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Few more results on Geometric Mean Labeling of graphs

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Abstract

A Graph with p vertices and q edges is said to be Geometric Mean Graph if it is possible to label vertices $x \in V$ with distinct elements $f(x)$ from $1, 2, \ldots, q+1$ in such a way that when edge $e = uv$ is labelled with $f^*(uv) = \left\lceil \sqrt{(f(u)f(v)} \right\rceil$ or $f^*(uv) = \left\lfloor \sqrt{(f(u)f(v)} \right\rfloor$ then the resulting edge labels are distinct. ~~In this case~~ f is called Geometric Mean Labeling of G. In this paper, we prove the following graphs $A(T_n) \bigodot K_1$, $T(T_n) \bigodot K_1$, $Q_n \bigodot \bar{K}_2$, $D(Q_n) \bigodot \bar{K}_2$, $C_n \bigodot P_2$ are Geometric Mean Graphs.

AMS subject classification: 05C78.
Keywords: Geometric mean labeling, Geometric Mean graphs.

1. Introduction

All graphs considered here are simple, finite, connected and undirected. Given a graph G, the symbols $V(G)$ and $E(G)$ denote the vertex set and the edge set of the graph G respectively. Let $G(p, q)$ be a graph with $p = |V(G)|$ vertices and $q = |E(G)|$ edges. A graph labeling is an assignment of integers to the vertices or edges or both,subject to certain conditions. There are various types of graph labeling and a detailed survey is available in [1]. The concept of Geometric mean labeling was introduced by S. Somasundaram, P. Vidhyarani and R. Ponraj in [2]. We use the following definitions in the subsequent sections.

Definition 1.1. An Alternate Triangular snake $A(T_n)$ is obtained from a path $u_1, u_2, \ldots, u_n$ by joining u_i and u_{i+1} alternatively to a new vertex v_i for $1 \leq i \leq n-1$.

Definition 1.2. A Triple Triangular snake $T(T_n)$ consists of three triangular snakes that have a common path. That is, a Triple Triangular snake is obtained from a path $u_1, u_2, \ldots, u_n$ by joining u_i and u_{i+1} to a new vertex v_i for $i = 1, 2, \ldots, n-1$ and also to new vertex w_i for $i = 1, 2, \ldots, n-1$ and also to a new vertex z_i for $i = 1, 2, \ldots, n-1$.

Definition 1.3. A Quadrilateral snake Q_n is obtained from a path $u_1, u_2, \ldots, u_n$ by joining u_i and u_{i+1} to two new vertices v_i and w_i $1 \leq i \leq n-1$ respectively and then joining v_i and w_i.

Definition 1.4. A Double Quadrilateral snake $D(Q_n)$ consists of two quadrilateral snakes that have a common path.

Definition 1.5. The Corona $G = G_1 \bigodot G_2$ of two graphs G_1 and G_2 is defined as a graph obtained by taking one copy of G_1 (with p vertices) and p copies of G_2 and then joining the i^{th} vertex of G_1 to every vertex of i^{th} copy of G_2.

2. Results

Theorem 2.1. $A(T_n) \bigodot K_1$ is a geometric mean graph.

Proof. Let $G = A(T_n) \bigodot K_1$ be the graph. We consider the following two cases.

Case 1: n is even
Subcase 1.1: The first triangle starts from the first vertex of the path
Let,

$$V(G) = \{u_i, v_i, w_i : 1 \leq i \leq n\}$$

$$E(G) = \{u_i w_i : 1 \leq i \leq n\} \cup \{u_i u_{i+1} : 1 \leq i \leq n-1\}$$

$$\cup \left\{u_{2i-1} v_{2i-1} : 1 \leq i \leq \frac{n}{2}\right\} \cup \left\{u_{2i} v_{2i-1} : 1 \leq i \leq \frac{n}{2}\right\} \cup \left\{v_{2i-1} v_{2i} : 1 \leq i \leq \frac{n}{2}\right\}$$

Then,
$|V(G)| = p = 3n$
$|E(G)| = q = \dfrac{7n-2}{2}$
Define a function

$$f : V(G) \to \{1, 2, \ldots, q+1\}$$

as follows:

$$\begin{aligned}
f(u_{2i-1}) &= 7i-5 & 1 \le i \le \frac{n}{2}\\
f(u_{2i}) &= 7i-1 & 1 \le i \le \frac{n}{2}\\
f(w_{2i-1}) &= 7i-6 & 1 \le i \le \frac{n}{2}\\
f(w_{2i}) &= 7i & 1 \le i \le \frac{n}{2}\\
f(v_{2i}) &= 7i-2 & 1 \le i \le \frac{n}{2}\\
f(v_{2i-1}) &= 7i-4 & 1 \le i \le \frac{n}{2}
\end{aligned}$$

