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A New Theorem for the Prime Counting Function in Number Theory

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Abstract

In this paper, I am presenting (A new formula and the proof for it's correctness for the prime counting function). The prime counting function find how many prime numbers under any magnitude (Prime counting function); A prime number can be divided by 1 and itself and they are (2, 3, 5, 7, 11, 13, 17, ...). The prime counting function was conjectured in the end of the 18th century by Gauss and by Legendre to be approximately (not the real value) X/In(x), but in this paper I am presenting the real formula and the proof it's correctness for the prime counting function.

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1. Introduction

The main problem in Number Theory is to understand the distribution of Prime numbers. Let $\pi(x)$ denote the Prime counting function,defined as the number of primes less than or equal to X. Many deep problems in analytic number theory can be expressed in terms of the prime counting function $\pi(x)$. For example (the Riemann Hypothesis). So what Gauss and Legendre approximation solution X/In(x) in the sense that

$$\pi(x) \sim \frac{X}{logX} \text{ as } X \to +\infty.$$

This statement is the prime number theorem. So till now their is no formula for the prime counting function $\pi(x)$ as you see from the end of 18th century till now, in this paper I am presenting the real formula and the proof for it's correctness for the prime counting function $\pi(x)$.

Theorem 1.1. (For the (prime counting function).) I will present now the the prime counting function which is related to the sum of the prime numbers under or equal to (X), and the sum of $\pi(n)$:
First let: $\pi(x)$: Prime counting function.
X: Any positive integer
n: Any positive integer $n \geq 2$
P: prime numbers

The Prime counting function

$$\pi(x) = \frac{\sum_{p=2}^{p \leq x} p + \sum_{n=2}^{x-1} \pi(n)}{X}$$

$\sum_{p=2}^{p \leq X} p$: the sum of prime numbers under or equal to x $\sum_{n=2}^{n=x-1} \pi(n)$: the sum of $\pi(n)$.

And the first few prime numbers: 2, 3, 5, 7, 11, 13, 17, 19, 23,...). And the first few values of $\pi(n)$, for example $n = 1$ to $n = 10$ they are :(0, 1, 2, 2, 3, 3, 4, 4, 4, 4).

Proof theorem 1.1. For this (formula) which is:

$$\pi(x) = \frac{\sum_{p=2}^{p \leq x} p + \sum_{n=2}^{x-1} \pi(n)}{X}$$

Step 1: $\pi(2) = 1$
Step 2: Assume true for all $n \leq N$
Step 3: Proven for $(N + 1)$
Step 4: Assume $(N + 1)$ is a prime
Step 5: As we assume $(N + 1)$ is a prime that's mean $\pi(N + 1) - \pi(N) = 1$.

$$\pi(N + 1) = \frac{\sum_{p=2}^{N+1} p + \sum_{n=2}^{N} \pi(n)}{N + 1}$$

$$\pi(N + 1) = \frac{(N + 1) + \sum_{p=2}^{N} p + \pi(N) + \sum_{n=2}^{N-1} \pi(n)}{N + 1}$$

$$\pi(N+1) = \frac{N+1}{N+1} + \frac{\pi(N)}{N+1} + \frac{\sum_{p=2}^{N} p + \sum_{n=2}^{N-1} \pi(n)}{(N+1)}$$

By multiplying the formula with $\left[\frac{N}{N}\right]$

$$\pi(N+1) = 1 + \frac{\pi(N)}{N+1} + \frac{\sum_{p=2}^{N} p + \sum_{n=2}^{N-1} \pi(n)}{(N+1)} * \left(\frac{N}{N}\right)$$

$$\pi(N+1) = 1 + \frac{\pi(N)}{N+1} + \left(\left(\frac{\sum_{p=2}^{N} p + \sum_{n=2}^{N-1} \pi(n)}{(N+1)} * \frac{N}{N}\right)\right)$$

$$\pi(N+1) = 1 + \frac{\pi(N)}{N+1} + \left(\left(\frac{\sum_{p=2}^{N} p + \sum_{n=2}^{N-1} \pi(n)}{N} * \frac{N}{N+1}\right)\right)$$

And

$$\pi(N) = \frac{\sum_{p=2}^{N} p + \sum_{n=2}^{N-1} \pi(n)}{N}$$

So we will have:

$$\pi(N+1) = 1 + \frac{\pi(N)}{N+1} + \left(\left(\pi(N) * \frac{N}{(N+1)}\right)\right)$$

$$\pi(N+1) = 1 + \frac{\pi(N) + \pi(N) * N}{(N+1)}$$

$$\pi(N+1) = 1 + \frac{\pi(N) * (N+1)}{(N+1)}$$

$$\pi(N+1) = 1 + \pi(N)$$

$$\pi(N+1) - \pi(N) = 1.$$

Example for the theorem (1.1):

$$\pi(x) = \frac{\sum_{p=2}^{p\leq x} p + \sum_{n=2}^{x-1} \pi(n)}{X}$$

For $X = 10$. The prime numbers under 10 are (2, 3, 5, 7). The first values of $\pi(n)$ are (0, 1, 2, 2, 3, 3, 4, 4, 4, 4).

$$\pi(10) = \frac{(2+3+5+7) + (\pi(2) + \pi(3) + \pi(4) + \pi(5) + \pi(6) + \pi(7) + \pi(8) + \pi(9))}{10}$$

$$\pi(10) = \frac{(17) + (1+2+2+3+3+4+4+4)}{10}$$

$$\pi(10) = \frac{(17+23)}{10}$$

$$\pi(10) = \frac{40}{10}$$

$$\pi(10) = 4$$

2. Results

The prime counting function has many applications in Number Theory ; and it's related to one of the famous problem in mathematics; for example; (The Riemann Hypothesis) because the prime counting function is related to Riemann's function, and it has many thousands of applications across science and mathematics.

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