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Some New Sets in Ideal Delta Space

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Abstract

In this paper, in an ideal delta space $(X, \tau, \tau^{\delta}, I)$, we introduce $L^{*\delta}$-perfect, $R^{*\delta}$-perfect and $C^{*\delta}$- perfect sets in an ideal delta spaces and study their properties.

Keywords: $R^{*\delta}$ - perfect, $L^{*\delta}$ - perfect, $C^{*\delta}$ - perfect, $R^{*\delta}$ - topology

1. Introduction and Preliminaries

The set δ (delta) has been introduced in topological space by C.Chottopadhyay and U.K.Roy [2]. In [2] and [1] C.Chottopadhyay have discussed the properties of this set in detail. The contributions of Hamlet and Jankovic[3-6] in ideal topological spaces initiated the generalization of some important properties in general topology via topological ideals. Shyampada Modak [7] introduced ideal delta space. By a space (X, τ), the system of open neighborhoods of x is denoted by $N(x) = \{U \in \tau : x \in U\}$. For a given subset A of a space (X, τ), $cl(A)$ and $\text{int}(A)$ are used to denote the closure of A and interior of A, respectively, with respect to the topology.

A subset A of a topological space (X, τ) is called a δ- set [2] if $\text{int}(cl(A)) \subset cl(\text{int}(A))$. This collection of all δ- sets in a topological space (X, τ) is denoted by τ^{δ}. This collection does not form a topology because arbitrary union of δ- sets may not be a δ- set in general [2]. Furthermore, it is intersecting that the finite union (intersection) of δ- sets is again a δ- set [1]. As $\tau \subset \tau^{\delta}$, then intersection of an open with a δ- set is a δ- set. Let A be a subset of a topological space (X, τ). The δ- closure of A [1] is denoted as $cl_{\delta}(A)$ and is defined as intersection of all δ- sets containing A.

A nonempty collection I of subsets of a set X is said to be an ideal on X, if it satisfies the following two conditions: (i) If $A \in I$ and $B \subseteq A$, then $B \in I$ (heredity); (ii) If $A \in I$ and $B \in I$, then $A \cup B \in I$ (finite addition). An ideal topological space (or ideal space) (X, τ, I) means a topological space (X, τ) with an ideal I defined on X. Let (X, τ) be a topological space with an ideal I defined on X. Then for any set A of X, $A^*(I, \tau) = \{x \in X / A \cap U \notin I$ for every $U \in N(x)\}$ is called the local function of A with respect to I and τ .If there is no ambiguity, we will write $A^*(I)$ or simply A^* for $A^*(I, \tau)$. Also $cl^*(A) = A \cup A^*$ defines a Kuratowski closure operator [5] for topology $\tau^*(I)$ (or simply τ^* which is finer than τ .An ideal I on a space (X, τ) is said to be condense ideal if and only if $\tau \cap I = \{\phi\}$. X^* is always a proper subset of X. Also $X = X^*$ if and only if the ideal is condense.

Let (X, τ) be a topological space and I be an ideal on X, then $(X, \tau, \tau^\delta, I)$ is called an ideal delta space[7]. Then for any set A of X, $A^{*\delta}(I, \tau^\delta) = \{x \in X / A \cap U_x \notin I,$ for every δ - set U_x containing $x\}$. This is simply called δ - local function and simply denoted as $A^{*\delta}$. Also $cl^{*\delta}(A) = A \cup A^{*\delta}$ defines a Kuratowski closure operator [7] for a topology $\tau^{*\delta}(I)$ (or simply $\tau^{*\delta}$) which is finer than τ^δ.