Then f induces a bijective function

$$f^* : E(G) \to \{1, 2, \dots, q\}$$

as follows:

$$\begin{aligned}
f^*(u_{2i-1}u_{2i}) &= 7i-3 & 1 \le i \le \frac{n}{2}\\
f^*(u_{2i}u_{2i+1}) &= 7i & 1 \le i \le \frac{n-2}{2}\\
f^*(u_{2i-1}w_{2i-1}) &= 7i-6 & 1 \le i \le \frac{n}{2}\\
f^*(u_{2i}w_{2i}) &= 7i-1 & 1 \le i \le \frac{n}{2}\\
f^*(u_{2i-1}v_{2i-1}) &= 7i-5 & 1 \le i \le \frac{n}{2}\\
f^*(v_{2i-1}v_{2i}) &= 7i-4 & 1 \le i \le \frac{n}{2}\\
f^*(v_{2i-1}u_{2i}) &= 7i-2 & 1 \le i \le \frac{n}{2}
\end{aligned}$$

We observe that all the edge labels are distinct. Also,
$|V(G)| = p = 3n$
$|E(G)| = q = \frac{7n-2}{2}$
Hence, f is Geometric Mean Labeling of G.
Subcase 1.2: The first triangle starts from the second vertex of the path.
Let,

$$V(G) = \{u_i, w_i : 1 \le i \le n\} \cup \{v_i : 1 \le i \le n-2\}$$

$$E(G) = \{u_i w_i : 1 \le i \le n\} \cup \{u_i u_{i+1} : 1 \le i \le n-1\} \cup \left\{u_{2i}v_{2i-1} : 1 \le i \le \frac{n-2}{2}\right\}$$
$$\cup \left\{u_{2i+1}v_{2i-1} : 1 \le i \le \frac{n-2}{2}\right\} \cup \left\{v_{2i-1}v_{2i} : 1 \le i \le \frac{n-2}{2}\right\}$$

Then,
$|V(G)| = p = 3n-2$

$|E(G)| = q = \dfrac{7n-8}{2}$

Define a function $f : V(G) \to \{1, 2, \ldots, q+1\}$ as follows:

$$\begin{aligned}
f(u_{2i}) &= 7i-4 & 1 \le i \le \frac{n}{2} \\
f(u_{2i-1}) &= 7i-5 & 1 \le i \le \frac{n}{2} \\
f(w_{2i}) &= 7i-3 & 1 \le i \le \frac{n}{2} \\
f(w_{2i-1}) &= 7i-6 & 1 \le i \le \frac{n}{2} \\
f(v_{2i}) &= 7i-1 & 1 \le i \le \frac{n-2}{2} \\
f(v_{2i-1}) &= 7i & 1 \le i \le \frac{n-2}{2}
\end{aligned}$$

Then f induces a bijective function $f^* : E(G) \to \{1, 2, \ldots, q\}$ as follows:

$$\begin{aligned}
f^*(u_{2i-1}u_{2i}) &= 7i-5 & 1 \le i \le \frac{n}{2} \\
f^*(u_{2i}w_{2i}) &= 7i-4 & 1 \le i \le \frac{n}{2} \\
f^*(u_{2i-1}w_{2i-1}) &= 7i-6 & 1 \le i \le \frac{n}{2} \\
f^*(u_{2i}u_{2i+1}) &= 7i-2 & 1 \le i \le \frac{n-2}{2} \\
f^*(u_{2i}v_{2i-1}) &= 7i-3 & 1 \le i \le \frac{n-2}{2} \\
f^*(u_{2i+1}v_{2i-1}) &= 7i & 1 \le i \le \frac{n-2}{2} \\
f^*(v_{2i-1}v_{2i}) &= 7i-1 & 1 \le i \le \frac{n-2}{2}
\end{aligned}$$

We observe that, all the edge labels are distinct.Also,
$|V(G)| = p = 3n-2$
$|E(G)| = q = \dfrac{7n-8}{2}$
Hence f is a Geometric Mean Labeling of G.

