On Soft Almost $\pi g$-continuous functions

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Abstract

The object of this paper is to introduce a new class of functions called soft almost $\pi g$-continuous functions. This class turns out to be the natural tool for studying different class of soft compact spaces. Further soft almost $\pi g$-open and soft almost $\pi g$-closed functions are obtained as generalizations of soft open and soft closed functions respectively.

Keywords: soft $\pi g$-closed set, soft $\pi g$-open set, soft $\pi g$-Continuity, soft almost $\pi g$-continuity, soft almost open $\pi g$-continuity, soft almost closed $\pi g$-continuity

1. Introduction
Molodtsov [8] initiated the concept of soft set theory as a new mathematical tool and presented the fundamental results of the soft sets. Recently Muhammad Shabir and Munazza Naz [10] introduced soft topological spaces which are defined over an initial universe with a fixed set of parameters. Kharal et al. [5] introduced soft function over classes of soft sets. Cigdem Gunduz Aras et al., [1] in 2013 studied and discussed the properties of Soft continuous mappings. In this paper, we give some characterizations of soft almost $\pi g$-continuous function and the relations of such function with other types of soft functions are obtained.

2. Preliminaries
Definition: 2.1[8]
Let $U$ be the initial universe and $P(U)$ denote the power set of $U$. Let $E$ denote the set of all parameters. Let $A$ be a non-empty subset of $E$. A pair $(F, A)$ is called a soft set over $U$, where $F$ is a mapping given by $F: A \rightarrow P(U)$. 
Definition: 2.2[7]
A subset \((A, E)\) of a topological space \(X\) is called soft generalized-closed (soft \(g\)-closed) if \(\text{cl}(A, E) \subseteq (U, E)\) whenever \((A, E) \not\subseteq (U, E)\) and \((U, E)\) is soft open in \(X\).

Definition: 2.3[2]
A subset \((A, E)\) of a topological space \(X\) is called soft regular closed, if \(\text{cl}(\text{int}(A, E)) = (A, E)\). The complement of soft regular closed set is soft regular open set.

Definition: 2.4[2]
The finite union of soft regular open sets is said to be soft \(\pi\)-open. The complement of soft \(\pi\)-open is said to be soft \(\pi\)-closed.

Definition: 2.5[2]
A subset \((A, E)\) of a topological space \(X\) is called soft \(\pi g\)-closed in a soft topological space \((X, \tau, E)\), if \(\text{cl}(A, E) \subseteq (U, E)\) whenever \((A, E) \not\subseteq (U, E)\) and \((U, E)\) is soft \(\pi\)-open in \(X\).

Definition: 2.6[1]
Let \((F, E)\) be a soft set over \(X\). The soft set \((F, E)\) is called soft point, denoted by \((x_e, E)\), if for element \(e \in E\), \(F(e) = \{x\}\) and \(F(e') = \emptyset\) for all \(e' \in E - \{e\}\).

Definition: 2.7[12]
Let \((X, \tau, E)\) and \((Y, \tau', E)\) be two topological spaces. A function \(f: (X, \tau, E) \rightarrow (Y, \tau', E)\) is said to be Soft Semi continuous(Soft pre-continuous, Soft \(\beta\)-continuous), if \(f^{-1}(G, E)\) is soft semi open(soft pre-open, soft \(\alpha\)-open, soft \(\beta\)-open) in \((X, \tau, E)\) for every soft open set \((G, E)\) of \((Y, \tau', E)\).

Definition: 2.8[3]
Let \((X, \tau, E)\) and \((Y, \tau', E)\) be two topological spaces. A function \(f: (X, \tau, E) \rightarrow (Y, \tau', E)\) is said to be Soft regular continuous(Soft \(\pi\)-continuous, Soft \(g\)-continuous, Soft \(\pi g\)-continuous), if \(f^{-1}(G, E)\) is soft regular open(soft \(\pi\)-open, soft \(g\)-open, soft \(\pi g\)-open) in \((X, \tau, E)\) for every soft open set \((G, E)\) of \((Y, \tau', E)\).

Definition: 2.9[3]
A function \(f: (X, \tau, E) \rightarrow (Y, \tau', E)\) is soft \(\pi g\)-irresolute, if \(f^{-1}(G, E)\) is soft \(\pi g\)-open in \((X, \tau, E)\) for every soft \(\pi g\)-open set \((G, E)\) of \((Y, \tau', E)\).

Definition: 2.10[2]
A space \((X, \tau, E)\) is called soft \(\pi g\)-T\(_{1\frac{1}{2}}\) [6], if every soft \(\pi g\)-closed set is soft closed, or equivalently every soft \(\pi g\)-open set is soft open.

Definition: 2.11[3]
A function \(f: (X, \tau, E) \rightarrow (Y, \tau', E)\) is called \(\delta\pi g\)-open, if image of each soft open set in \(X\) is \(\delta\pi g\)-open in \(Y\).
Definition: 2.12 [4]
A function \( f: (X, \tau, E) \rightarrow (Y, \tau', E) \) is called soft contra \( \pi g \)-continuous, if \( f^{-1}(F, E) \) is soft \( \pi g \)-closed in \( X \) for every soft open set \( (F, E) \) of \( Y \).

Definition: 2.13 [4]
A space \( (X, \tau, E) \) is said to be soft \( \pi g \)-compact, if every soft \( \pi g \)-open cover of \( X \) has a finite sub cover.

Definition: 2.14 [4]
A space \( (X, \tau, E) \) is said to be soft countably \( \pi g \)-compact, if every soft \( \pi g \)-open countably cover of \( X \) has a finite subcover.

Definition: 2.15 [4]
A space \( (X, \tau, E) \) is said to be soft \( \pi g \)-Lindelöf, if every soft \( \pi g \)-open cover of \( X \) has a countable subcover.

Definition: 2.16 [4]
A space \( (X, \tau, E) \) is said to be soft \( \pi g \)-connected provided that \( X \) cannot be written as the union of two disjoint non-empty soft \( \pi g \)-open sets.

Definition: 2.17 [4]
A space \( (X, \tau, E) \) is said to be \( \pi g \)-\( T_2 \) if for each pair of distinct soft points \( x \) and \( y \) in \( X \), there exist \( \mathcal{S} \pi g \)-open \( (F, E) \in \mathcal{S} \pi g \text{GO}(X, x) \) and \( (G, E) \in \mathcal{S} \pi g \text{GO}(X, y) \) such that \( (F, E) \cap (G, E) = \emptyset \).

Definition: 2.18 [10]
A space \( (X, \tau, E) \) is said to be soft Hausdorff, if for each pair of distinct points \( x \) and \( y \) in \( X \), there exist soft open sets \( (A, E) \) and \( (B, E) \) containing \( x \) and \( y \) such that \( (A, E) \cap (B, E) = \emptyset \).