Lemma 1. (see [7]) Let $(X, \tau, \tau^\delta, I)$ be an ideal delta space, and let A, B be subsets of X. Then

(i) $\phi^{*\delta} = \phi$

(ii) $A \subset B \Rightarrow A^{*\delta} \subset B^{*\delta}$

(iii) $I_1 \subseteq I_2 \Rightarrow A^{*\delta}(I_1) \subseteq A^{*\delta}(I_2)$

(iv) $A^{*\delta} \subseteq A^*$

(v) $A^{*\delta} \cup B^{*\delta} = (A \cup B)^{*\delta}$

(vi) for every $J \in I, (A \cup J)^{*\delta} = A^{*\delta} = (A - J)^{*\delta}$

A perfect set in topological space is a set without isolated points (dense in itself) and closed. Hayashi introduced $*$ - perfect sets [9] in ideal topological spaces. Later, Manoharan[8] introduced R^* - perfect sets. Muhammad Shabir [10] introduced and studied soft topological spaces. Rodyna A.Hosny [11] extended the idea of perfect sets to soft ideal topological spaces. In this Chapter, we introduce, $L^{*\delta}$ - perfect sets, $R^{*\delta}$ - perfect sets and $C^{*\delta}$ - perfect sets, study their properties.

Definition 2. A subset A of an ideal space (X,τ,I) is said to be

(i) τ^* - closed [5] if $A^* \subseteq A$

(ii) $*$ - dense in itself [9] if $A \subseteq A^*$

(iii) $*$ - perfect [9] if $A = A^*$

Definition 3. (See [8]) Let (X,τ,I) be an ideal topological space. A subset A of X is said to be

(i) L^* - perfect if $A - A^* \in I$

(ii) R^* - perfect if $A^* - A \in I$

(iii) C^* - perfect if A is both L^* - perfect and R^* - perfect

The collection of L^* - perfect sets, R^* - perfect sets and C^* - perfect sets in (X,τ,I) is denoted by **L**, **R** and **C** respectively.

2. $L^{*\delta}$ - perfect sets, $R^{*\delta}$ - perfect sets and $C^{*\delta}$ - perfect sets

In this section, we define three collections of subsets $\mathbf{L}_\delta$, $\mathbf{R}_\delta$ and $\mathbf{C}_\delta$ in an ideal delta space and study some their properties.

Definition 4. A subset A of an ideal delta space (X,τ,τ^δ,I) is said to be

(i) $\tau^{*\delta}$ - closed if $A^{*\delta} \subseteq A$

(ii) $*\delta$ - dense in itself if $A \subseteq A^{*\delta}$

(iii) $*\delta$ - perfect if $A = A^{*\delta}$

Definition 5. Let (X,τ,τ^δ,I) be an ideal delta space. A subset A of X is said to be

(i) $L^{*\delta}$ - perfect if $A - A^{*\delta} \in I$

(ii) $R^{*\delta}$ - perfect if $A^{*\delta} - A \in I$

(iii) $C^{*\delta}$ - perfect if A is both $L^{*\delta}$ - perfect and $R^{*\delta}$ - perfect

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The collection of $L^{*\delta}$ - perfect sets, $R^{*\delta}$ - perfect sets and $C^{*\delta}$ - perfect sets in (X,τ,τ^δ,I) is denoted by $\mathbf{L}_\delta$, $\mathbf{R}_\delta$ and $\mathbf{C}_\delta$ respectively.

Remark 6. If $I=\{\phi\}$, then

(i) $\mathbf{L}_\delta=\{A\subseteq X: A-A^{*\delta}\in I=\{\phi\}\}$

$=\{A\subseteq X: A-A^{*\delta}=\phi\}$

$=\{A\subseteq X: A\subseteq A^{*\delta}\}$

= The collection of $*\delta$- dense in itself

(ii) $\mathbf{R}_\delta=\{A\subseteq X: A^{*\delta}-A\in I=\{\phi\}\}$

$=\{A\subseteq X: A^{*\delta}-A=\phi\}$

$=\{A\subseteq X: A^{*\delta}\subseteq A\}$

= The collection of all $\tau^{*\delta}$ - closed sets.

(iii) $\mathbf{C}_\delta=\{A\subseteq X: A-A^{*\delta}\in I$ and $A^{*\delta}-A\in I\}$

$=\{A\subseteq X: A\subseteq A^{*\delta}$ and $A^{*\delta}\subseteq A\}$

$=\{A\subseteq X: A=A^{*\delta}\}$

= The collection of $*\delta$- perfect sets.

Remark 7. If $I=\mathcal{P}(X)$, then $A^{*\delta}=\{x\in X: A\cap U_x\notin \mathcal{P}(X)$ for every δ- set U_x containing $x\}=\phi$. Which implies $\mathbf{L}_\delta=\{A\subseteq X: A-A^{*\delta}\in I=\mathcal{P}(X)\}=\{A\subseteq X: A\in \mathcal{P}(X)\}=\mathcal{P}(X)$.