Case 2: n is odd
Subcase 2.1: The first triangle starts from the first vertex of the path. Let,

$$V(G) = \{u_i, w_i : 1 \le i \le n\} \cup \{v_i : 1 \le i \le n-1\}$$

$$E(G) = \{u_i w_i : 1 \le i \le n\} \cup \{u_i u_{i+1} : 1 \le i \le n-1\}$$

$$\cup \left\{ u_{2i-1}v_{2i-1}, u_{2i}v_{2i-1}, v_{2i-1}v_{2i} : 1 \le i \le \frac{n-1}{2} \right\}$$

Then,
$|V(G)| = p = 3n - 1$
$|E(G)| = q = \dfrac{7n-5}{2}$
Define a function $f : V(G) \to \{1, 2, \ldots, q+1\}$ as follows:

$$\begin{aligned}
f(u_{2i-1}) &= 7i - 5 \quad 1 \le i \le \frac{n+1}{2} \\
f(w_{2i-1}) &= 7i - 6 \quad 1 \le i \le \frac{n+1}{2} \\
f(u_{2i}) &= 7i - 1 \quad 1 \le i \le \frac{n-1}{2} \\
f(w_{2i}) &= 7i \quad 1 \le i \le \frac{n-1}{2} \\
f(v_{2i}) &= 7i - 2 \quad 1 \le i \le \frac{n-1}{2} \\
f(v_{2i-1}) &= 7i - 4 \quad 1 \le i \le \frac{n-1}{2}
\end{aligned}$$

Then f induces a bijective function $f^* : E(G) \to \{1, 2, \ldots, q\}$ as follows:

$$\begin{aligned}
f^*(u_{2i-1}w_{2i-1}) &= 7i - 6 \quad 1 \le i \le \frac{n+1}{2} \\
f^*(u_{2i-1}u_{2i}) &= 7i - 4 \quad 1 \le i \le \frac{n-1}{2} \\
f^*(u_{2i}u_{2i+1}) &= 7i \quad 1 \le i \le \frac{n-1}{2} \\
f^*(u_{2i}w_{2i}) &= 7i - 1 \quad 1 \le i \le \frac{n-1}{2} \\
f^*(u_{2i-1}v_{2i-1}) &= 7i - 5 \quad 1 \le i \le \frac{n-1}{2} \\
f^*(u_{2i}v_{2i-1}) &= 7i - 2 \quad 1 \le i \le \frac{n-1}{2} \\
f^*(v_{2i}v_{2i-1}) &= 7i - 3 \quad 1 \le i \le \frac{n-1}{2}
\end{aligned}$$

We observe that all the edge labels are distinct. Also,
$|V(G)| = p = 3n - 1$
$|E(G)| = q = \dfrac{7n-5}{2}$
Hence, f is Geometric Mean Labeling of G.
Subcase 2.2: The first triangle starts from the second vertex of the path
Let,

$$V(G) = \{u_i, w_i : 1 \le i \le n\} \cup \{v_i : 1 \le i \le n-1\}$$

$$E(G) = \{u_i w_i : 1 \leq i \leq n\} \cup \{u_i u_{i+1} : 1 \leq i \leq n-1\}$$
$$\cup \left\{u_{2i} v_{2i-1}, u_{2i+1} v_{2i-1}, v_{2i-1} v_{2i} : 1 \leq i \leq \frac{n-1}{2}\right\}$$

Then,
$|V(G)| = p = 3n - 1$
$|E(G)| = q = \frac{7n-5}{2}$
Define a function $f : V(G) \to \{1, 2, \ldots, q+1\}$as follows:

$$\begin{aligned}
f(u_{2i-1}) &= 7i - 5 \quad 1 \leq i \leq \frac{n+1}{2} \\
f(w_{2i-1}) &= 7i - 6 \quad 1 \leq i \leq \frac{n+1}{2} \\
f(u_{2i}) &= 7i - 4 \quad 1 \leq i \leq \frac{n-1}{2} \\
f(v_{2i-1}) &= 7i \quad 1 \leq i \leq \frac{n-1}{2} \\
f(v_{2i}) &= 7i - 1 \quad 1 \leq i \leq \frac{n-1}{2} \\
f(w_{2i}) &= 7i - 3 \quad 1 \leq i \leq \frac{n-1}{2}
\end{aligned}$$