Definition: 2.19 [4]
A function \( f: (X, \tau, E) \rightarrow (Y, \tau', E) \) is said to be soft R-Map, if \( f^{-1}(A, E) \) is soft regular closed in \( X \) for every soft regular closed \( (A, E) \) of \( Y \).

Definition: 2.20 [4]
A space \( (X, \tau, E) \) is said to be soft submaximal, if each soft dense subset of \( X \) is soft open.

3. Soft Almost \( \pi g \)-continuous functions

Definition: 3.1
A function \( f: (X, \tau, E) \rightarrow (Y, \tau', E) \) is said to be soft almost \( \pi g \)-continuous, if \( f^{-1}(A, E) \) is soft \( \pi g \)-open in \( X \) for every soft regular open \( (A, E) \) of \( Y \).
**Theorem: 3.2**

The following statements are equivalent for a function $f : (X, \tau, E) \to (Y, \tau', E)$:

1. $f$ is soft almost $\pi g$-continuous.
2. $f^{-1}(F, E) \in \mathcal{S}\pi GC(X)$ for every soft $(F, E) \in \mathcal{S}RC(Y)$.
3. For each $x \in X$ and each soft regular closed set $(F, E)$ in $Y$ containing $f(x)$, there exists soft $\pi g$-closed set $(U, E)$ in $X$ containing $x$ such that $f(U, E) \subset (F, E)$.
4. For each $x \in X$ and each soft regular open set $(F, E)$ in $Y$ containing $f(x)$, there exists soft $\pi g$-open set $(K, E)$ in $X$ not containing $x$ such that $f^{-1}(F, E) \subset (K, E)$.
5. $f^{-1}(\text{int}(\text{cl}(G, E))) \in \mathcal{S}\pi GO(X)$ for every soft open subset $(G, E)$ of $Y$.
6. $f^{-1}(\text{cl}(\text{int}(F, E))) \in \mathcal{S}\pi GC(X)$ for every soft closed subset $(F, E)$ of $Y$.

**Proof:**

(1) $\Rightarrow$ (2)

Let $(F, E) \in \mathcal{S}RC(Y)$. Then $Y \setminus (F, E) \in \mathcal{S}RO(Y)$ By (1) $f^{-1}(Y \setminus (F, E)) = X \setminus f^{-1}(F, E) \in \mathcal{S}\pi GO(X)$. Thus $f^{-1}(A, E) \subset \mathcal{S}\pi GC(X)$.

(2) $\Rightarrow$ (3)

Let $(F, E)$ be a soft regular closed set in $Y$ containing $f(x)$. Then $f^{-1}(F, E) \in \mathcal{S}\pi GC(X)$ and $x \in f^{-1}(F, E)$ by (2). Take $(U, E) = f^{-1}(F, E)$. Then $f(U, E) \subset (F, E)$.

(3) $\Rightarrow$ (2)

Let $(F, E) \in \mathcal{S}RC(Y)$ and $x \in f^{-1}(F, E)$. From (3) there exists a soft $\pi g$-closed set $(U, E)$ in $X$ containing $x$ such that $f(U, E) \subset (F, E)$. We have $f^{-1}(F, E) = \bigcup \{(U, E) : x \in f^{-1}(F, E)\}$. Thus $f^{-1}(F, E)$ is soft $\pi g$-closed set.

(3) $\Rightarrow$ (4)

Let $(F, E)$ be a soft regular open set in $Y$ not containing $f(x)$. Then $Y \setminus (F, E)$ is a soft regular closed set containing $f(x)$. By (3) there exists a soft $\pi g$-closed set $(U, E)$ in $X$ containing $x$ such that $f(U, E) \subset Y \setminus (F, E)$. Hence $(U, E) \subset f^{-1}(Y \setminus (F, E)) \subset X \setminus f^{-1}(F, E)$. Then $f^{-1}(F, E) \subset X \setminus (U, E)$. Take $(K, E) = X \setminus (U, E)$. Then we obtain a soft $\pi g$-open set $(K, E)$ in $X$ not containing $x$ such that $f^{-1}(F, E) \subset (K, E)$.

(4) $\Rightarrow$ (3)

Let $(F, E)$ be a soft regular closed set in $Y$ containing $f(x)$. Then $Y \setminus (F, E)$ is a soft regular open set in $Y$ not containing $f(x)$. By (4) there exists a soft $\pi g$-open set $(K, E)$ in $X$ not containing $x$ such that $f^{-1}(Y \setminus (F, E)) \subset (K, E)$. That is $X \setminus f^{-1}(F, E) \subset (K, E)$ implies $X \setminus (K, E) \subset f^{-1}(F, E)$. Hence $f(X \setminus (K, E)) \subset (F, E)$. Take $(U, E) = X \setminus (K, E)$. Then $(U, E)$ is soft $\pi g$-closed set in $X$ containing $x$ such that $f(U, E) \subset (F, E)$.

(1) $\Rightarrow$ (5)

Let $(G, E)$ be a soft open subset of $Y$. Since $\text{int}(\text{cl}(G, E))$ is soft regular open then by (1) $f^{-1}(\text{int}(\text{cl}(G, E))) \in \mathcal{S}\pi GO(X)$.

(5) $\Rightarrow$ (1)

Let $(G, E) \in \mathcal{S}RO(Y)$. Then $(G, E)$ is open in $Y$. By (5) $f^{-1}(\text{int}(\text{cl}(G, E))) \in \mathcal{S}\pi GO(X)$ implies $f^{-1}(G, E) \in \mathcal{S}\pi GO(X)$. Hence $f$ is soft almost $\pi g$-continuous.

(2) $\iff$ (6) is similar as (1) $\iff$ (5).
Theorem: 3.3
If \( f: (X, \tau, E) \rightarrow (Y, \tau', E) \) is a soft almost \( \pi_g \)-continuous function then the following properties hold:

1. \( \exists \pi_g\text{-cl}(f^{-1}(\text{cl}(\text{int}(\text{cl}(B, E)))) \supseteq f^{-1}(\text{cl}(B, E)) \) for every soft subset (B, E) of Y.
2. \( \exists \pi_g\text{-cl}(f^{-1}(\text{cl}(\text{int}(F, E)))) \supseteq f^{-1}(F, E) \) for every soft closed set (F, E) of Y.
3. \( \exists \pi_g\text{-cl}(f^{-1}(\text{cl}(V, E))) \supseteq f^{-1}(\text{cl}(V, E)) \) for every soft open set (V, E) of Y.

Theorem: 3.4
Every restriction of a soft almost \( \pi_g \)-continuous function is soft almost \( \pi_g \)-continuous.