The following Theorem 8 shows that the relation between $*\delta$- perfect set and $C^{*\delta}$- perfect set.

Theorem 8. In an ideal delta space (X,τ,τ^δ,I), every $*\delta$- perfect set is $C^{*\delta}$- perfect.

Proof. Let A be an $*\delta$- perfect set. Then $A=A^{*\delta}$, which implies $A-A^{*\delta}=A^{*\delta}-A=\phi\in I$. Therefore A is both $R^{*\delta}$- perfect and $L^{*\delta}$- perfect and hence A is $C^{*\delta}$- perfect.

The following Example 9 shows that the converse of the above Theorem 8 is not true.

Example 9. Let (X,τ,τ^{δ},I) be an ideal delta space with $X=\{a,b,c,d\}$, $\tau=\{\phi,\{c\},\{a,b\},\{a,b,c\},X\}$, $I=\{\phi,\{a\}\}$. Then $\tau^{\delta}=\{\phi,\{c\},\{a,b\},\{c,d\},\{a,b,c\},\{a,b,d\},X\}$. The set $\{a\}$ is both $R^{*\delta}$ - perfect and $L^{*\delta}$ - perfect and hence $C^{*\delta}$ - perfect but not a $*\delta$ - perfect set.

The following Theorem 10 gives the relation between $\tau^{*\delta}$ - closed set and $R^{*\delta}$ - perfect.

Theorem 10. In an ideal delta space (X,τ,τ^{δ},I), every $\tau^{*\delta}$ - closed set is $R^{*\delta}$ - perfect.

Proof. Let A be a $\tau^{*\delta}$ - closed set. Then $A^{*\delta}\subseteq A$. Therefore $A^{*\delta}-A=\phi\in I$ and hence A is an $R^{*\delta}$ - perfect.

Corollary 11. In an ideal delta space (X,τ,τ^{δ},I) ,

(i) X and ϕ are $R^{*\delta}$ - perfect

(ii) every τ - closed set is $R^{*\delta}$ - perfect

(iii) every τ^{*} - closed set is $R^{*\delta}$ - perfect

(iv) every τ^{δ} - closed set is $R^{*\delta}$ - perfect

Proof. The proof follows from Theorem 10.

The following Example shows that the converses of Theorem 10 and Corollary 11 are not true.

Example 11. Let (X,τ,τ^{δ},I) be an ideal delta space with $X=\{a,b,c,d\}$, $\tau=\{\phi,\{c\},\{a,b\},\{a,b,c\},X\}$, $I=\{\phi,\{a\}\}$. Then $\tau^{\delta}=\{\phi,\{c\},\{a,b\},\{c,d\},\{a,b,c\},\{a,b,d\},X\}$. The set $\{b,c,d\}$ is $R^{*\delta}$ - perfect but not τ - closed, τ^{*} - closed, τ^{δ} - closed and $\tau^{*\delta}$ - closed

Theorem 12. If a subset A of an ideal delta space (X,τ,τ^{δ},I) is $C^{*\delta}$ - perfect, then $A\Delta A^{*\delta}\in I$.

Proof. Since A is both $L^{*\delta}$ - perfect and $R^{*\delta}$ - perfect, $A - A^{*\delta} \in I$ and $A^{*\delta} - A \in I$ .By finite additive property of ideals, $(A - A^{*\delta}) \cup (A^{*\delta} - A) \in I$, which implies $A \Delta A^{*\delta} \in I$.

The following Theorem 13 shows that every element of an ideal is $C^{*\delta}$ - perfect.

Theorem 13. If a subset A of an ideal delta space $(X, \tau, \tau^{\delta}, I)$ is such that $A \in I$, then A is $C^{*\delta}$ - perfect.

Proof. Since $A \in I$, $A^{*} = \phi$ and $A^{*\delta} \subseteq A^{*}$,which implies $A^{*\delta} = \phi$, $A - A^{*\delta} = A \in I$ and $A^{*\delta} - A = \phi \in I$. Then A is both $L^{*\delta}$ - perfect and $R^{*\delta}$ - perfect. Therefore A is $C^{*\delta}$ - perfect.

