Some Results On Root Square Mean Graphs

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Abstract

A graph $G = (V, E)$ with $p$ vertices and $q$ edges is said to be a Root Square Mean graph if it is possible to label the vertices $x \in V$ with distinct elements $f(x)$ from $1, 2, \ldots, q + 1$ in such a way that when each edge $e = uv$ is labeled with $f(e = uv) = \left\lfloor \sqrt{\frac{f(u)^2 + f(v)^2}{2}} \right\rfloor$ or $\left\lceil \frac{f(u)^2 + f(v)^2}{2} \right\rceil$, then the resulting edge labels are distinct. In this case $f$ is called a Root Square Mean labeling of $G$. In this paper we investigate some results on Root Square Mean labeling of graphs.

Key Words: Graph, Root Square Mean graph, Path, Cycle, Complete graph, complete bipartite graph.

1. Introduction

The graph considered here will be finite, undirected and simple. The vertex set is denoted by $V(G)$ and the edge set is denoted by $E(G)$. For all detailed survey of graph labeling we refer to Gallian [1]. For all other standard terminology and notations we follow Harary[2]. S.S.Sandhya, S.Somasundaram and S.Anusa introduced the concept of Root Square Mean labeling of graphs in [3]. In this paper we investigate some results on Root Square Mean labeling of graphs. The definitions and other information’s which are useful for the present investigation are given below.

Definition1.1: A walk in which $u_1u_2 \ldots u_n$ are distinct is called a path. A path on $n$ vertices is denoted by $P_n$.

Definition1.2: A closed path is called a cycle. A cycle on $n$ vertices is denoted by $C_n$.

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**Definition 1.3:** The Corona of two graphs $G_1$ and $G_2$ is the graph $G = G_1 \odot G_2$ formed by one copy of $G_1$ and $|G_1|$ copies of $G_2$ where the $i^{th}$ vertex of $G_1$ is adjacent to every vertex in the $i^{th}$ copy of $G_2$.

**Definition 1.4:** Any cycle with a pendent edge attached at each vertex is called a crown. It is denoted by $C_n \odot K_1$.

**Definition 1.5:** A Dragon is formed by joining the end point of a path to a cycle. It is denoted by $C_n @ P_m$.

**Definition 1.6:** A graph $G$ is said to be complete if every pair of its distinct vertices are adjacent. A Complete graph on $n$ vertices is denoted by $K_n$.

**Definition 1.7:** The Complement $\bar{G}$ of a graph $G$ has $V(G)$ as its vertex set, but two vertices are adjacent in $\bar{G}$ if and only if they are not adjacent in $G$.

**Definition 1.8:** An $(n, t)$-kite graph consists of a cycle of length $n$ with $t$ edges path attached to one vertex of a cycle.

**Definition 1.9:** A complete bipartite graph is a bipartite graph with bipartition $(V_1, V_2)$ such that every vertex of $V_1$ is joined to all the vertices of $V_2$. It is denoted by $K_{m,n}$, where $|V_1| = m$ and $|V_2| = n$.

**Theorem 1.10:** Any path is a Root Square Mean graph.

**Theorem 1.11:** Any Cycle is a Root Square Mean graph.

**Theorem 1.12:** Combs are Root Square Mean graphs.

**Theorem 1.13:** Complete graph $K_n$ is a Root Square Mean graph if and only if $n \leq 4$.

**Theorem 1.14:** $K_{1,n}$ is a Root Square Mean graph if and only if $n \leq 6$.

### 2. Main Results

**Theorem 2.1:** Let $P_n$ be the path and $G$ be the graph obtained from $P_n$ by attaching $C_3$ in both the end edges of $P_n$. Then $G$ is a Root Square Mean graph.

**Proof:** Let $P_n$ be the path $u_1 u_2 \ldots u_n$ and $v_1 u_1 u_2$, $v_2 u_{n-1} u_n$ be the triangles which are connected to the path at the end. Define a function $f : V(G) \to \{1, 2, \ldots, q + 1\}$ by

- $f(u_i) = i + 1, 1 \leq i \leq n - 1$
- $f(u_n) = n + 3$
- $f(v_1) = 1, f(v_2) = n + 2$

The edges are labeled as
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\[ f(u_1v_1) = 1, f(u_2v_1) = 2 \]
\[ f(u_{n-1}v_2) = n + 1, f(u_nv_2) = n + 3 \]
\[ f(u_{n-1}u_n) = n + 2 \]
\[ f(u_iu_{i+1}) = i + 2, 1 \leq i \leq n - 1. \]

Hence \( f \) is a Root Square Mean labeling.

**Example 2.2:** Root Square Mean labeling of \( G \) obtained from \( P_7 \) is given below.

![Figure 1](image1.png)

**Theorem 2.3:** The Crown \( C_n \odot K_1 \) is a Root Square Mean graph.

**Proof:** Let \( C_n \) be the cycle \( u_1u_2 ... u_nu_1 \) and \( v_i \) be the pendent vertices adjacent to \( u_i, 1 \leq i \leq n. \)
Define a function \( f: V(C_n \odot K_1) \rightarrow \{1,2,3,...,q+1\} \) by
\[ f(u_i) = 2i - 1, 1 \leq i \leq n \]
\[ f(v_i) = 2i, 1 \leq i \leq n \]

Then we get distinct edge labels. Hence \( f \) is a Root Square Mean labeling.