Then f induces a bijective function $f^* : E(G) \to \{1, 2, \ldots, q\}$ as follows:

$$\begin{aligned}
f^*(u_{2i-1} w_{2i-1}) &= 7i - 6 \quad 1 \leq i \leq \frac{n+1}{2} \\
f^*(u_{2i-1} u_{2i}) &= 7i - 5 \quad 1 \leq i \leq \frac{n-1}{2} \\
f^*(u_{2i} u_{2i+1}) &= 7i - 2 \quad 1 \leq i \leq \frac{n-1}{2} \\
f^*(u_{2i} w_{2i}) &= 7i - 4 \quad 1 \leq i \leq \frac{n-1}{2} \\
f^*(u_{2i+1} v_{2i-1}) &= 7i \quad 1 \leq i \leq \frac{n-1}{2} \\
f^*(v_{2i-1} v_{2i}) &= 7i - 1 \quad 1 \leq i \leq \frac{n-1}{2} \\
f^*(u_{2i} v_{2i-1}) &= 7i - 3 \quad 1 \leq i \leq \frac{n-1}{2}
\end{aligned}$$

We observe that all the edge labels are distinct. Also,
$|V(G| = p = 3n - 1$
$|E(G)| = q = \frac{7n-5}{2}$
Hence, f is Geometric Mean Labeling of G. ■

Example 2.2. Geometric Mean Labeling of $A(T_6) \bigodot K_1$ where the first triangle starts from first vertex is shown in Figure 1.

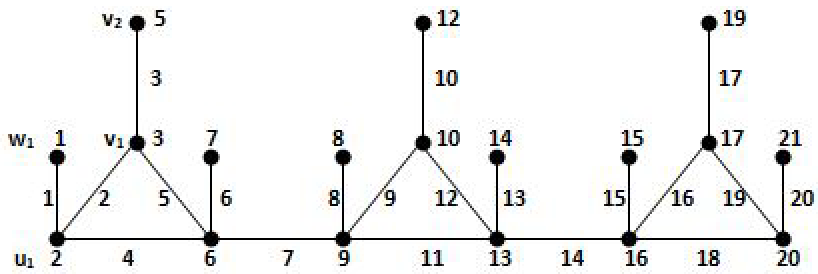

Figure 1:

Example 2.3. Geometric Mean Labeling of $A(T_6) \bigodot K_1$ where the first triangle starts from the second vertex is shown in Figure 2.

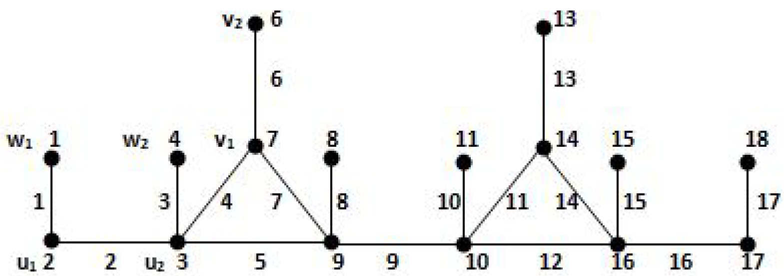

Figure 2:

Theorem 2.4. *$T(T_n) \bigodot K_1$ is a geometric mean graph.*

Proof. Let, $G = T(T_n) \bigodot K_1$ be the graph.
Let,

$$V(G) = \{u_i, u_{i1} : 1 \leq i \leq n\} \cup \{v_i, v_{i1}, z_i, z_{i1}, t_i, t_{i1} : 1 \leq i \leq n-1\}$$

$$E(G) = \{u_i u_{i1} : 1 \leq i \leq n\}$$

$$\cup\{u_i u_{i+1}, v_i v_{i1}, z_i z_{i1}, t_i t_{i1}, u_i v_i, u_i t_i, u_i z_i, u_{i+1} v_i, u_{i+1} t_i, u_{i+1} z_i : 1 \leq i \leq n-1\}$$

Then,
$|V(G)| = p = 8n - 6$
$|E(G)| = q = 11n - 10$

Define a function $f : V(G) \rightarrow \{1, 2, \ldots, q+1\}$ as follows:

$$\begin{aligned}
f(u_i) &= 11i - 9 && 1 \leq i \leq n \\
f(u_{i1}) &= 11i - 10 && 1 \leq i \leq n \\
f(v_i) &= 11i - 3 && 1 \leq i \leq n-1 \\
f(v_{i1}) &= 11i - 4 && 1 \leq i \leq n-1 \\
f(z_i) &= 11i - 8 && 1 \leq i \leq n-1 \\
f(z_{i1}) &= 11i - 7 && 1 \leq i \leq n-1 \\
f(t_i) &= 11i - 1 && 1 \leq i \leq n-1 \\
f(t_{i1}) &= 11i - 2 && 1 \leq i \leq n-1
\end{aligned}$$