Corollary 14. Let A be an subset of an ideal delta space $(X, \tau, \tau^{\delta}, I)$. Consider the following

(i) If $A \in I$, then every subset of A is $C^{*\delta}$ - perfect.

(ii) If A is $R^{*\delta}$ - perfect, then $A^{*\delta} - A$ is $C^{*\delta}$ - perfect.

(iii) If A is $L^{*\delta}$ - perfect, then $A - A^{*\delta}$ is $C^{*\delta}$ - perfect.

(iv) A is $C^{*\delta}$ - perfect, then $A \Delta A^{*\delta}$ is $C^{*\delta}$ - perfect.

Proof. The Proof follows from Theorem 13.

Theorem 15. In an ideal delta space $(X, \tau, \tau^{\delta}, I)$, every $*\delta$ - dense in itself set is an $L^{*\delta}$ - perfect set.

Proof. Let A be a $*\delta$ - dense in itself set of X . Then $A \subseteq A^{*\delta}$. Therefore $A - A^{*\delta} = \phi \in I$, which implies A is an $L^{*\delta}$ - perfect set.

The following Remark 16 shows that the converse of Theorem 15 need not be true.

Remark 16. The members of an ideal delta space are $L^{*\delta}$ - perfect, but the non-empty members of the ideal are not $*\delta$ - dense in itself. Therefore the converse of Theorem 15 need not be true.

Theorem 17. In an ideal delta space (X,τ,τ^{δ},I) ,

(i) empty set is an $L^{*\delta}$ - perfectset

(ii) X is an $L^{*\delta}$ - perfect set if the ideal is condense.

Proof.(i) Since $\phi-\phi^{*}=\phi\in I$, the empty set is an $L^{*\delta}$ - perfect. (ii) We know that $X=X^{*\delta}$ if and only if the ideal I is condense. Then $X-X^{*\delta}=\phi\in I$, which implies X is an $L^{*\delta}$ - perfect set

3. Main Results

In this section, we prove that finite union and intersection of $R^{*\delta}$ - perfect sets are again $R^{*\delta}$ - perfect set. Using these results, we obtain a new topology for the finite topological space which is finer than $\tau^{*\delta}$ - topology.

In ideal delta spaces, usually $A\subseteq B$ implies $A^{*\delta}\subseteq B^{*\delta}$. We observe that there are some sets A and B such that $A\subseteq B$ but $A^{*\delta}=B^{*\delta}$

Example 18. Let (X,τ,τ^{δ},I) be an ideal delta space with $X=\{a,b,c,d\}$, $\tau=\{\phi,\{c\},\{a,b\},\{a,b,c\},X\}$, $I=\{\phi,\{a\},\{c\},\{d\},\{a,c\},\{a,d\},\{c,d\},\{a,c,d\}\}$. Then $\tau^{\delta}=\{\phi,\{c\},\{a,b\},\{c,d\},\{a,b,c\},\{a,b,d\},X\}$. Here $A=\{b,d\}$ and $B=\{b,c,d\}$ are such that $A\subseteq B$ but $A^{*\delta}=B^{*\delta}=\{a,b,d\}$

Theorem 19. . Let (X,τ,τ^{δ},I) be an ideal delta space. Let A and B be two subsets of X such that $A\subseteq B$ and $A^{*\delta}=B^{*\delta}$, then

(i) B is $R^{*\delta}$ - perfect if A is $R^{*\delta}$ - perfect

(ii) A is $L^{*\delta}$ - perfect if B is $L^{*\delta}$ - perfect

Proof. (i) Let A be an $R^{*\delta}$ - perfect set. Then $A-A^{*\delta}\in I$ .Now $B^{*\delta}-B=A^{*\delta}-B\subseteq A^{*\delta}-A$. By heredity property of ideals, $B^{*\delta}-B\in I$. Hence B is $R^{*\delta}$ - perfect.

(ii) Let B be an $L^{*\delta}$ - perfect set. Then $B - B^{*\delta} \in I$ .Now $A - A^{*\delta} = A - B^{*\delta} \subseteq B - B^{*\delta}$. By heredity property of ideals, $A^{*\delta} - A \in I$. Hence A is $L^{*\delta}$ - perfect.

