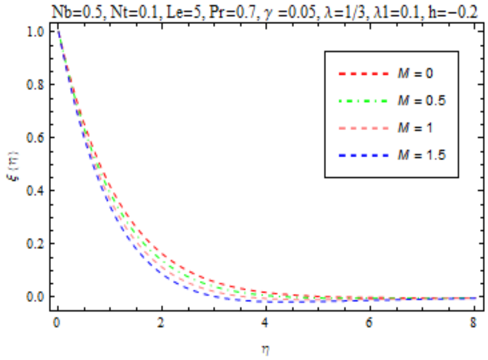

Figure 14: Effect of Magnetic field parameter M on gyrotactic microorganism profile

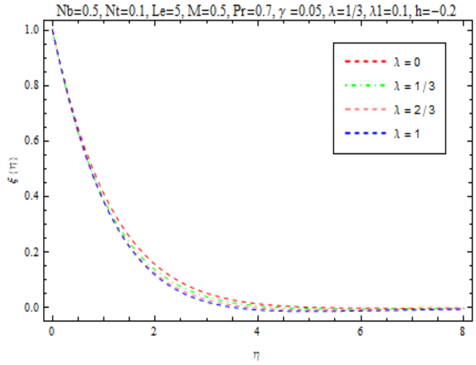

Figure 15: Effect of power exponent parameter λ on gyrotactic microorganism profile

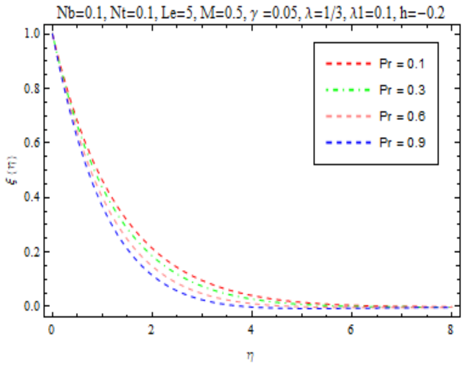

Figure 16: Effect of Prandtl number Pr on gyrotactic microorganism profile

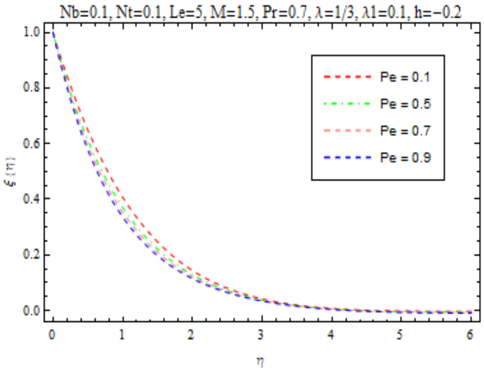

Figure 17: Effect of Peclet number Pe on gyrotactic microorganism profile

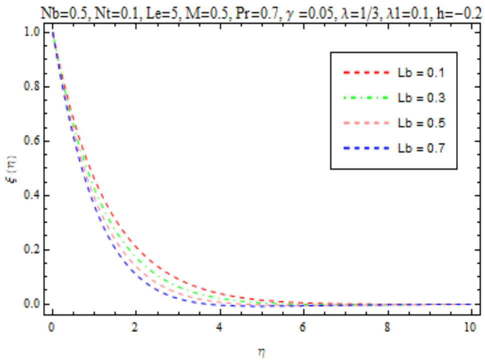

Figure 18: Effect of bioconvection Lewis number Lb on gyrotactic microorganism profile

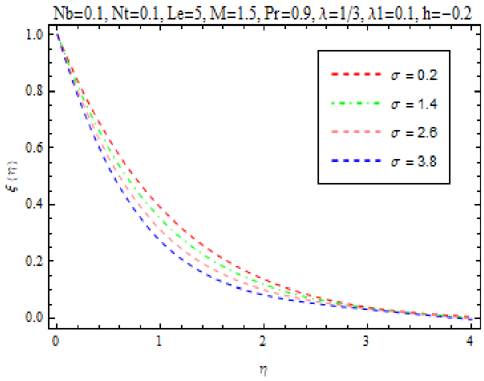

Figure 19: Effect of bioconvection parameter σ on gyrotactic microorganism profile

