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A Study on Some Curvature Properties of Almost C(λ) Manifold

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Abstract

The plan of the present paper is to study some curvature properties of almost c(λ) manifolds. We shall consider Einstein semi symmetric almost c(λ) manifolds and such manifolds satisfying E.S =0 , S.E =0 , E.R =0 , where E is the Einstein tensor , R is the Riemannian curvature tensor , S is the Ricci tensor of the manifold .we shall also consider ϕ- Ricci symmetric almost c(λ) manifolds.

Keywords and Phrases: Almost contact manifolds, almost c(λ) manifolds , Einstein tensor , ϕ Ricci symmetry, Riemannian curvature tensor.

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SECTION -1

INTRODUCTION

The notion of almost c(λ) manifolds was first given by Janssen and Vanhecke [6] . Again in the paper [7] conformally flat almost c(λ) manifolds have been studied. Recently in the papers [1] , [2] , [3] , A. Akbar has studied some curvature properties of almost c(λ) manifolds .The notion of Einstein tensor has been introduced to study curvature properties in the paper [3] . ϕ- Ricci symmetric sasakian manifolds have been studied in the paper [5].

In this paper we would like to study, some curvature properties of almost c(λ) manifolds . The present paper is organised as follows : we give some preliminary formulae in section- 2. In section- 3 we study Einstein semisymmetric almost c(λ) manifolds. Section 4 contains the study of almost c(λ) manifolds satisfying E.R = 0 , where E is the Einstein tensor , R is the Riemannian curvature tensor , S is the Ricci curvature tensor of the manifolds . The last section contains the study of ϕ- Ricci symmetric almost c(λ) manifolds.

SECTION- 2

Preliminaries : An odd dimensional differentiable manifold is called almost contact manifolds if th ere exist a 1-1 tensor η , a vetor field X_i and a Riemannian metric g such that [4] .

$\phi^2 X = X + \eta(X)\xi$, $\eta(\xi) = 1$ --(2.1)

$g(\phi X, \phi Y) = g(X,Y) - \eta(X)\,\eta(Y)$ --(2.2)

$\phi(\xi) = 0$, $\eta(\phi(X)) = 0$ -------------------------------(2.3)

Here X, Y are differentiable vector fields defined on the manifolds . An almost contact manifolds is called an almost c(λ) manifolds if its curvature tensor R is given by [7]

$R(X, Y)Z = R(\phi X, \phi y)z - \lambda[Xg(X, Y) - g(X, Y)Y - \phi Xg(X, Y) + g(X, Y)\phi Y]$------(2.4)

From above equation we also have

$R(X,Y)\xi = R(\phi X, \phi Y)\xi - \lambda[\eta(Y)X - \eta(X)Y]$ -------------------(2.5)

$R(\xi, Y)Z = -\lambda[g(Y,Z)\xi - \eta(Z)Y]$---------------------------(2.6)

$R(\xi,Y)\xi = -\lambda[\eta(Y)\xi - Y]$ ---------------------------------------(2.7)

$R(\xi,\xi)Z = 0$--(2.8)

The Ricci tensor of almost c(λ) manifolds was deduced in the paper [1]. The Ricci tensor is given below

$S(X,Y) = -\lambda[(2n-1)g(X,Y) + \eta(X)\eta(Y)]$-------------------------------------(2.9)

Here the dimension of the manifold is considered 2n+1 from above we get the Ricci operator Q as follows

$QX = \lambda(1-2n)X - \lambda\eta(X)\xi$---------------------------------------(2.10)

The Einstein tensor of a manifold is defined as

$E(X,Y) = S(X,Y) - r/2g(X,Y)$--------------------------------------(2.11)

Where S is the Ricci curvature tensor and r is the scaler curvature tensor of the manifold.

SECTION- 3

Definition 3.1

An almost c(λ) manifold will be called Einstein semisymmetric if it satisfies

$R(X,Y)Z .E(U,V) = 0.$ --(3.1)

Let us consider an almost c(λ) manifolds is Einstein semisymmetric then

$R(X,Y)Z.E(U,V) = 0$.

Now from (2.11) $E(U,V) = S(U,V) – r/2g(U,V)$. Again using we have $E(U,V) = -\lambda [(2n-1)g(U,V) +\eta(U)\eta(V)] - r/2g(U,V)$. ---------------------------(3.2) Now $R(X,Y)Z.E(U,V) =0$, means

$E(R(X,Y)Z,U) +E(Z,R(X,Y)U) = 0.$

Using (2.11) in the above equation we get $S(R(X,Y)Z,U) – r/2\, g(R(X,Y)Z,U) + S(Z,R(X,Y)U) – r/2g(Z,R(X,Y)U) = 0$.

Putting $Z= \xi$ in the above equation we get $S(R(X,Y)\xi,U) -r/2g(R(X,Y)\xi,U) +S (\xi, R(X,Y)U) -r/2g(\xi,R(X,Y)U) = 0$.