Corollary 20. Let $(X, \tau, \tau^{\delta}, I)$ be an ideal delta space. Let A and B be two subsets of X such that $A \subseteq B \subseteq cl^{*\delta}(A)$, then

(i) B is $R^{*\delta}$ - perfect if A is $R^{*\delta}$ - perfect

(ii) A is $L^{*\delta}$ - perfect if B is $L^{*\delta}$ - perfect

Proof. Since $A \subseteq B \subseteq cl^{*\delta}(A)$, $A^{*\delta} \subseteq B^{*\delta} \subseteq (cl^{*\delta}(A))^{*\delta} = (A \cup A^{*\delta})^{*\delta} = A^{*\delta} \cup (A^{*\delta})^{*\delta} = A^{*\delta}$. Hence $A^{*\delta} = B^{*\delta}$. Therefore the result follows from Theorem 19.

Theorem 21. Let A be an subset of an ideal delta space $(X, \tau, \tau^{\delta}, I)$ such that A is $L^{*\delta}$ - perfect and $A \cap A^{*\delta}$ is $R^{*\delta}$ - perfect, then both A and $A \cap A^{*\delta}$ are $C^{*\delta}$ - perfect.

Proof. Since A is $L^{*\delta}$ - perfect, $A - A^{*\delta} \in I$,by lemma1.(vi) for every $J \in I, (A \cup J)^{*\delta} = A^{*\delta} = (A - J)^{*\delta}$. Therefore $A \cup (A - A^{*\delta}) = A^{*\delta} = A - (A - A^{*\delta})$, which implies $A^{*\delta} = (A \cap A^{*\delta})^{*\delta}$. Therefore $A \cap A^{*\delta} \subseteq A$ with $(A \cap A^{*\delta})^{*\delta} = A^{*\delta}$, By Theorem 19 A is $R^{*\delta}$ - perfect if $A \cap A^{*\delta}$ is $R^{*\delta}$ - perfect

and $A \cap A^{*\delta}$ is $L^{*\delta}$ - perfect if A is $L^{*\delta}$ - perfect. Hence A is $R^{*\delta}$ - perfect and $A \cap A^{*\delta}$ is $L^{*\delta}$ - perfect. Therefore A and $A \cap A^{*\delta}$ are $C^{*\delta}$ - perfect.

Theorem 22. If a subset A of an ideal delta space $(X, \tau, \tau^{\delta}, I)$ is $R^{*\delta}$ - perfect and $A^{*\delta}$ is $L^{*\delta}$ - perfect, then $A \cap A^{*\delta}$ is $L^{*\delta}$ - perfect.

Proof. Since A is $R^{*\delta}$ - perfect, $A^{*\delta} - A \in I$,by lemma1.(vi) for every $J \in I, (A \cup J)^{*\delta} = A^{*\delta} = (A - J)^{*\delta}$. Therefore $(A^{*\delta} \cup (A^{*\delta} - A)^{*\delta} = (A^{*\delta})^{*\delta} = (A^{*\delta} - (A^{*\delta} - A))^{*\delta}$,this implies $(A^{*\delta})^{*\delta} = (A \cap A^{*\delta})^{*\delta}$. Therefore, we have $A \cap A^{*\delta} \subseteq A$ with $(A \cap A^{*\delta})^{*\delta} = A^{*\delta}$. By Theorem 19 (ii), $A \cap A^{*\delta}$ is $L^{*\delta}$ - perfect if $A^{*\delta}$ is $L^{*\delta}$ - perfect. Hence $A \cap A^{*\delta}$ is $L^{*\delta}$ - perfect.

Theorem 23. If A and B are $R^{*\delta}$ - perfect sets, then $A\cup B$ is an $R^{*\delta}$ - perfect set.

Proof. Let A and B be $R^{*\delta}$ - perfect sets. Then $A^{*\delta}-A\in I$ and $B^{*\delta}-B\in I$. By finite additive property of ideals $(A^{*\delta}-A)\cup(B^{*\delta}-B)\in I$. Since $(A^{*\delta}\cup B^{*\delta})-(A\cup B)\subseteq(A^{*\delta}-A)\cup(B^{*\delta}-B)$, by heredity property of ideal $(A^{*\delta}\cup B^{*\delta})-(A\cup B)\in I$. Hence $(A\cup B)^{*\delta}-(A\cup B)\in I$. Therefore $A\cup B$ is an $R^{*\delta}$ - perfect set.