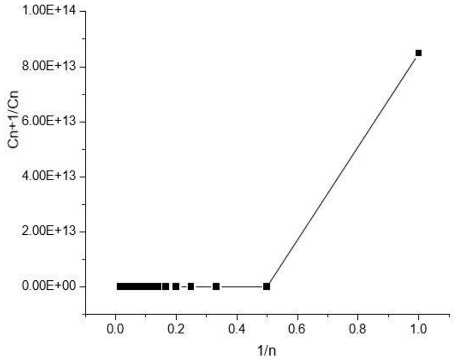

Figure 20: Domb-Sykes Velocity Plot with R=5.1342

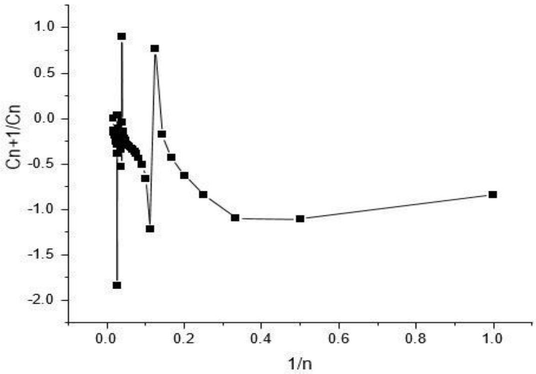

Figure 21: Domb-Sykes Temperature Plot with R=7.0816

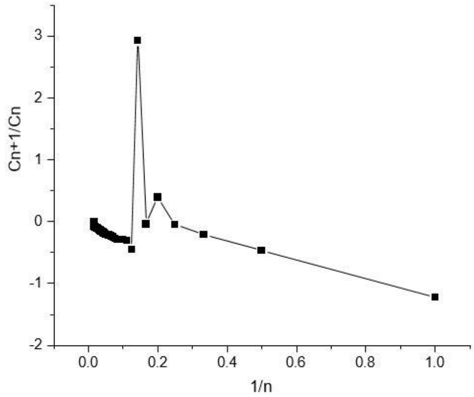

Figure 22: Domb-Sykes Concentration Plot with R=7.6923

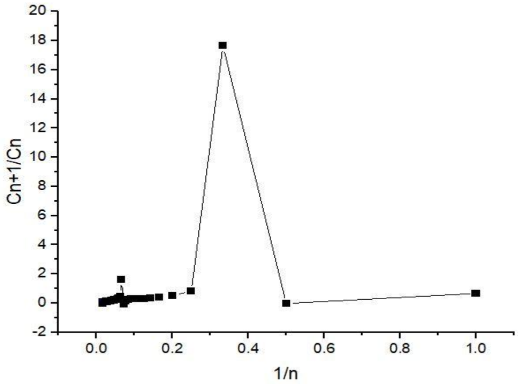

Figure 23: Domb-Sykes gyrotactic microorganism Plot with R=7.0816

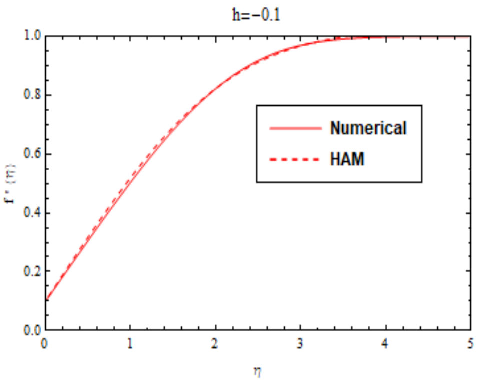

Figure 24: Comparision of HAM solution and numerical solution for Williamson parameter $\gamma = 1$ on velocity profile

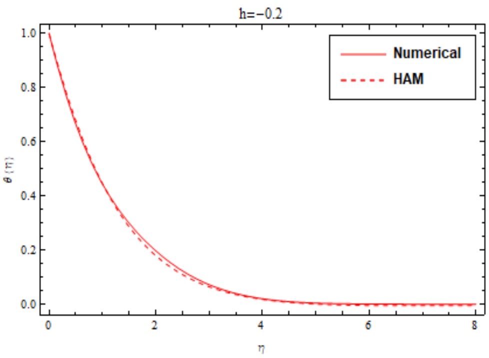

Figure 25: Comparision of HAM solution and numerical solution for Magnetic parameter $M = 0$ on temperature profile

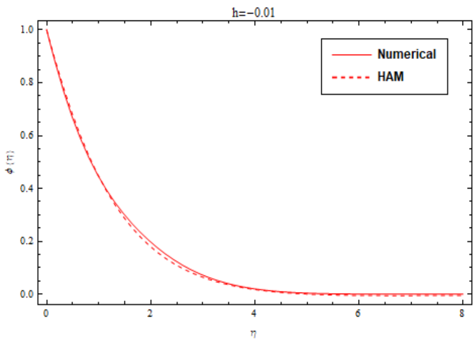

Figure 26: Comparision of HAM solution and numerical solution for Lewis number Le = 7 on concentration profile

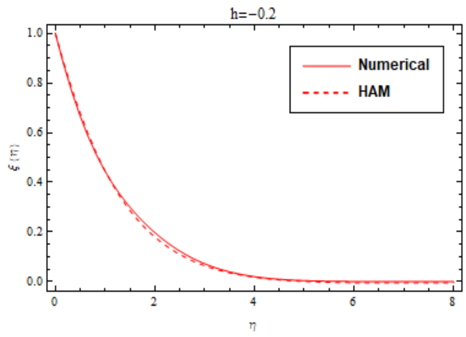

Figure 27: Comparision of HAM solution and numerical solution for Magnetic parameter $M = 0$ on gyrotactic microorganism profile

Table 1: Comparision of $f''(0)$ when $\gamma = M = 0$ and $h = 0.26156$

Parameter	$f''(0)$		
λ_1	Haile (R K integration technique) [28]	Present work (HAM)	Present work (R K Method)
0.1	0.46251	0.462509	0.462533
1	0.00000	0.0000	0.000000

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