Then f induces a bijective function $f^* : E(G) \rightarrow \{1, 2, \ldots, q\}$ as follows:

$$\begin{aligned}
f^*(u_i u_{i1}) &= 11i - 10 && 1 \leq i \leq n \\
f^*(u_i v_i) &= 11i - 7 && 1 \leq i \leq n-1 \\
f^*(u_i u_{i+1}) &= 11i - 5 && 1 \leq i \leq n-1 \\
f^*(u_i t_i) &= 11i - 6 && 1 \leq i \leq n-1 \\
f^*(v_i v_{i1}) &= 11i - 3 && 1 \leq i \leq n-1 \\
f^*(z_i z_{i1}) &= 11i - 8 && 1 \leq i \leq n-1 \\
f^*(t_i t_{i1}) &= 11i - 2 && 1 \leq i \leq n-1 \\
f^*(u_{i+1} v_i) &= 11i - 1 && 1 \leq i \leq n-1 \\
f^*(u_{i+1} t_i) &= 11i && 1 \leq i \leq n-1 \\
f^*(u_i z_i) &= 11i - 9 && 1 \leq i \leq n-1 \\
f^*(u_{i+1} z_i) &= 11i - 4 && 1 \leq i \leq n-1
\end{aligned}$$

We observe that, all the edge labels are distinct.
Also,
$|V(G)| = p = 8n - 6$
$|E(G)| = q = 11n - 10$
Hence, f is Geometric Mean Labeling of G. ■

Example 2.5. Geometric Mean Labeling of $T(T_3) \bigodot K_1$ is shown in figure 3.

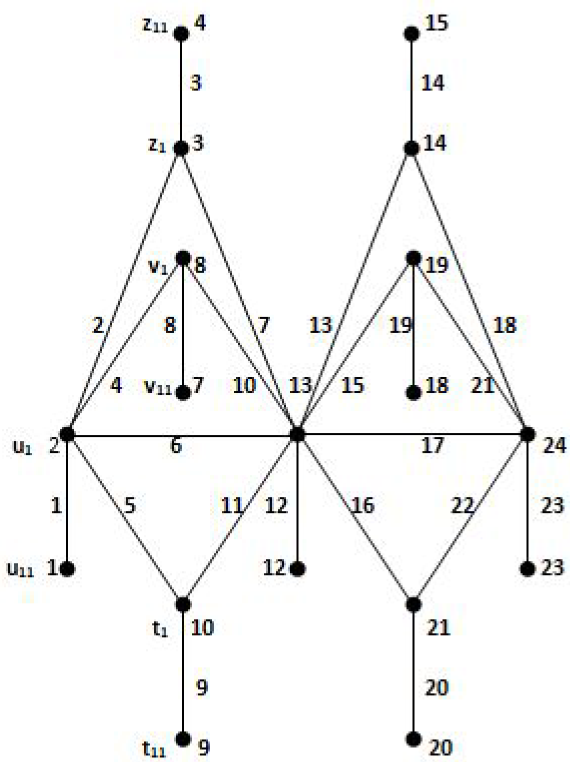

Figure 3:

Theorem 2.6. $Q_n \bigodot \bar{K}_2$ is a geometric mean graph.

Proof. Let, $G = Q_n \bigodot \bar{K}_2$.
Let,

$$V(G) = \{u_{ij} : 1 \leq i \leq n, 1 \leq j \leq 3\} \cup \{v_{ij}, w_{ij} : 1 \leq i \leq n-1; 1 \leq j \leq 3\}$$

$$E(G) = \{u_{i1}u_{i+1,1} : 1 \leq i \leq n-1\} \cup \{u_{i1}u_{ij} : 1 \leq i \leq n, 2 \leq j \leq 3\}$$