**Example 2.4:** The Root Square Mean labeling of \( C_6 \odot K_1 \) is given below.

![Figure 2](image2.png)
Theorem 2.5: Dragon $C_n \circ P_m$ is a Root Square Mean graph.

Proof: Let $v_1 v_2 \ldots v_m$ be the path $P_m$ be the cycle $C_n$ and $v_1 v_2 \ldots v_n$ be the path $P_m$.
Here $u_n = v_1$. Define a function $f : V(C_n \circ P_m) \rightarrow \{1, 2, 3, \ldots, q + 1\}$ by

$f(u_i) = i, 1 \leq i \leq n,$
$f(v_i + 1) = n + i, 1 \leq i \leq m.$

Then the edge labels are distinct. Hence $f$ is a Root Square Mean labeling.

Example 2.6: Root Square mean labeling of $C_6 \circ P_6$ is given below.

![Figure 3]

Theorem 2.7: Let $G$ be a graph obtained by attaching a pendent edge to both sides of each vertex of a path $P_n$. Then $G$ is a Root Square Mean graph.

Proof: Let $G$ be the graph obtained by attaching pendent edges to both sides of each vertex of a path $P_n$. Let $u_i, v_i$ and $w_i, 1 \leq i \leq n$ be the new vertices of $G$. Define a function $f : V(G) \rightarrow \{1, 2, 3, \ldots, q + 1\}$ by

$f(u_i) = 3i - 1, 1 \leq i \leq n$
$f(v_i) = 3i, 1 \leq i \leq n$
$f(w_i) = 3i - 2, 1 \leq i \leq n.$

Then the edges are labeled as

$f(u_i u_{i+1}) = 3i, 1 \leq i \leq n - 1,$
$f(u_i v_i) = 3i - 1, 1 \leq i \leq n$
$f(u_i w_i) = 3i - 2, 1 \leq i \leq n.$

Hence $f$ is a Root Square Mean labeling Then the edge labels are distinct.

Example 2.8: The graph obtained from $P_6$ is given below.
Theorem 2.9: $C_n \odot \overline{K_2}$ is a Root Square Mean graph for all $n \geq 3$.

Proof: Let $C_n$ be the cycle $u_1u_2 \ldots u_nu_1$ and $v_i, w_i$ be the vertices adjacent to $u_i, 1 \leq i \leq n$.

Define a function $f: V(C_n \odot \overline{K_2}) \rightarrow \{1, 2, 3, \ldots, q + 1\}$ by

$f(u_i) = 3i - 1, 1 \leq i \leq n$

$f(v_i) = 3i - 2, 1 \leq i \leq n$

$f(w_i) = 3i, 1 \leq i \leq n$.

Then the edge labels are distinct. Hence $f$ is a Root Square Mean labeling.

Example 2.10: Here we display the Root Square Mean labeling of $C_6 \odot \overline{K_2}$

Theorem 2.11: $C_n \odot \overline{K_3}$ is a Root Square Mean graph for all $n \geq 3$.

Proof: Let $C_n$ be the cycle $u_1u_2 \ldots u_nu_1$ and $v_i, w_i, t_i$ be the vertices adjacent to $u_i, 1 \leq i \leq n$.

Define a function $f: V(C_n \odot \overline{K_3}) \rightarrow \{1, 2, 3, \ldots, q + 1\}$ by
\[ f(u_i) = 4i - 2, 1 \leq i \leq n \]
\[ f(v_i) = 4i - 3, 1 \leq i \leq n \]
\[ f(w_i) = 4i - 1, 1 \leq i \leq n \]
\[ f(t_i) = 4i, 1 \leq i \leq n. \]

Then the edge labels are distinct. Hence \( f \) is a Root Square Mean labeling.

**Example 2.12**: Here we display the Root Square Mean labeling of \( C_6 \odot \overline{K}_3 \).

![Figure 6](image)

**Theorem 2.13**: Let \( G \) be a graph obtained by attaching each vertex of \( P_n \) to the central vertex of \( K_{1,2} \). Then \( G \) is a Root Square Mean graph.

**Proof**: Let \( P_n \) be the path \( u_1u_2 \ldots u_n \) and let \( v_i, w_i \) be the vertices of \( K_{1,2} \) which are attached to the vertex \( u_i \) of \( P_n \).