Corollary 24. Finite union of $R^{*\delta}$ - perfect sets is an $R^{*\delta}$ - perfect set.

Proof. The proof follows from Theorem 23.

Theorem 25. If A and B are $L^{*\delta}$ - perfect sets, then $A\cup B$ is an $L^{*\delta}$ - perfect set.

Proof. Since A and B are $L^{*\delta}$ - perfect sets, $A-A^{*\delta}\in I$ and $B-B^{*\delta}\in I$. Hence by finite additive property of ideals $(A-A^{*\delta})\cup(B-B^{*\delta})\in I$. Since $(A\cup B)-(A^{*\delta}\cup B^{*\delta})\subseteq(A-A^{*\delta})\cup(B-B^{*\delta})$, by heredity property of ideal $(A\cup B)-(A^{*\delta}\cup B^{*\delta})\in I$. This proves $A\cup B$ is an $L^{*\delta}$ - perfect set.

Corollary 26. Finite union of $L^{*\delta}$ - perfect sets is an $L^{*\delta}$ - perfect set.

Proof . The proof follows from Theorem 25.

Theorem 27. If A and B are $R^{*\delta}$ - perfect sets, then $A\cap B$ is an $R^{*\delta}$ - perfect set.

Proof. Suppose that A and B are $R^{*\delta}$ - perfect sets. Then $A-A^{*\delta}\in I$ and $B-B^{*\delta}\in I$. By finite additive property of ideals $(A^{*\delta}-A)\cup(B^{*\delta}-B)\in I$ .Since $(A^{*\delta}\cap B^{*\delta})-(A\cap B)\subseteq(A^{*\delta}-A)\cup(B^{*\delta}-B)$, by heredity property $(A^{*\delta}\cap B^{*\delta})-(A\cap B)\in I$. Also $(A\cap B)^{*\delta}-(A\cap B)\subseteq(A^{*\delta}\cap B^{*\delta})-(A\cap B)\in I$. This proves the result.

Corollary 28. Finite intersection of $R^{*\delta}$ - perfect sets is an $R^{*\delta}$ - perfect set.

Proof. The proof follows from Theorem 27.

Theorem 29. Finite union of $C^{*\delta}$ - perfect sets is a $C^{*\delta}$ - perfect set.

Proof. From Corollaries 24 and 26, finite union of $C^{*\delta}$ - perfect sets is a $C^{*\delta}$ - perfect set.

Theorem 30. If (X,τ,τ^{δ},I) is an ideal delta space with X being finite, then the collection of $R^{*\delta}$ sets forms a topology which is finer than the topology of $\tau^{*\delta}$ - closed sets.

Proof. By corollary 11, X and ϕ are $R^{*\delta}$ - perfects. By corollary 24. Finite union of $R^{*\delta}$ - perfect sets is an $R^{*\delta}$ - perfect set and by corollary 28 finite intersection of $R^{*\delta}$ - perfect sets is $R^{*\delta}$ - perfect set. Hence the collection $\mathbf{R}_{\delta}$ is a topology if X is finite. Also by Theorem 10, every $\tau^{*\delta}$ - closed set is $R^{*\delta}$ - perfect. Hence the topology $\mathbf{R}_{\delta}$ is finer than the topology of $\tau^{*\delta}$ - closed sets.

Theorem 31. In an ideal delta space (X,τ,τ^{δ},I) ,($\tau^{*\delta}$ - closed set) $\cup I \subseteq \mathbf{R}_{\delta}$.

Proof. The proof follows from Theorem 10 and 13.

The following Example 32 shows that ($\tau^{*\delta}$ - closed set) $\cup I \neq \mathbf{R}_{\delta}$.