Proof:
Let \( f: (X, \tau, E) \rightarrow (Y, \tau', E) \) be a soft almost \( \pi_g \)-continuous function of X into Y and (A, E) be any soft open subset of X. For any soft regular open subset (F, E) of Y, \( (f|(A, E))^{-1}(F, E) = (A, E) \cap f^{-1}(F, E) \). Since f is almost \( \pi_g \)-continuous \( f^{-1}(F, E) \in \exists \pi_g\text{GO}(X) \). Hence \( (A, E) \cap f^{-1}(F, E) \) relatively soft \( \pi_g \)-open subset of (A, E). That is \( (f|(A, E))^{-1}(F, E) \) is soft \( \pi_g \)-open subset of (A, E). Hence f|(A, E) is soft almost \( \pi_g \)-continuous.

Theorem: 3.5
If \( f: (X, \tau, E) \rightarrow (Y, \tau', E) \) is a soft function of X into Y and \( X = (A, E) \cup (B, E) \) where (A, E) and (B, E) are soft \( \pi_g \)-closed and f|(A, E) and f|(B, E) are soft almost \( \pi_g \)-continuous, then f is soft almost \( \pi_g \)-continuous.

Proof:
Let (F, E) be any soft regular closed set of Y. Since f|(A, E) and f|(B, E) are soft almost \( \pi_g \)-continuous, \( (f|(A, E))^{-1}(F, E) \) and \( (f|(B, E))^{-1}(F, E) \) are soft \( \pi_g \)-closed in (A, E) and (B, E) respectively. Since (A, E) and (B, E) are soft \( \pi_g \)-closed subsets of X, \( (f|(A, E))^{-1}(F, E) \) and \( (f|(B, E))^{-1}(F, E) \) are soft \( \pi_g \)-closed subsets of X. Also \( f^{-1}(F, E) = (f|(A, E))^{-1}(F, E) \cup (f|(B, E))^{-1}(F, E) \) is soft \( \pi_g \)-closed in X. Hence f is soft almost \( \pi_g \)-continuous.

Theorem: 3.6
If \( f: (X, \tau, E) \rightarrow (Y, \tau', E) \) is a soft function of X into Y and \( X = (A, E) \cup (B, E) \) and f|(A, E) and f|(B, E) are both soft almost \( \pi_g \)-continuous at a point x belonging to (A, E) \( \cap (B, E) \), then f is soft almost \( \pi_g \)-continuous at x.

Proof:
Let (U, E) be any soft regular open set containing f(x). Since x \( \in (A, E) \cap (B, E) \) and f|(A, E), f|(B, E) are both soft almost \( \pi_g \)-continuous at x, therefore there exist soft \( \pi_g \)-open sets (F, E) and (G, E) such that x \( \in (A, E) \cap (F, E) \) and f|(A, E) \( \cap (F, E) \) \( \exists \pi_g\text{GO}(U, E) \) and x \( \in (B, E) \cap (G, E) \) and f|(B, E) \( \cap (G, E) \) \( \exists \pi_g\text{GO}(U, E) \). Since X = (A, E) \( \cup (B, E) \), f|(A, E) \( \cap (B, E) \) = f|(A, E) \( \cap (F, E) \) \( \cap (G, E) \) \( \exists \pi_g\text{GO}(U, E) \) and f|(B, E) \( \cap (G, E) \) \( \exists \pi_g\text{GO}(U, E) \). Thus (F, E) \( \cap (G, E) = (K, E) \) is a soft \( \pi_g \)-
open set containing $x$ such that $f(K, E) \not\subseteq (U, E)$. Hence $f$ is soft almost $\pi g$-continuous at $x$.

**Theorem: 3.7**

If a function $f: X \to \prod Y_i$ is soft almost $\pi g$-continuous, then $P_i \circ f: X \to Y_i$ is soft almost $\pi g$-continuous for each $i \in I$, where $P_i$ is the projection of $\prod Y_i$ onto $Y_i$.

**Proof:**

Let $(V_i, E)$ be any soft regular open set of $Y_i$. Since $P_i$ is a soft continuous open, it is a soft $R$-map. Hence $P_i^{-1}(V_i, E)$ is soft regular open in $\prod Y_i$. Thus $(P_i \circ f)^{-1}(V_i, E) = f^{-1}(P_i^{-1}(V_i, E))$ is soft $\pi g$-open in $X$. Therefore $P_i \circ f$ is soft almost $\pi g$-continuous.

**Theorem: 3.8**

If a function $f: \prod X_i \to \prod Y_i$ is soft almost soft $\pi g$-continuous, then $f_i: X_i \to Y_i$ is soft almost $\pi g$-continuous for each $i \in I$.

**Proof:**

Let $k$ be an arbitrarily fixed index and $(V_k, E)$ be any soft regular open set of $Y_k$. Then $\prod Y_k \times (V_k, E)$ is soft regular open in $\prod Y_i$ where $i \not\in I$ and $j \not= k$. Hence $f^{-1}((\prod Y_k \times (V_k, E))) = \prod Y_k \times f_k^{-1}(V_k, E)$ is soft $\pi g$-open in $\prod X_i$. Thus $f_k^{-1}(V_k, E)$ is soft $\pi g$-open in $\prod X_k$. Hence $f_k$ is soft almost $\pi g$-continuous.

**Definition: 3.9**

A function $f: (X, \tau, E) \to (Y, \tau', E)$ is called

1. Soft almost $g$-continuous, if $f^{-1}(A, E)$ is soft $g$-closed in $X$ for every soft regular closed $(A, E)$ of $Y$.
2. Soft almost $\pi$-continuous, if $f^{-1}(A, E)$ is soft $\pi$-closed in $X$ for every soft regular closed $(A, E)$ of $Y$.
3. Soft completely continuous, if $f^{-1}(A, E)$ is soft regular closed in $X$ for every soft closed set $(A, E)$ of $Y$.

**Theorem: 3.10**

1. Every soft $R$-Map is soft almost $\pi$-continuous.
2. Every soft almost $\pi$-continuous is soft almost-continuous.
3. Every soft almost $\pi$-continuous is soft almost $\pi g$-continuous.
4. Every soft almost continuous is soft almost $g$-continuous.
5. Every soft almost $g$-continuous is soft almost $\pi g$-continuous.

