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Certain Quadruple Series Equations with Jacobi Polynomials as Kernels

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The solution of Quadruple equations involving Jacobi polynomials has been obtained in this paper.

1. INTRODUCTION:

In this paper the solution of following four series equations has been obtained:

$$\sum_{n=0}^{\infty} \frac{\Gamma(\mu+n+l+1) A_n (1+H_n)}{\Gamma(\beta+n+l+1)} P_{n+l}^{(\alpha,\beta)}(x) = \begin{cases} f_1(x), -1 < x < a, \\ f_3(x), b < x < c, \end{cases} \tag{1.1}$$

$$\sum_{n=0}^{\infty} \frac{\Gamma(\lambda+n+l+1) A_n}{\Gamma(\gamma+n+l+1)} P_{n+l}^{(\gamma,\delta)}(x) = \begin{cases} f_2(x), a < x < b, \\ f_4(x), c < x < 1, \end{cases} \tag{1.2}$$

Where l is an arbitrary non-negative integer, $f_1(x)$, $f_2(x)$, $f_3(x)$, $f_4(x)$ are prescribed functions, the sequence $\{A_n\}$ is to be determined, H_n is a suitably restricted Known Coefficient, and in general:

$$\min\{\alpha, \beta, \gamma, \delta, \lambda, \mu\} > -1 \tag{1.3}$$

Recently, Dwivedi, Gupta and Gupta (1964) have solved the above equations in the particular vase when $H_n = 0$.

These equations arises in the four part boundary value problems of electrostatics, elasticity and other fields of mathematical physics.

2. SOLUTION OF FOUR SERIES EQUATIONS

By employing the familiar technique for solving four series equations, we get the solution of equations (1.1) and (1.2) as:

$$A_n = \frac{(n+l)!\ \Gamma(\gamma+\delta+2n+2l+1)\Gamma(\gamma+\delta+n+l+1)}{2^{\gamma+\delta+1}\Gamma(\lambda+n+l+1)\Gamma(\delta+n+l+1)} \bullet$$

$$\left[\int_{-1}^{a} g(x) + \int_{a}^{b} f_2(x) + \int_{b}^{c} h(x) + \int_{c}^{1} f_4(x)\right](1-\xi)^{\gamma}(1+\xi)^{\delta} P_{n+l}^{(\gamma,\delta)}(x)dx \tag{2.1}$$

Where the unknown functions g(x) and h(x) are to be determined by the following set of equations:

$$\eta(t)G(t) = P_3(t) + Q_1(t) + \int_{b}^{c} G(\xi)\{K(\xi,t) + Z(\xi,t)\}dt,\ b < t < c \tag{2.2}$$

Where

$$P_3(t) = -\frac{31n(1+\alpha-\gamma-\rho)\pi}{\pi(t-b)^{r+\rho-\alpha}} \int_{-1}^{a} \frac{P_1(r)(b-r)^{\gamma+\rho-\alpha}}{(t-r)} dr \tag{2.3}$$

$$P_1(t) = \frac{\sin(1+\alpha-\gamma-\rho)\pi}{\pi(1+x)^{-\beta} a_n^*} \frac{d}{dt} \int_{-1}^{t} \frac{P(x)dx}{(t-x)^{\gamma+\rho-\alpha}} \tag{2.4}$$

$$P(x) = f_1(x) - \left[\int_{a}^{b} f_2(\xi) - \int_{c}^{1} f_4(\xi)\right](1-\xi)^{\gamma}(1+\xi)^{\delta}\{M(x,\xi) + N(x,\xi)\}d\xi \tag{2.5}$$

$$M(x,\xi) = \sum_{n=0}^{\infty} \frac{\Gamma(\mu+n+l+1)(n+l)!(\gamma+\delta+2n+2l+1)}{\Gamma(\beta+n+l+1)\Gamma(\lambda+n+l+1)2^{\gamma+\sigma+1}} \bullet \frac{\Gamma(\gamma+\delta+n+l+1)}{\Gamma(\delta+n+l+1)} P_{n+l}^{(\alpha,\beta)}(x)\ P_{n+l}^{(\gamma,\delta)}(\xi)$$

$$= (1+\xi)^{-\delta}(1+x)^{-\beta} a_n^* \int_{-1}^{W} \eta(t)(\xi-t)^{\rho-1}(x-t)^{\gamma+\rho-\alpha-1}dt \tag{2.6}$$

$$\eta(t) = (1+t)^{\delta-\rho}(1-t)^{-\gamma-\rho} \tag{2.7}$$

$$N(x,\xi) = \sum_{n=0}^{\infty} \frac{\Gamma(\mu+n+l+1)(n+l)!(\gamma+\delta+2n+2l+1)}{\Gamma(\beta+n+l+1)\Gamma(\lambda+n+l+1)2^{\gamma+\sigma+1}} \bullet \frac{\Gamma(\gamma+\delta+n+l+1)}{\Gamma(\delta+n+l+1)} H_n P_{n+l}^{(\alpha,\beta)}(x) P_{n+l}^{(\gamma,\delta)}(\xi) \tag{2.8}$$

$$Q_1(t) = \frac{\sin(1+\alpha-\gamma-\rho)\pi}{\pi(1+x)-\beta a_n^*} \bullet \frac{d}{dt}\int_b^t \frac{Q(x)dx}{(t-x)^{\gamma+\rho-\alpha}} \tag{2.9}$$

$$Q(x) = f_3(x) - \left[\int_a^b f_2(\xi) + \int_c^1 f_4(\xi)\right](1-\xi)^{\gamma}(1+\xi)^{\delta}\{M(x,\xi)+N(x,\xi)\}d\xi \tag{2.10}$$

$$K(\xi,t) = \frac{\sin(1-\rho)\pi \sin(1+\alpha-\gamma-\rho)\pi}{\pi^2(t-b)^{\gamma+\rho-\alpha}(\xi-b)} \int_a^b \frac{\eta(r)(b-r)^{\gamma+2\rho-\alpha}}{(t-r)(\xi-r)} dr \tag{2.11}$$

$$Z(\xi,t) = \frac{\sin(1-\rho)\pi \bullet \sin(1+\alpha-\gamma-\rho)\pi}{\pi^2\rho^{-1}} \int_b^{\xi} \frac{dx}{(\xi-x)^{\rho+1}} \bullet \frac{d}{dt}\int_b^t \frac{N(x,\xi)d\xi}{(t-\xi)^{\gamma+\rho-\alpha}} \tag{2.12}$$

$$h(\xi) = -\frac{\sin(1-\rho)\pi}{\pi} \bullet \frac{d}{d\xi}\int_{\xi}^{c} \frac{G(t)dt}{(t-\xi)^{\rho}} \tag{2.13}$$

The equation (2.2) is a Fredholm integral equation of the second kind. From this equation, we can determine G(t). Knowing G(t), h(t) can be found out from the equation (2.13) and the corresponding coefficients A_n from the equation (2.1) and hence the solution follows.

PARTICULAR CASES

(1) When $H_n = 0,\ l = 0,\ \lambda = \alpha+\beta-\mu,\ \gamma = \alpha+\beta-\delta,\ \delta = \beta,\ \mu = \mu' - \beta$, and $\mu' = \beta + \gamma_2$, the four series equations (1.1) to (1.2) would correspond to another set of four series equations which are extensions of those of Srivastava's triple equations (1964).

(2) If we take $H_n = 0$ in equations. (1.1) and (1.2), we obtain the solution of four series equations considered recently by Dwivedi, Gupta and Gupta (1984).

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