$$\cup \{v_{i1}v_{ij}, w_{i1}w_{ij} : 1 \leq i \leq n-1, 2 \leq j \leq 3\}$$

$$\cup \{u_{i1}v_{i1}, v_{i1}w_{i1}, u_{i+1,1}w_{i1} : 1 \leq i \leq n-1\}$$

Then,
$|V(G)| = p = 9n - 6$
$|E(G)| = q = 10n - 8$

Define a function $f : V(G) \to \{1, 2, \ldots, q+1\}$ as follows:

$$\begin{aligned}
f(u_{i1}) &= 10i - 7 & 1 \le i \le n \\
f(u_{i2}) &= 10i - 8 & 1 \le i \le n \\
f(u_{i3}) &= 10i - 9 & 1 \le i \le n \\
f(v_{i2}) &= 10i - 4 & 1 \le i \le n-1 \\
f(v_{i1}) &= 10i - 5 & 1 \le i \le n-1 \\
f(v_{i3}) &= 10i - 6 & 1 \le i \le n-1 \\
f(w_{i2}) &= 10i & 1 \le i \le n-1 \\
f(w_{i1}) &= 10i - 1 & 1 \le i \le n-1 \\
f(w_{i3}) &= 10i - 2 & 1 \le i \le n-1
\end{aligned}$$

Then f induces a bijective function $f^* : E(G) \to \{1, 2, \ldots, q\}$ as follows:

$$\begin{aligned}
f^*(u_{i1}u_{i3}) &= 10i - 9 & 1 \le i \le n \\
f^*(u_{i1}u_{i2}) &= 10i - 8 & 1 \le i \le n \\
f^*(v_{i1}v_{i3}) &= 10i - 6 & 1 \le i \le n-1 \\
f^*(v_{i1}v_{i2}) &= 10i - 5 & 1 \le i \le n-1 \\
f^*(w_{i1}w_{i3}) &= 10i - 2 & 1 \le i \le n-1 \\
f^*(w_{i1}w_{i2}) &= 10i - 1 & 1 \le i \le n-1 \\
f^*(u_{i1}u_{i+1,1}) &= 10i - 3 & 1 \le i \le n-1 \\
f^*(u_{i1}v_{i1}) &= 10i - 7 & 1 \le i \le n-1 \\
f^*(v_{i1}w_{i1}) &= 10i - 4 & 1 \le i \le n-1 \\
f^*(w_{i1}u_{i+1,1}) &= 10i & 1 \le i \le n-1
\end{aligned}$$

We observe that, all the edge labels are distinct.
Also,
$|V(G)| = p = 9n - 6$
$|E(G)| = q = 10n - 8$
Hence, f is Geometric Mean Labeling of G. ■

Example 2.7. Geometric Mean Labeling of $Q_3 \odot \bar{K}_2$ is shown in figure 4.

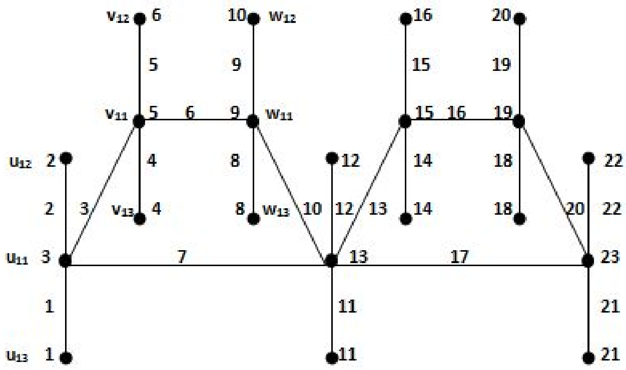

Figure 4:

Theorem 2.8. $D(Q_n) \bigodot \bar{K}_2$ is a geometric mean graph.

Proof. Let, $G = D(Q_n) \bigodot \bar{K}_2$ be the graph.
Let,

$$V(G) = \{u_{ij} : 1 \leq i \leq n, 1 \leq j \leq 3\} \cup \{v_{ij}, w_{ij}, t_{ij}, p_{ij} : 1 \leq i \leq n-1, 1 \leq j \leq 3\}$$

$$E(G) = \{u_{i1}u_{i+1,1} : 1 \leq i \leq n-1\} \cup \{u_{i1}u_{ij} : 1 \leq i \leq n, 2 \leq j \leq 3\}$$
$$\cup \{v_{i1}v_{ij}, w_{i1}w_{ij}, t_{i1}t_{ij}, p_{i1}p_{ij} : 1 \leq i \leq n-1, 2 \leq j \leq 3;\}$$
$$\cup \{u_{i1}v_{i1}, u_{i1}t_{i1}, v_{i1}w_{i1}, t_{i1}p_{i1}, u_{i+1,1}w_{i1}, u_{i+1,1}p_{i1} : 1 \leq i \leq n-1\}$$