**Remark: 3.11**

The following diagram holds for the above implications. Also none of the results are reversible as seen in the following examples.
1. Soft almost $\pi$-continuous
2. Soft R-map
3. Soft almost $g$-continuous
4. Soft almost continuous
5. Soft almost $\pi g$-continuous

Example 3.12
Let $X = \{a, b, c, d\}$, $Y = \{a, b, c\}$, $E = \{e_1, e_2\}$. Let $F_1, F_2, F_3, F_4, F_5, F_6$ and $G_1, G_2, G_3, G_4, G_5, G_6, G_7$ are functions from $E$ to $P(X)$ and $E$ to $P(Y)$ are defined as follows:

$F_1(e_1) = \{c\}$, $F_1(e_2) = \{a\}$; $F_2(e_1) = \{d\}$, $F_2(e_2) = \{b\}$; $F_3(e_1) = \{c, d\}$, $F_3(e_2) = \{a, b\}$; $F_4(e_1) = \{a, d\}$, $F_4(e_2) = \{b, d\}$ and $G_1(e_1) = \{b\}$, $G_1(e_2) = \{a\}$; $G_2(e_1) = \{a, c\}$, $G_2(e_2) = \{b, c\}$, $G_3(e_1) = \{b\}$, $G_3(e_2) = X$, $G_4(e_1) = \emptyset$, $G_4(e_2) = \{a\}$, $G_5(e_1) = \{a, c\}$, $G_5(e_2) = X$, $G_6(e_1) = \emptyset$, $G_6(e_2) = \{b, c\}$, $G_7(e_1) = \emptyset$, $G_7(e_2) = X$. Then $\tau = \{\emptyset, X\}$, $(F_1, E)$, $(F_2, E)$, $(F_3, E)$, $(F_4, E)$, $(F_5, E)$, $(F_6, E)$, $(F_7, E)$, $\tau'$ is a soft topological space over $X$ and $\tau' = \{\emptyset, \bar{Y}\}$, $(G_1, E)$, $(G_2, E)$, $(G_3, E)$, $(G_4, E)$, $(G_5, E)$, $(G_6, E)$, $(G_7, E)$, $\tau'$ is a soft topological space over $Y$. If the function $f: (X, \tau, E) \rightarrow (Y, \tau', E)$ is defined as $f(a) = b$, $f(b) = a$, $f(c) = c$, $f(d) = d$, then $f$ is soft almost continuous but not soft almost $\pi$-continuous.

Example 3.13
Let $X = \{a, b, c, d\}$, $Y = \{a, b, c\}$, $E = \{e_1, e_2\}$. Let $F_1, F_2, F_3, F_4, F_5, F_6$ and $G_1, G_2, G_3, G_4, G_5, G_6, G_7$ are functions from $E$ to $P(X)$ and $E$ to $P(Y)$ are defined as follows:

$F_1(e_1) = \{c\}$, $F_1(e_2) = \{a\}$; $F_2(e_1) = \{d\}$, $F_2(e_2) = \{b\}$; $F_3(e_1) = \{c, d\}$, $F_3(e_2) = \{a, b\}$; $F_4(e_1) = \{a, d\}$, $F_4(e_2) = \{b, d\}$ and $G_1(e_1) = \{b\}$, $G_1(e_2) = \{a\}$; $G_2(e_1) = \{a, c\}$, $G_2(e_2) = \{b, c\}$, $G_3(e_1) = \{b\}$, $G_3(e_2) = X$, $G_4(e_1) = \emptyset$, $G_4(e_2) = \{a\}$, $G_5(e_1) = \{a, c\}$, $G_5(e_2) = X$, $G_6(e_1) = \emptyset$, $G_6(e_2) = \{b, c\}$, $G_7(e_1) = \emptyset$, $G_7(e_2) = X$. Then $\tau = \{\emptyset, \bar{X}\}$, $(F_1, E)$, $(F_2, E)$, $(F_3, E)$, $(F_4, E)$, $(F_5, E)$, $(F_6, E)$, $(F_7, E)$, $\tau'$ is a soft topological space over $X$ and $\tau' = \{\emptyset, \bar{Y}\}$, $(G_1, E)$, $(G_2, E)$, $(G_3, E)$, $(G_4, E)$, $(G_5, E)$, $(G_6, E)$, $(G_7, E)$, $\tau'$ is a soft topological space over $Y$. If the function $f: (X, \tau, E) \rightarrow (Y, \tau', E)$ is defined as $f(a) = b$, $f(b) = d$, $f(c) = c$, $f(d) = a$, then $f$ is soft almost $\pi g$-continuous but not soft almost $\pi$-continuous.
Example 3.14
Let \( X = \{a, b, c, d\} \), \( Y = \{a, b, c\} \), \( E = \{e_1, e_2\} \). Let \( F_1, F_2, F_3, F_4 \) and \( G_1, G_2, G_3, G_4, G_5, G_6, G_7 \) are functions from \( E \) to \( P(X) \) and \( E \) to \( P(Y) \) are defined as follows:

\[
\begin{align*}
F_1(e_1) &= \{a\}, \\
F_1(e_2) &= \{d\}; \\
F_2(e_1) &= \{b\}, \\
F_2(e_2) &= \{c\}; \\
F_3(e_1) &= \{a, b\}, \\
F_3(e_2) &= \{c, d\}; \\
F_4(e_1) &= \{b, c, d\}, \\
F_4(e_2) &= \{a, b, c\} \\
\end{align*}
\]

\[
\begin{align*}
G_1(e_1) &= \{b\}, \\
G_1(e_2) &= \{a\}; \\
G_2(e_1) &= \{a, c\}, \\
G_2(e_2) &= \{b, c\}; \\
G_3(e_1) &= \{b\}, \\
G_3(e_2) &= X; \\
G_4(e_1) &= \{a\}, \\
G_4(e_2) &= \{a, c\}; \\
G_5(e_1) &= \{a\}, \\
G_5(e_2) &= X; \\
G_6(e_1) &= \{b, c\}, \\
G_6(e_2) &= \{a\}; \\
G_7(e_1) &= \{b\}, \\
G_7(e_2) &= X.
\end{align*}
\]

Then \( \mathcal{T} = \{\emptyset, \bar{X}, (F_1, E), (F_2, E), (F_3, E), (F_4, E), (F_5, E), (F_6, E)\} \) is a soft topological space over \( X \) and \( \mathcal{T}' = \{\emptyset, \bar{Y}, (G_1, E), (G_2, E), (G_3, E), (G_4, E), (G_5, E), (G_6, E), (G_7, E)\} \) is a soft topological space over \( Y \). If the function \( f: (X, \tau, E) \to (Y, \tau', E) \) is defined as \( f(a) = b, f(b) = d, f(c) = c, f(d) = a \) then \( f \) is soft almost \( g \)-continuous but not soft almost continuous.

Example 3.15
In example 3.13 we see that \( f \) is soft almost \( \pi g \)-continuous but not soft almost \( g \)-continuous, since \( f^{-1}(G_2, E) \) is not soft \( g \)-closed in \( X \).

Lemma: 3.16
Let \( (X, \tau, E) \) be a soft topological space. If \( (U, E), (V, E) \in \tilde{S}\pi\text{GO}(X) \) and \( X \) is a soft submaximal space then \( (U \times V, E) \in \tilde{S}\pi\text{GO}(X) \).