Example 32. Let (X,τ,τ^{δ},I) be an ideal delta space with $X=\{a,b,c,d\}$, $\tau=\{\phi,\{c\},\{a,b\},\{a,b,c\},X\}$, $I=\{\phi,\{a\}\}$. Then $\tau^{*\delta}$ - closed set = $\{\phi,\{a\},\{d\},\{a,b\},\{a,d\},\{c,d\},\{a,b,d\},\{a,c,d\},X\}$ and $\mathbf{R}_{\delta}$ = $\{\phi,\{a\},\{b\},\{d\},\{a,b\},\{b,d\},\{c,d\},\{a,b,d\},\{a,c,d\},\{b,c,d\},X\}$. Hence clearly ($\tau^{*\delta}$ - closed set) $\cup I \neq \mathbf{R}_{\delta}$.

Theorem 33. Let (X,τ,τ^{δ},I) be an ideal delta space and $A \subseteq X$. Then the set A is $R^{*\delta}$ - perfect if and only if $F \subseteq A^{*\delta} - A$ implies that $F \in I$.

Proof. Assume that A is an $R^{*\delta}$ - perfect set. Then $A^{*\delta} - A \in I$. By heredity property of ideals, every set $F \subseteq A^{*\delta} - A$ in X is also in I . Conversely assume that $F \subseteq A^{*\delta} - A$ in X implies that $F \in I$. Since $A^{*\delta} - A$ is a subset of itself, by assumption $A^{*\delta} - A \in I$. Hence A is $R^{*\delta}$ - perfect.

Theorem 34. Let (X, τ) be a topological space and $A \subseteq X$. Let I_1 and I_2 be two ideals in X with $I_1 \subseteq I_2$. Then A is $R^{*\delta}$ - perfect with respect to I_2 if it is $R^{*\delta}$ - perfect with respect to I_1 .

Proof. Since $I_1 \subseteq I_2$, $A^{*\delta}(I_2) \subseteq A^{*\delta}(I_1)$, by lemma 1 (iii). Let A be $R^{*\delta}$ - perfect with respect to I_1 . Then $A^{*\delta}(I_1) - A \in I_1$. Also $A^{*\delta}(I_2) - A \subseteq A^{*\delta}(I_1) - A$. Hence by heredity property of ideals $A^{*\delta}(I_1) - A \in I_1 \subseteq I_2$. Therefore A is $R^{*\delta}$ - perfect with respect to I_2 .

4. $R^{*\delta}$ - topology

By corollary 10 and Theorem 27, we observe that the collection $\mathbf{R}_\delta$ satisfies the conditions of being a basis for some topology and it will be called as $R^{*\delta}(\tau^\delta, I)$. We define $R^{*\delta}(\tau^\delta, I) = \quad A \subseteq X / X - A \in R^{*\delta}(\tau^\delta, I)\}$ on a non empty set X . Clearly $R^{*\delta}(\tau^\delta, I)$ is a topology if X is finite. The members of the collection $R^{*\delta}(\tau^\delta, I)$ will be called $R^{*\delta}$ - open sets. If there is no confusion about the topology τ^δ and ideal I , then we call $R^{*\delta}(\tau^\delta, I)$ as $R^{*\delta}$ - topology when X is finite.

Definition 35. A subset A of an ideal delta topological space $(X, \tau, \tau^\delta, I)$ is said to be $R^{*\delta}$ - closed if it is a complement of an $R^{*\delta}$ - open set.

Definition 36. Let A be a subset of an ideal delta space $(X, \tau, \tau^\delta, I)$. One defines $R^{*\delta}$ - interior of the set A as the largest $R^{*\delta}$ - open set contained in A . One will denote $R^{*\delta}$ - interior of the set A by $R^{*\delta}$ - $\text{int}(A)$.

Definition 37. Let A be a subset of an ideal delta space $(X, \tau, \tau^\delta, I)$. A point $x \in A$ is said to be an $R^{*\delta}$ -interior point of A if there exist an $R^{*\delta}$ - open set U of x such that $x \in U \subseteq A$.

Definition 38. Let $(X, \tau, \tau^{\delta}, I)$ be an ideal delta space and $x \in X$. One defines $R^{*\delta}$ - neighborhood of x as an $R^{*\delta}$ - open set containing x. One denotes the set of all $R^{*\delta}$ - neighborhood of x by $R^{*\delta}$ - $N(x)$.