Then,
$|V(G)| = p = 15n - 12;$
$|E(G)| = q = 17n - 15$
Define a function $f : V(G) \to \{1, 2, \ldots, q+1\}$ as follows:

$$\begin{aligned}
f(u_{i1}) &= 17i - 14 & 1 \leq i \leq n \\
f(u_{i2}) &= 17i - 15 & 1 \leq i \leq n \\
f(u_{i3}) &= 17i - 16 & 1 \leq i \leq n \\
f(v_{i1}) &= 17i - 12 & 1 \leq i \leq n-1 \\
f(v_{i2}) &= 17i - 11 & 1 \leq i \leq n-1 \\
f(v_{i3}) &= 17i - 13 & 1 \leq i \leq n-1
\end{aligned}$$

$$\begin{aligned}
f(w_{i1}) &= 17i - 8 & 1 \leq i \leq n-1 \\
f(w_{i2}) &= 17i - 7 & 1 \leq i \leq n-1 \\
f(w_{i3}) &= 17i - 9 & 1 \leq i \leq n-1 \\
f(t_{i1}) &= 17i - 5 & 1 \leq i \leq n-1 \\
f(t_{i2}) &= 17i - 6 & 1 \leq i \leq n-1 \\
f(t_{i3}) &= 17i - 4 & 1 \leq i \leq n-1 \\
f(p_{i1}) &= 17i - 1 & 1 \leq i \leq n-1 \\
f(p_{i2}) &= 17i - 2 & 1 \leq i \leq n-1 \\
f(p_{i3}) &= 17i & 1 \leq i \leq n-1
\end{aligned}$$

Then f induces a bijective function $f^* : E(G) \to \{1, 2, \ldots, q\}$ as follows:

$$\begin{aligned}
f^*(u_{11}u_{21}) &= 8 \\
f^*(w_{11}w_{12}) &= 10 \\
f^*(w_{11}w_{13}) &= 9 \\
f^*(v_{11}w_{11}) &= 7 \\
f^*(u_{11}t_{11}) &= 6 \\
f^*(u_{i1}u_{i2}) &= 17i - 15 \quad 1 \le i \le n \\
f^*(u_{i1}u_{i3}) &= 17i - 16 \quad 1 \le i \le n \\
f^*(v_{i1}v_{i2}) &= 17i - 12 \quad 1 \le i \le n-1 \\
f^*(v_{i1}v_{i3}) &= 17i - 13 \quad 1 \le i \le n-1
\end{aligned}$$

$$\begin{aligned}
f^*(t_{i1}t_{i2}) &= 17i - 6 \quad 1 \le i \le n-1 \\
f^*(t_{i1}t_{i3}) &= 17i - 5 \quad 1 \le i \le n-1 \\
f^*(p_{i1}p_{i2}) &= 17i - 2 \quad 1 \le i \le n-1 \\
f^*(p_{i1}p_{i3}) &= 17i - 1 \quad 1 \le i \le n-1 \\
f^*(t_{i1}p_{i1}) &= 17i - 4 \quad 1 \le i \le n-1 \\
f^*(u_{i1}v_{i1}) &= 17i - 14 \quad 1 \le i \le n-1 \\
f^*(w_{i1}u_{i+1,1}) &= 17i - 3 \quad 1 \le i \le n-1 \\
f^*(p_{i1}u_{i+1,1}) &= 17i \quad 1 \le i \le n-1 \\
f^*(u_{i1}t_{i1}) &= 17i - 10 \quad 2 \le i \le n-1 \\
f^*(u_{i1}u_{i+1,1}) &= 17i - 7 \quad 2 \le i \le n-1 \\
f^*(w_{i1}w_{i2}) &= 17i - 8 \quad 2 \le i \le n-1 \\
f^*(w_{i1}w_{i3}) &= 17i - 9 \quad 2 \le i \le n-1 \\
f^*(v_{i1}w_{i1}) &= 17i - 11 \quad 2 \le i \le n-1
\end{aligned}$$

We observe that, all the edge labels are distinct.
Also,
$|V(G)| = p = 15n - 12$
$|E(G)| = q = 17n - 15$
Hence, f is Geometric Mean Labeling of G. ■

Example 2.9. Geometric Mean Labeling of $D(Q_3) \odot \bar{K}_2$ is shown in figure 5.