Theorem: 3.17
Let \( f: (X, \tau, E) \to (Y, \tau', E) \) be soft function and let \( g: (X, \tau, E) \to (X \times Y, \tau \times \tau', E) \) be the soft graph function of \( f \), defined by \( g(x) = (x, f(x)) \) for every \( x \in X \). Suppose that \( X \) be soft submaximal space. Then \( g \) is soft almost \( \pi g \)-continuous, if and only if \( f \) is soft almost continuous.

Proof:
Let \( x \in X \) and \((V, E) \in \tilde{S}\text{RO}(Y) \) containing \( f(x) \). Then we have \( g(x) = (x, f(x)) \in X \times (V, E) \in \tilde{S}\text{RO}(X \times Y) \). Since \( g \) is soft almost \( \pi g \)-continuous, \( g^{-1}(X \times (V, E)) \in \tilde{S}\pi\text{GO}(X) \). Thus \( f \) is soft almost \( \pi g \)-continuous.

Conversely let \( x \in X \) and \((W, E) \in \tilde{S}\text{RO}(X \times Y) \) containing \( g(x) \). Then there exists \((U, E) \in \tilde{S}\text{RO}(X) \) and \((V, E) \in \tilde{S}\text{RO}(Y) \) such that \((x, f(x)) \in \tilde{S} (U \times V, E) \in \tilde{S}(W, E) \). Since \( f \) is soft almost \( \pi g \)-continuous, \( f^{-1}(V, E) \in \tilde{S}\pi\text{GO}(X) \). Say \((A, E) = f^{-1}(V, E) \) and take \((B, E) = (U, E) \cap (A, E) \). By previous lemma \((B, E) \in \tilde{S}\pi\text{GO}(X) \) and \( g(B, E) \in \tilde{S}(U \times V, E) \in \tilde{S}(W, E) \). This shows that \( g \) is soft almost \( \pi g \)-continuous.

Theorem: 3.18
Let \( f: (X, \tau, E) \to (Y, \tau', E) \) and \( g: (Y, \tau', E) \to (Z, \tau'', E) \) be soft almost \( \pi g \)-continuous and \( Y \) is soft Hausdorff. If \( X \) is soft submaximal then the set \( \{x \in X: f(x) = g(x)\} \) is soft \( \pi g \)-closed in \( X \).

Proof:
Let \((A, E) = \{x \in X: f(x) = g(x)\} \) and \( x \in X \setminus (A, E) \). Then \( f(x) \neq g(x) \). Since \( Y \) is soft Hausdorff, there exist soft open sets \((U, E) \) and \((V, E) \) of \( Y \), such that \( f(x) \in (U, E) \),
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$g(x) \in (U, E)$ and $(U, E) \cap (A, E) = \emptyset$. Since $f$ and $g$ are soft almost $\pi g$-continuous, $(G, E) = f^{-1}(\text{int}(\text{cl}(U, E))) \in \tilde{\pi} \text{GO}(X, x)$ and $(H, E) = g^{-1}(\text{int}(\text{cl}(V, E))) \in \tilde{\pi} \text{GO}(X, x)$. Take $(W, E) = (G, E) \cap (H, E)$ then $(W, E) \in \tilde{\pi} \text{GO}(X, x)$ and $f(W, E) \cap g(W, E) = \emptyset$. Therefore $(W, E) \cap (A, E) = \emptyset$. Hence $x \in X \tilde{\pi}g\text{-cl} (A, E)$. This shows that $(A, E)$ is soft $\pi g$-closed in $X$.

**Theorem: 3.19**
Let $f: (X, \tau, E) \to (Y, \tau', E)$ and $g: (Y, \tau', E) \to (Z, \tau'', E)$ be functions. Then the following properties hold:

1. If $f$ is soft almost $\pi g$-continuous and $g$ is soft R-map, then $g \circ f: (X, \tau, E) \to (Z, \tau'', E)$ is soft almost $\pi g$-continuous.
2. If $f$ is soft $\pi g$-irresolute and $g$ is soft $\pi g$-continuous, then $g \circ f: (X, \tau, E) \to (Z, \tau'', E)$ is soft almost $\pi g$-continuous.
3. If $f$ is soft almost $\pi g$-continuous and $g$ is soft completely continuous, then $g \circ f: (X, \tau, E) \to (Z, \tau'', E)$ is soft almost $\pi g$-continuous.
4. If $f$ is soft almost $\pi g$-continuous and $g$ is soft almost continuous, then $g \circ f: (X, \tau, E) \to (Z, \tau'', E)$ is soft almost $\pi g$-continuous.

**Definition: 3.20**
A function $f: (X, \tau, E) \to (Y, \tau', E)$ is said to be soft weakly $\pi g$-continuous, if $(\text{cl}(A, E))$ is soft $\pi g$-open in $X$ for every soft open set $(A, E)$ of $Y$.

**Theorem: 3.21**
Let $f: (X, \tau, E) \to (Y, \tau', E)$ be a soft function. Suppose that $X$ is soft $\pi g$-space and $Y$ is soft regular space. Then the following properties equivalent:

1. $f$ is soft $\pi g$-continuous
2. $f$ is soft almost $\pi g$-continuous
3. $f$ is soft weakly $\pi g$-continuous

**Proof:**
$(1) \implies (2) \implies (3)$. This is obvious.

**Theorem: 3.22**
If for each pair of distinct points $x$ and $y$ in a soft space $X$, there exists a function $f$ of $X$ into a soft Hausdorff space $Y$ such that

1. $f(x) \neq f(y)$
2. $f$ is soft weakly $\pi g$-continuous at $x$ and
3. $f$ is soft almost $\pi g$-continuous at $y$, then $X$ is soft $\pi g$-T$_2$.

**Proof:**
Since $Y$ is soft Hausdorff, there exists soft open sets $(U, E)$ and $(V, E)$ of $Y$ such that $f(x) \in (U, E)$ and $f(y) \in (V, E)$ and $(U, E) \cap (V, E) = \emptyset$. Hence $\text{cl}(U, E) \cap (\text{int}(\text{cl}(V, E))) = \emptyset$. Since $f$ is soft weakly $\pi g$-continuous at $x$, there exists $(A, E) \in \tilde{\pi} \text{GO}(X, x)$ such that $f(A, E) \notin \text{cl}(U, E)$. Since $f$ is soft almost $\pi g$-continuous at $y$, $f^{-1}(\text{int}(\text{cl}(V, E))) = \emptyset$. Hence $x \in X \tilde{\pi}g\text{-cl} (A, E)$.
(B, E) ∈ \(\tilde{\mathcal{F}}\pi_{GO}(X, y)\). Therefore we obtain \((A, E) \cap (B, E) = \emptyset\). This shows that \(X\) is soft \(\pi g-T_2\).