Using(2.5) and (2.9) in the above equation we get ,

$S(R(\phi X, \phi Y)\xi -\lambda[\eta(Y)X – \eta(X)Y] , U) – r/2g(R(\phi X, \phi Y)\xi -\lambda[\eta(Y)X -\eta(X)Y],U) - \lambda[(2n-1)g(\xi,R(X,Y)U) +\eta(\xi)\eta(R(X,Y)U)] - r/2\, \eta(R (X,Y)U) = 0,$

$S(R(\phi X, \phi Y)\xi -\lambda[\eta(Y)X – \eta(X)Y] , U) – r/2g(R(\phi X, \phi Y)\xi,U)+\lambda r/2\, \eta(Y)g (X,U)- \lambda r/2\eta(X)g(Y,U)-\lambda[(2n-1)\eta(R(X,Y)U)+\eta((R(X,Y)U)]-r/2\, \eta(R(X,Y)U=0,$

$S(R(\phi X, \phi Y)\xi -\lambda[\eta(Y)X – \eta(X)Y] , U) – r/2g(R(\phi X, \phi Y)\xi,U)+ \lambda r/2\, \eta(Y)g (X,U)- \lambda r/2\eta(X)g(Y,U)-2n\lambda\, \eta(R(X,Y)U)- r/2\, \eta(R(X,Y)U=0.$

Putting $X=\xi$,We have

$S(-\lambda[\eta(Y)\xi-Y],U)+ \lambda r/2\eta(Y)\eta(U)- \lambda r//2)g(Y,U)- 2n\lambda\, \eta(R(\xi,Y)U)-r/2\, \eta(R(\xi,Y)U=0$

$-\lambda S(\phi^2Y,U)-\lambda r/2[g(Y,U)- \eta(Y)\eta(U)]-(2n\lambda+r/2)\, \eta(R(\xi,Y)U)=0,$

$-\lambda S(\phi^2Y,U)-)-\lambda r/2[g(Y,U)- \eta(Y)\eta(U))]-(2n\lambda+r/2)\eta(-\lambda\{ g(Y,U)\xi-\eta(U)Y\})=0,$

$\lambda^2[(2n-1)g(\phi^2Y,U)+ \eta((\phi^2Y)\, \eta(U)])-\lambda r/2[g(Y,U)- \eta(Y)\eta(U)] -(2n\lambda+r/2)\eta(- \lambda\{ g(Y,U)\xi-\eta(U)Y\})=0,$

$\lambda^2(2n-1)g(\phi^2Y,U))-\lambda r/2[g(Y,U)- \eta(Y)\eta(U)] -(2n\lambda+r/2)\eta(-\lambda\{ g(Y,U)\xi-\eta(U)Y\})=0,$

$\lambda^2(2n-1)g(\phi^2Y,U))- \lambda r/2[g(Y,U)- \eta(Y)\eta(U)]-(2n\lambda+r/2) (-\lambda\{ g(Y,U)- \eta(Y)\eta(U\})]=0,$

$\lambda^2(2n-1)g(\phi^2Y,U)+2\, \lambda^2n[g(Y,U)- \eta(Y)\eta(U)]=0,$

$\lambda^2(2n-1)g(\phi^2Y,U)+ 2\lambda^2n\, g(\phi Y, \phi U)=0,$

$-\lambda^2(2n-1). g(\phi Y, \phi U)+ 2\lambda^2 n g(\phi Y, \phi U)=0,$

$\lambda^2 g(\phi Y, \phi U)=0$,this implies $\lambda=0$. As $g(\phi Y, \phi U)$ is not zero.

Hence we can state the following theorem.

Theorem 3.1

If an almost c(λ) manifolds is Einstein semisymmetric , then λ is necessarily zero.

But the converse may not be true always.

SECTION-4

In this section we like to study almost c(λ) manifold satisfying E.R = 0 where E is Einstein tensor and R is Riemannian curvature tensor.Let us consider an almost c(λ) , manifold satisfying E.R = 0. now E.R =0 means, $E(U,R(X,Y)Z)+E(R(X,Y),V)=0$. $S(U,R(X,Y)Z)-r/2g(U,R(X,Y)Z) +S(V,R(X,Y)Z)-r/2g(V,R(X,Y)Z)=0$.