Theorem 39. In an ideal delta space $(X, \tau, \tau^{\delta}, I)$, every $\tau^{*\delta}$ - open set is $R^{*\delta}$ - open set.

Proof. Let A be a $\tau^{*\delta}$ - open set. Therefore $X - A$ is $\tau^{*\delta}$ - closed set. This implies $X - A$ is an $R^{*\delta}$ - closed set. Hence A is an $R^{*\delta}$ - open set.

Corollary 40. The topology $R^{*\delta}(\tau^{\delta}, I)$ on a finite set X is finer than the topology $\tau^{*\delta}(\tau, I)$.

Proof. The proof follows from theorem 39.

Corollary 41. For any subset A of an ideal delta space $(X, \tau, \tau^{\delta}, I)$, $\text{int}_{\delta}(A)$ is an $R^{*\delta}$ - open set.

Proof. The proof follows from Theorem 39.

Remark 42. (i) Since every δ - open set is an $R^{*\delta}$ - open set, every δ - neighborhood U of a point $x \in X$ is an $R^{*\delta}$ neighborhood of x.

(ii) If $x \in X$ is an δ - interior point of a subset A of X, then x is an $R^{*\delta}$ -interior point of A.

(iii) From (ii), we have $\text{int}_{\delta}(A) \subseteq \text{int}^{*\delta}(A) \subseteq R^{*\delta}$ - $\text{int}(A)$, where $\text{int}^{*\delta}$ denotes interior of A with respect to the topology $\tau^{*\delta}$.

Theorem 43. Let A and B be subsets of an ideal delta space $(X, \tau, \tau^{\delta}, I)$ with X being finite. Then the following properties hold.

(i) $R^{*\delta}$ - $\text{int}(A) = \cup\{U / U \subseteq X$ and U is an $R^{*\delta}$ - open set$\}$.

(ii) $R^{*\delta}$ - $\text{int}(A)$ is the largest $R^{*\delta}$ - open set of X contained in A.

(iii) A is $R^{*\delta}$ - open if and only if $A = R^{*\delta}$ - $\text{int}(A)$.

(iv) $R^{*\delta}$- $\text{int}(R^{*\delta}\text{-}\text{int}(A)) = R^{*\delta}$- $\text{int}(A)$.

(v) If $A \subseteq B$, then $R^{*\delta}$- $\text{int}(A) \subseteq R^{*\delta}$- $\text{int}(B)$.

Proof. The proof follows from Definition 35,36 and 37.

Definition 44. Let A be a subset of an ideal delta space $(X, \tau, \tau^{\delta}, I)$. One defines $R^{*\delta}$-closure of the set A as the smallest $R^{*\delta}$-closed set containing A. One will denote $R^{*\delta}$-closure of a set A by $R^{*\delta}\text{-}cl(A)$.

Remark 45. For any subset A of an ideal delta space $(X, \tau, \tau^{\delta}, I)$, $R^{*\delta}\text{-}cl(A) \subseteq cl^{*\delta}(A) \subseteq cl_{\delta}(A)$.

Theorem 46. Let A and B be subsets of an ideal delta space $(X, \tau, \tau^{\delta}, I)$ with X being finite. Then the following properties hold.

(i) $R^{*\delta}\text{-}cl(A) = \cap\{F / A \subseteq F$ and F is $R^{*\delta}$-closed set$\}$.

(ii) A is $R^{*\delta}$-closed if and only if $A = R^{*\delta}\text{-}cl(A)$.

(iii) $R^{*\delta}\text{-}cl(R^{*\delta}\text{-}cl(A)) = R^{*\delta}\text{-}cl(A)$.

(iv) If $A \subseteq B$, then $R^{*\delta}\text{-}cl(A) \subseteq R^{*\delta}\text{-}cl(B)$.

Proof. The proof follows from Definition 44.

Theorem 47. Let A be a subset of an ideal delta space $(X, \tau, \tau^{\delta}, I)$. Then the following properties hold.

(i) $R^{*\delta}$- $\text{int}(X - A) = X - R^{*\delta}\text{-}cl(A)$.

(ii) $R^{*\delta}$- $cl(X - A) = X - R^{*\delta}\text{-}\text{int}(A)$.

Proof. The proof follows from Definitions 35,36 and 44.

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