**Theorem: 3.23**
If \(f: (X, \tau, E) \to (Y, \tau', E)\) is soft almost \(\pi g\)-continuous surjective function and \(X\) is soft \(\pi g\)-connected space, then \(Y\) is soft connected.

**Proof:**
Suppose \(Y\) is not soft connected. Then there exist non-empty disjoint soft open subsets \((U, E)\) and \((V, E)\) of \(Y\) such that \(Y = (U, E) \cup (V, E)\). Since \(f\) is soft almost \(\pi g\)-continuous, then \(f^{-1}(U, E)\) and \(f^{-1}(V, E)\) are non-empty disjoint soft \(\pi g\)-clopen sets in \(X\). Then we have \(X = f^{-1}(U, E) \cup f^{-1}(V, E)\) such that \(f^{-1}(U, E)\) and \(f^{-1}(V, E)\) are disjoint. This shows that \(X\) is not soft \(\pi g\)-connected which is a contradiction. Hence \(Y\) is soft connected.

**Definition: 3.24**
A function \(f: (X, \tau, E) \to (Y, \tau', E)\) has a soft \(\pi g\)-\((r)\)-graph if for each \((x, y) \in X \times Y \setminus G(f)\), there exists \((U, E) \in \tilde{\mathcal{F}}\pi_{GO}(X, x)\) and a regular open set \((V, E)\) of \(Y\) containing \(y\) such that \((U \times V, E) \cap G(f) = \emptyset\).

**Lemma: 3.25**
A function \(f: (X, \tau, E) \to (Y, \tau', E)\) has a soft \(\pi g\)-\((r)\)-graph if and only if for each \((x, y) \in X \times Y\) such that \(y \neq f(x)\), there exists a soft \(\pi g\)-open set \((U, E)\) and a regular open set \((V, E)\) containing \(x\) and \(y\) respectively such that \(f(U, E) \cap (V, E) = \emptyset\).

**Theorem: 3.26**
If \(f: (X, \tau, E) \to (Y, \tau', E)\) is a soft almost \(\pi g\)-continuous function and \(Y\) is soft Hausdorff then \(f\) has a soft \(\pi g\)-\((r)\)-graph.

**Proof:**
Let \((x, y) \in X \times Y\) such that \(y \neq f(x)\). Then there exists a soft open sets \((U, E)\) and \((V, E)\) such that, \(y \in (U, E), f(x) \in (V, E)\) and \((U, E) \cap (V, E) = \emptyset\). Hence \(\text{int}(\text{cl}(U, E)) \cap \text{int}(\text{cl}(V, E)) = \emptyset\). Since \(f\) is soft almost \(\pi g\)-continuous, \(f^{-1}(\text{int}(\text{cl}(U, E))) = (W, E) \in \tilde{\mathcal{F}}\pi_{GO}(X, x)\). This implies that \(f(W, E) \cap \text{int}(\text{cl}(U, E)) = \emptyset\). Therefore \(f\) has a soft \(\pi g\)-\((r)\)-graph.

**Definition: 3.27**
A space \((X, \tau, E)\) is said to be:
1. Soft nearly compact, if every soft regular open cover of \(X\) has a finite soft subcover.
2. Soft nearly countably compact, if every countable soft cover of \(X\) by soft regular open sets has a finite soft subcover.
3. Soft nearly Lindelof, if every cover of \(X\) by soft regular open sets has a countable soft subcover.
Theorem: 3.28
Let \( f: (X, \tau, E) \rightarrow (Y, \tau', E) \) be a soft almost \( \pi g \)-continuous surjection. Then the following statements hold:

1. If \( X \) is soft \( \pi g \)-compact, then \( Y \) is soft nearly compact.
2. If \( X \) is soft \( \pi g \)-Lindelöf, then \( Y \) is soft nearly Lindelöf.
3. If \( X \) is soft countably \( \pi g \)-compact, then \( Y \) is soft nearly countably compact.

4. Soft almost \( \pi g \)-open function and soft almost \( \pi g \)-closed function

Definition: 4.1
A function \( f: (X, \tau, E) \rightarrow (Y, \tau', E) \) is called soft almost open(Soft almost closed), if the image of every soft regular open subset of \( X \) is soft open(soft regular closed) subset of \( Y \).

Definition: 4.2
A function \( f: (X, \tau, E) \rightarrow (Y, \tau', E) \) is called soft almost \( \pi g \)-open(Soft almost \( \pi g \)-closed), if the image of every soft regular open subset of \( X \) is soft \( \pi g \)-open (soft \( \pi g \)-closed) subset of \( Y \).

Remark: 4.3
A one to one soft function is soft almost \( \pi g \)-open if and if it is soft almost \( \pi g \)-closed.

Remark: 4.4
Every soft \( \pi g \)-open function is soft almost \( \pi g \)-open. But the converse is not true in general.

Example: 4.5
Let \( X = \{a, b, c, d\} \), \( Y = \{a, b, c, d\} \), \( E = \{e_1, e_2\} \). Let \( F_1, F_2, F_3, F_4, F_5, F_6 \) and \( G_1, G_2, G_3, G_4 \) are functions from \( E \) to \( P(X) \) and \( E \) to \( P(Y) \) are defined as follows:

\[
\begin{align*}
F_1(e_1) &= \{c\}, \quad F_1(e_2) = \{a\}; \\
F_2(e_1) &= \{d\}, \quad F_2(e_2) = \{b\}; \\
F_3(e_1) &= \{c, d\}, \quad F_3(e_2) = \{a, b\}; \\
F_4(e_1) &= \{a, d\}, \quad F_4(e_2) = \{b, d\}; \\
F_5(e_1) &= \{a, c, d\}, \quad F_5(e_2) = \{a, b, c\}; \\
F_6(e_1) &= \{a, c, d\}, \quad F_6(e_2) = \{a, b, d\}; \\
G_1(e_1) &= \{a\}, \quad G_1(e_2) = \{d\}; \\
G_2(e_1) &= \{b\}, \quad G_2(e_2) = \{c\}; \\
G_3(e_1) &= \{a, b\}, \quad G_3(e_2) = \{c, d\}; \\
G_4(e_1) &= \{b, c, d\}, \quad G_4(e_2) = \{a, b, c\}.
\end{align*}
\]

Then \( \tau = (\mathcal{G}, \mathcal{X}, (F_1, E), (F_2, E), (F_3, E), (F_4, E), (F_5, E), (F_6, E)) \) is a soft topological space over \( X \) and \( \tau' = (\mathcal{G}, \mathcal{Y}, (G_1, E), (G_2, E), (G_3, E), (G_4, E)) \) is a soft topological space over \( Y \). If the function \( f: (X, \tau, E) \rightarrow (Y, \tau', E) \) is defined as \( f(a) = d, f(b) = a, f(c) = c, f(d) = b \), then \( f \) is soft almost \( \pi g \)-open but not soft \( \pi g \)-open.