Putting Z= ξ,We get, . $S(U,R(X,Y) \xi)-r/2g(U,R(X,Y) \xi) +S(V,R(X,Y) \xi)-r/2g(V,R(X,Y) \xi)=0,$

$-\lambda [(2n-1)g(U,R(X,Y)\xi) + \eta(U) \eta(R(X,Y)\xi)]-r/2 g(R(X,Y)\xi,U)- \lambda [(2n-1)g(R(X,Y)\xi,V) + \eta(V) \eta(R(X,Y)\xi)]-r/2 g(R(X,Y)\xi,V)=0,$

$-\lambda [(2n-1)g(U,R(\phi X, \phi Y)\xi])- \lambda[\eta(Y)X- \eta(X)Y)]+ \eta(U) \eta(R(\phi X, \phi Y)\xi)- \lambda[\eta(Y)X- \eta(X)Y)]) -r/2g(U,R(\phi X, \phi Y)\xi])- \lambda[\eta(Y)X- \eta(X)Y)])- \lambda [(2n-1)g(V,R(\phi X, \phi Y)\xi])- \lambda[\eta(Y)X- \eta(X)Y)]+ \eta(V) \eta(R(\phi X, \phi Y)\xi)- \lambda[\eta(Y)X- \eta(X)Y)])- r/2g(V,R(\phi X, \phi Y)\xi])- \lambda[\eta(Y)X- \eta(X)Y)])=0.$

Putting X= ξ,We have,

$-\lambda [(2n-1)g(U, -\lambda(\eta(Y)\xi-Y)+ \eta(U) \eta(-\lambda(\eta(Y)\xi-Y)]-r/2g(U, -\lambda(\eta(Y)\xi-Y)) -\lambda [(2n-1)g(V, -\lambda(\eta(Y)\xi-Y)+ \eta(V) \eta(-\lambda(\eta(Y)\xi-Y)]-r/2)g(V, -\lambda(\eta(Y)\xi-Y))=0,$

$\lambda^2(2n-1) g(\phi^2 Y,U))+r \lambda /2g(\phi^2 Y,U) +\lambda^2(2n-1)g(\phi^2 Y,V))+ r \lambda /2 g(\phi^2 Y,V)=0,$

$(2n \lambda^2 - \lambda^2+r \lambda /2)(g(\phi^2 Y,U)+ g(\phi^2 Y,V))=0$. Finally we get $\lambda=0$.

Thus we are in a situation to state the following ,

Theorem 4.1

If an almost c(λ) manifold satisfies E.R = 0 then λ is necessarily zero.

But the converse may not be true always.

SECTION -5

ϕ -Ricci symmetric almost c(λ) manifold:

The notion of ϕ- Ricci symmetric sasakian manifolds was introduced by U.C. De and A. Sarkar in the paper [5] following this paper in this section we study ϕ -Ricci symmetric almost c(λ) manifolds.

Definition 5.1

An almost c(λ) manifold will be called ϕ- Ricci symmetric if $\phi^2(\nabla_w Q)X = 0$.

The manifold will be called Ricci symmetric if the vector field W and X are orthogonal to ξ.

Let us consider an almost c(λ) manifold is ϕ-Ricci symmetric.

Now from (2.10) we get $QX = \lambda(1-2n)X - \lambda\eta(X)\xi$ now

$(\nabla_w Q)X = \nabla_w(QX) - Q(\nabla_w X)$

$= \nabla_w(\lambda(1-2n)X - \lambda\eta(X)\xi) - \lambda(1-2n)\nabla_w X + \lambda\eta(\nabla_w X)\xi$

$= (1-2n)(\lambda\nabla_w X + X\nabla_w \lambda) - (\nabla_w \lambda)\eta(X)\xi - \lambda((\nabla_w \eta(X))\xi$

$+ (\nabla_w \xi)\eta(X)) - \lambda(1-2n)\nabla_w X - \lambda\eta(\nabla_w X)\xi$

$= (1-2n)X\nabla_w \lambda - (\nabla_w \lambda)\eta(X)\xi - \lambda((\nabla_w \eta)X\xi + \eta(\nabla_w X)\xi + (\nabla_w \xi)\eta(X))$

$+ \lambda\eta(\nabla_w X)\xi$

$= (1-2n)X\nabla_w \lambda - (\nabla_w \lambda)\eta(X)\xi - \lambda((\nabla_w \eta)X\xi - \lambda(\nabla_w \xi)\eta(X)$

Since the manifold is locally ϕ - Ricci symmetric by definition

$(1-2n)X\nabla_w \lambda - (\nabla_w \lambda)\eta(X)\xi - \lambda((\nabla_w \eta)X\xi - \lambda(\nabla_w \xi)\eta(X) = 0,$

$\nabla_w \lambda(X - 2nX - \eta(X)\xi) - \lambda((\nabla_w \eta)X\xi + (\nabla_w \xi)\eta(X)) = 0.$

Replacing X by ϕ X

$(\nabla_w \lambda)(QX - 2n\,\phi X) - \lambda((\nabla_w \eta)(\phi X)\xi = 0.$

If λ is constant then λ must be zero. Thus we are in a position to state the following theorem.

Theorem 5.1

There exist no ϕ - Ricci symmetric almost c(λ) manifiold with λ as a non zero constant.

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