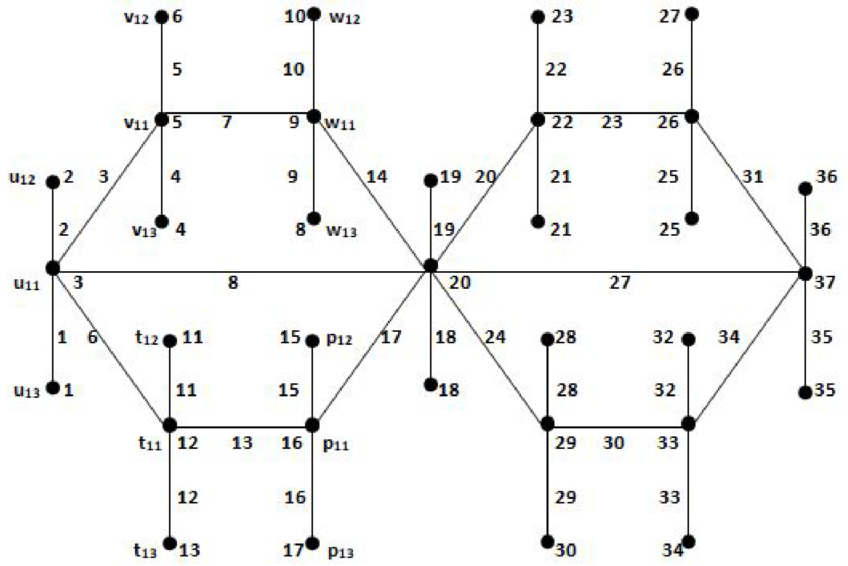

Figure 5:

Theorem 2.10. $C_n \bigodot P_2$ is a geometric mean graph.

Proof. Let, $G = C_n \bigodot P_2$.
Let,
$s = 4n,\ t = \lceil \sqrt{12n-3} \rceil$
$V(G) = \{v_i : 1 \leq i \leq 4n / v_{4j} = v_{4j-3}, 1 \leq j \leq n\}$
$E(G) = \{v_i v_{i+1} : 1 \leq i \leq s-1\} \cup \{v_s v_1\}$
Then,
$|V(G)| = p = 3n$
$|E(G)| = q = 4n$
Define a function $f : V(G) \to \{1, 2, \ldots, q+1\}$ as follows:

$$\begin{aligned} f(v_{4i-3}) &= 4i-1 & 1 \leq i \leq n \\ f(v_{4i-2}) &= 4i-3 & 1 \leq i \leq n \\ f(v_{4i-1}) &= 4i & 1 \leq i \leq n \end{aligned}$$

Then f induces a bijective function $f^* : E(G) \to \{1, 2, \ldots, q\}$ as follows:

$$\begin{aligned} f^*(v_i v_{i+1}) &= i & 1 \leq i \leq t-1 \\ f^*(v_i v_{i+1}) &= i+1 & t \leq i \leq 4n-1 \\ f^*(v_s v_1) &= t \end{aligned}$$

We observe that, all the edge labels are distinct.
Also,
$|V(G)| = p = 3n$

$|E(G)| = q = 4n$
Hence,f is Geometric Mean Labeling of G. ∎

Example 2.11. Geometric Mean Labeling of $C_4 \bigodot P_2$ is shown in figure 6.

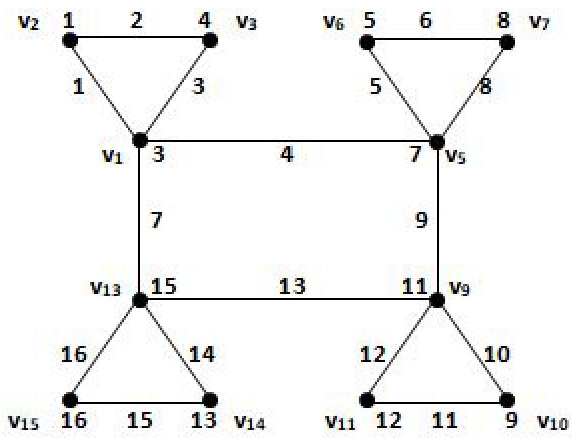

Figure 6:

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