Theorem: 4.6
If \( f: (X, \tau, E) \rightarrow (Y, \tau', E) \) is a soft almost closed function of \( X \) onto \( Y \) then for every soft regular open subset \( (G, E) \) of \( X \) and for every soft point \( y \in Y \) such that \( f^{-1}(y) \notin (G, E) \) we have \( y \in \text{int}(f(G, E)) \).
Proof:
Since \((G, E)\) is soft regular open, \(X(G, E)\) is soft regular closed. Since \(f\) is soft almost \(\pi g\)-continuous, \(f(X(G, E))\) is soft \(\pi g\)-closed. Since \(f^{-1}(y) \supseteq (G, E), y \notin f(X(G, E))\). Hence there must exist a soft open set \((U, E)\) containing \(y\) such that \((U, E) \cap f(X(G, E)) = \emptyset\). Then \(y \in (U, E) \supseteq (f(G, E))\). This shows that \(y\) is a soft interior point of \(f(G)\).

Theorem: 4.7
A surjection \(f: (X, \tau, E) \to (Y, \tau', E)\) is soft almost \(\pi g\)-closed if and only if for each subset \((A, E)\) of \(Y\) and each \((U, E) \in \mathcal{SR}(X)\) containing \(f^{-1}(A, E)\) there exists a soft \(\pi g\)-open set \((V, E)\) of \(Y\) such that \((A, E) \supseteq (U, E)\) and \(f^{-1}(V, E) \supseteq (U, E)\).

Proof:
Suppose that \(f\) is soft almost \(\pi g\)-closed. Let \((A, E)\) be a subset of \(Y\) and \((U, E) \in \mathcal{SR}(X)\) containing \(f^{-1}(A, E)\). If \((V, E) = Y(f(U, E))\) then \((V, E)\) is soft \(\pi g\)-open set of \(Y\) such that \((A, E) \supseteq (U, E)\) and \(f^{-1}(V, E) \supseteq (U, E)\).

Conversely let \((F, E)\) be any soft regular closed set of \(X\). Then \(f^{-1}(Y(f(F, E)) \supseteq X(f(F, E))\) and \(X(f(F, E)) \in \mathcal{SR}(X)\). Then there exists a soft \(\pi g\)-open set \((V, E)\) of \(Y\) such that \(Y(f(F, E)) \supseteq (V, E)\) and \(f^{-1}(V, E) \supseteq X(f(F, E))\). Therefore \(Y(f(F, E)) \supseteq f(X(f^{-1}(V, E)) \supseteq Y(V, E)\). Hence we obtain \(f(F, E) = Y(V, E)\) and \(f(F, E)\) is soft \(\pi g\)-closed in \(Y\) which shows that \(f\) is soft \(\pi g\)-closed.

Definition: 4.8
A space \((X, \tau, E)\) is said to be soft quasi-normal, if for any two disjoint soft \(\pi\)-closed sets \((A, E)\) and \((B, E)\) in \((X, \tau, E)\), there exists disjoint soft open sets \((U, E)\) and \((V, E)\) such that \((A, E) \supseteq (U, E)\) and \((B, E) \supseteq (V, E)\).

Definition: 4.9
A function \(f: (X, \tau, E) \to (Y, \tau', E)\) is called
1. Soft \(\pi\)-closed injection, if \(f(A, E)\) is soft \(\pi\)-closed in \(Y\) for every soft \(\pi\)-closed set \((A, E)\) of \(X\).
2. Soft almost \(\pi\)-continuous, if \(f^{-1}(A, E)\) is soft \(\pi\)-closed in \(X\) for every soft regular closed set \((A, E)\) of \(Y\).
3. Soft \(\pi\)-irresolute, if \(f^{-1}(A, E)\) is soft is soft \(\pi\)-closed in \(X\) for every soft \(\pi\)-closed set \((A, E)\) of \(Y\).

Theorem: 4.10
If \(f: (X, \tau, E) \to (Y, \tau', E)\) is soft almost \(\pi g\)-continuous, soft \(\pi\)-closed injection and \(Y\) is soft quasi-normal space then \(X\) is soft quasi-normal.

Proof:
Let \((A, E)\) and \((B, E)\) be any disjoint soft \(\pi\)-closed sets of \(X\). Since \(f\) is a soft \(\pi\)-closed injection, \(f(A, E)\) and \(f(B, E)\) are disjoint soft \(\pi\)-closed sets of \(Y\). Since \(Y\) is soft quasi-normal there exists disjoint soft open sets \((U, E)\) and \((V, E)\) of \(Y\) such that \(f(A, E) \supseteq (U, E)\) and \(f(B, E) \supseteq (V, E)\). Now if \((G, E) = \text{intcl}(U, E)\) and \((H, E) = \text{intcl}(V, E)\), then \(f(G, E) = f(H, E)\).
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(G, E) and (H, E) are disjoint soft regular open sets such that $f(A, E) \not\subseteq (G, E)$ and $f(B, E) \not\subseteq (H, E)$. Since $f$ is soft almost $\pi g$-continuous, $f^{-1}(G, E)$ and $f^{-1}(H, E)$ are disjoint soft $\pi g$-open sets containing $(A, E)$ and $(B, E)$ which shows that $X$ is soft quasi-normal.

**Lemma: 4.11**

A surjection $f: (X, \tau, E) \rightarrow (Y, \tau', E)$ is soft almost closed if and only if for each subset $(A, E)$ of $Y$ and each $(U, E) (U, E) \in SO(X)$ containing $f^{-1}(A, E)$ there exists a soft open set $(V, E)$ of $Y$ such that $(A, E) \not\subseteq (V, E)$ and $f^{-1}(V, E) \not\subseteq (U, E)$.

**Theorem: 4.12**

If $f: (X, \tau, E) \rightarrow (Y, \tau', E)$ is soft almost $\pi$-continuous, soft almost closed surjection and $X$ is soft quasi-normal space then $Y$ is soft quasi-normal.