Define a function \( f: V(G) \rightarrow \{1, 2, 3, \ldots, q + 1\} \) by

\[ f(u_i) = 3i - 2, 1 \leq i \leq n, \]
\[ f(v_i) = 3i - 1, 1 \leq i \leq n, \]
\[ f(w_i) = 3i, 1 \leq i \leq n. \]

Then the edges are labeled as

\[ f(u_iu_{i+1}) = 3i, 1 \leq i \leq n - 1 \]
\[ f(u_iv_i) = 3i - 2, 1 \leq i \leq n \]
\[ f(u_iw_i) = 3i - 1, 1 \leq i \leq n \]

Hence \( f \) is a Root Square Mean labeling. Then the edge labels are distinct.

**Example 2.14**: Root Square Mean labeling of \( G \) obtained from \( P_4 \) is given below.
Theorem 2.15: Let $G$ be a graph obtained by attaching each vertex of $P_n$ to the central vertex of $K_{1,3}$. Then $G$ is a Root Square Mean graph.

Proof: Let $P_n$ be the path $u_1 u_2 ... u_n$ and $v_i, w_i$ and $t_i$ be the vertices of $K_{1,3}$ which are attached to the vertices $u_i$ of $P_n$.

Define a function $f: V(G) \rightarrow \{1, 2, 3, ..., q+1\}$ by

- $f(u_i) = 4i - 3, 1 \leq i \leq n$
- $f(v_i) = 4i - 2, 1 \leq i \leq n$
- $f(w_i) = 4i - 1, 1 \leq i \leq n$
- $f(t_i) = 4i, 1 \leq i \leq n$.

Then the edges are labeled as

- $f(u_i u_{i+1}) = 4i, 1 \leq i \leq n - 1$
- $f(u_i v_i) = 4i - 3, 1 \leq i \leq n$
- $f(u_i w_i) = 4i - 2, 1 \leq i \leq n$
- $f(u_i t_i) = 4i - 1, 1 \leq i \leq n$

Then $f$ is a Root Square Mean labeling.

Example 2.16: The Root Square mean labeling of $P_4 \odot K_{1,3}$ is given below.

Theorem 2.17: Let $G$ be a graph obtained by joining a pendent vertex with a vertex of degree two of a comb graph. Then $G$ is a Root Square Mean graph.
**Proof:** Let the vertex set of the comb be \(\{u_1, u_2, ..., u_n, v_1, v_2, ..., v_n\}\). Let \(P_n\) be the path \(u_1 u_2 ... u_n\). Join a vertex \(v_i\) to \(u_i\), \(1 \leq i \leq n\). Let \(G\) be a graph obtained by joining a pendent vertex \(w\) to \(u_n\). Define a function \(f: V(G) \rightarrow \{1, 2, 3, ..., q + 1\}\) by

\[
\begin{align*}
    f(u_i) &= 2i - 1, \quad 1 \leq i \leq n \\
    f(v_i) &= 2i, \quad 1 \leq i \leq n \\
    f(w) &= 2n + 1.
\end{align*}
\]

Then the edges are labeled as

\[
\begin{align*}
    f(u_i u_{i+1}) &= 2i, \quad 1 \leq i \leq n - 1 \\
    f(u_i v_i) &= 2i - 1, \quad 1 \leq i \leq n \\
    f(u_n w) &= 2n.
\end{align*}
\]

This gives a Root Square Mean labeling of \(G\).

**Example 2.18:** Root Square Mean labeling of \(G\) with 11 vertices and 10 edges is given below

![Figure 9](image)

In the similar manner, we can see the Root Square Mean labeling of \(G\) obtained by joining a pendent vertex with a vertex of degree two on both ends of a comb graph. Root Square Mean labeling of \(G\) with 10 vertices and 9 edges is shown below.

![Figure 10](image)

**Theorem 2.19:** A \((m, n)\) – Kite graph is a Root Square Mean graph.

**Proof:** Let \(u_1 u_2 ... u_m u_1\) be the given cycle of length \(m\) and \(v_1, v_2 ... v_n\) be the given path of length \(n\). Define a function \(f: V(G) \rightarrow \{1, 2, 3, ..., q + 1\}\) by

\[
\begin{align*}
    f(u_i) &= i, \quad 1 \leq i \leq m \\
    f(v_i) &= m + i, \quad 1 \leq i \leq n
\end{align*}
\]
Then the edge labels are distinct. Hence \((m, n) – \) Kite graph is a Root Square Mean graph.

**Example 2.20:** The Root Square Mean labeling of \((5,6) – \) Kite graph is shown below.

![Figure 11](image)

**Theorem 2.21:** \(K_{n,n}\) is a Root Square Mean graph if and only if \(n \leq 2\).

**Proof:** Clearly \(K_{1,1}\) is a Root Square Mean graph. The labeling pattern of \(K_{1,1}\) and \(K_{2,2}\) are shown below.

![Figure 12](image)

Here the edge labels of \((5,10), (9,6)\) are repeated. Hence \(K_{n,n}\) is a Root Square Mean graph if and only if \(n \leq 2\).

**Conclusion**

All graphs are not Root Square Mean graphs. It is very interesting to investigate graphs which admit Root Square Mean labeling. In this paper, we proved that Dragon,
Crown, Carona of some graphs are Root Square graphs. It is possible to investigate similar results for several other graphs.

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