**Proof:**

Let $(A, E)$ and $(B, E)$ be any two disjoint soft closed sets of $Y$. Then $f^{-1}(A, E)$ and $f^{-1}(B, E)$ are disjoint soft $\pi$-closed sets of $X$. Since $X$ is soft quasi-normal there exists a disjoint soft open sets $(U, E)$ and $(V, E)$ of $X$ such that $f^{-1}(A, E) \not\subseteq (U, E)$ and $f^{-1}(B, E) \not\subseteq (V, E)$. Let $(G, E) = \text{intcl} (U, E)$ and $(H, E) = \text{intcl}(V, E)$. Then $(G, E)$ and $(H, E)$ are disjoint soft regular open sets of $X$ such that $f^{-1}(A, E) \not\subseteq (G, E)$ and $f^{-1}(B, E) \not\subseteq (H, E)$. Take $(K, E) = f((X \setminus (G, E))$ and $(L, E) = f((X \setminus (H, E))$. By previous lemma, $(K, E)$ and $(L, E)$ are soft open sets of $Y$ such that $(A, E) \not\subseteq (K, E)$ and $(B, E) \not\subseteq (L, E)$, $f^{-1}(K, E) \not\subseteq (G, E)$ and $f^{-1}(L, E) \not\subseteq (H, E)$. Since $(G, E)$ and $(H, E)$ are disjoint, $(K, E)$ and $(L, E)$ are disjoint. Since $(K, E)$ and $(L, E)$ are soft open, we obtain $(A, E) \not\subseteq \text{int}(K, E)$, $(B, E) \not\subseteq \text{int}(L, E)$ and $(A, E) \not\subseteq \text{int}(K, E) \cap (B, E) \not\subseteq \text{int}(L, E) = \emptyset$. Therefore $X$ is soft quasi-normal.

**Lemma: 4.13**

A subset $(A, E)$ of a space $X$ is soft $\pi g$-open if and only if $(F, E) \not\subseteq \text{int}(A, E)$ whenever $(F, E)$ is soft $\pi$-closed and $(F, E) \not\subseteq (A, E)$

**Theorem: 4.14**

Let $f: (X, \tau, E) \rightarrow (Y, \tau', E)$ be a soft $\pi$-continuous and soft almost $\pi g$-closed surjection. If $X$ is soft quasi-normal space then $Y$ is soft quasi-normal.

**Proof:**

Let $(A, E)$ and $(B, E)$ be any two disjoint soft $\pi$-closed sets of $Y$. Since $f$ is soft $\pi$-continuous, $f^{-1}(A, E)$ and $f^{-1}(B, E)$ are disjoint soft $\pi$-closed sets of $X$. Since $X$ is soft quasi-normal there exists a disjoint soft open sets $(U, E)$ and $(V, E)$ of $X$ such that $f^{-1}(A, E) \not\subseteq (U, E)$ and $f^{-1}(B, E) \not\subseteq (V, E)$. Let $(G, E) = \text{intcl} (U, E)$ and $(H, E) = \text{intcl}(V, E)$. Then $(G, E)$ and $(H, E)$ are disjoint soft regular open sets of $X$ such that $f^{-1}(A, E) \not\subseteq (G, E)$ and $f^{-1}(B, E) \not\subseteq (H, E)$. Then by theorem: 4.10 there exists soft $\pi g$-open sets $(K, E)$ and $(L, E)$ of $Y$ such that $(A, E) \not\subseteq (K, E)$ and $(B, E) \not\subseteq (L, E)$, $f^{-1}(K, E) \not\subseteq (G, E)$ and $f^{-1}(L, E) \not\subseteq (H, E)$. Since $(G, E)$ and $(H, E)$ are disjoint, $(K, E)$
and \((L, E)\) are disjoint. By previous we obtain \((A, E) \not\subseteq \text{int} (K, E), (B, E) \not\subseteq \text{int}(L, E)\) and \((A, E) \not\subseteq \text{int}(K, E) \cap (B, E) \not\subseteq \text{int}(L, E) = \emptyset\). Therefore \(Y\) is soft quasi-normal.

**Definition: 4.15**
A function \(f: (X, \tau, E) \rightarrow (Y, \tau', E)\) is said to be soft quasi \(\pi g\)-compact, if it is onto and if \((A, E)\) is soft \(\pi g\)-open (soft \(\pi g\)-closed) whenever \(f^{-1} (A, E)\) is soft open (soft closed).

**Definition: 4.16**
A function \(f: (X, \tau, E) \rightarrow (Y, \tau', E)\) is said to be soft almost quasi \(\pi g\)-compact, if it is onto and if \((A, E)\) is soft \(\pi g\)-open (soft \(\pi g\)-closed) whenever \(f^{-1} (A, E)\) is soft regular open (soft regular closed).

**Theorem: 4.17**
A function \(f: (X, \tau, E) \rightarrow (Y, \tau', E)\) of \(X\) onto \(Y\) is soft almost quasi \(\pi g\)-compact if and only if the image of every soft regular open inverse is soft \(\pi g\)-open.

**Proof:**
Let \(f\) be a soft almost quasi \(\pi g\)-compact. Let \((A, E)\) be any soft regular open inverse set. Then since \(f^{-1} ((f(A, E)) = (A, E)\) is soft regular open, \(f(A, E)\) is soft \(\pi g\)-open.
Conversely, if \(f^{-1} (F, E)\) be soft regular open, then \(f^{-1} (A, E)\) is soft regular inverse set. Therefore \(f(f^{-1} (A, E))\) is soft \(\pi g\)-open. That is \((F, E)\) is soft \(\pi g\)-open. Hence \(f\) is soft almost quasi \(\pi g\)-compact.

**Corollary: 4.18**
A function \(f: (X, \tau, E) \rightarrow (Y, \tau', E)\) of \(X\) onto \(Y\) is soft almost quasi \(\pi g\)-compact if and only if the image of every soft regular closed inverse is soft \(\pi g\)-closed.

**Theorem: 4.19**
If \(f: (X, \tau, E) \rightarrow (Y, \tau', E)\) is one to one functions of \(X\) onto \(Y\) then the following properties are equivalent:
1. \(f\) is soft almost \(\pi g\)-open
2. \(f\) is soft almost \(\pi g\)-closed.
3. \(f\) is soft almost quasi \(\pi g\)-compact
4. \(f^{-1}\) is soft almost \(\pi g\)-continuous

**Theorem: 4.20**
Suppose that \(f: (X, \tau, E) \rightarrow (Y, \tau', E)\) and \(g: (Y, \tau', E) \rightarrow (Z, \tau'', E)\) be functions. Then the following properties:
1. if \(f\) is soft almost \(\pi g\)-continuous and if \(g \circ f\) is soft \(\pi g\)-open then \(g\) is soft almost \(\pi g\)-open.
2. if \(f\) is soft almost \(\pi g\)-continuous and if \(g \circ f\) is soft \(\pi g\)-closed then \(g\) is soft almost \(\pi g\)-closed.
3. if \(f\) is soft almost \(\pi g\)-continuous and if \(g \circ f\) is soft quasi \(\pi g\)-compact then \(g\) is soft almost quasi \(\pi g\)-compact.
References
