Some Results On Super Geometric Mean Labeling

S.S.Sandhya, E.Ebin Raja Merly and B.Shiny

1. Assistant Professor in Mathematics, Sree Ayyappa College for Women Chunkankadai-629 003 Kanyakumari District. E-mail:sssandhya2009@gmail.com
2. Assistant Professor in Mathematics, Nesamony Memorial Christian College, Marthandam- 629 165 Kanyakumari District E-mail:ebinmerly@gmail.com
3. Assistant Professor in Mathematics, DMI Engineering College, Aralvaimozhi- 629 301, Kanyakumari District E-mail: shinymaths89@gmail.com

ABSTRACT

Let \( f: V(G) \rightarrow \{1, 2, \ldots, p+q\} \) be an injective function. For a vertex labeling “f” the induced edge labeling \( f^*(e=uv) \) is defined by, 
\[
f^*(e) = \sqrt{f(u)f(v)} \quad \text{or} \quad \sqrt{f(u)+f(v)}
\]
then “f” is called a Super Geometric Mean labeling if \( \{f(V(G)) \cup f(e): e \in E(G)\} = \{1, 2, \ldots, p+q\} \). A graph which admits Super Geometric Mean labeling is called Super Geometric Mean Graph.
In this paper we investigate Super Geometric Mean Labeling for Some Standard Graphs.

Key words: Graph, Mean graph, Geometric mean graph, Super Geometric mean graph, Ladder, Comb.

1. Introduction

We begin with simple finite connected and undirected graph \( G=(V,E) \) with \( p \) vertices and \( q \) edges. The vertex set is denoted by \( V(G) \) and the edge set is denoted by \( E(G) \). A Path of length ‘n’ is denoted by \( P_n \) and the Cycle of length ‘n’ is denoted by \( C_n \). For all other standard terminology and notations we follow Harary[2] and for the detailed survey of graph labeling we follow Gallian J.A[1].
The Concept of "Geometric Mean Labeling" has introduced by S. Somasundaram, R. Ponraj, and P. Vidhyarani in [6]. Somasundaram. S and Ponraj.R introduced “Mean Labeling” in [4]. The concept of
“Harmonic Mean Labeling” has been introduced by Somasundaram.S, Ponraj.R and Sandhya.S.S in [5]. Jeyasekaran.C, Sandhya.S.S, and David Raj.C have introduced “Super Harmonic Mean Labeling” in [3]. In this paper, we discuss “Super Geometric Mean Labeling” behavior for some standard graphs. The definitions which are useful for the present investigation are given below.

**Definition 1.1:**
A graph G=(V,E) with p vertices and q edges is called a Geometric Mean Graph if it is possible to label vertices x∈V with distinct label f(x) from 1,2,…, q+1 in such a way that when each edge e=uv is labeled with,

\[ f'(e = uv) = \sqrt{f(u)f(v)} \]

then the edge labels are distinct. In this case, “f’” is called Geometric Mean Labeling of G.

**Definition 1.2:**
Let f:V(G)→{1,2,…, p+q} be an injective function. For a vertex labeling “f”, the induced edge labeling f*(e=uv) is defined by,

\[ f^{*}(e) = \sqrt{f(u)f(v)} \]

Then “f” is called a Super Geometric Mean labeling if

\[ \{f(V(G)) \cup \{f(e):e\in E(G)\}\} = \{1,2,…,p+q\}. \]

A graph which admits Super Geometric Mean labeling is called Super Geometric Mean Graph.

**Example 1.3:** A Super Geometric Mean labeling of a graph G is shown below.

![Figure 1](image-url)
Definition 1.4
The **Ladder** $L_n$, $n \geq 2$, is the product graph $P_n \times P_2$ and contains $2n$ vertices and $3n-2$ edges.

Definition 1.5
The graph obtained by joining a single pendant edge to each vertex of a Path is called a **Comb** $(P_n \odot K_1)$.

Now we shall use frequent reference to the following theorems.

**Theorem 1.6[6]**
Crowns are Geometric mean graphs.

**Theorem 1.7[6]**
Ladders are Geometric mean graphs.

**Theorem 1.8[6]**
Combs are Geometric mean graphs.

**Theorem 1.9[3]**
Crowns are Super Harmonic mean graphs.

**Theorem 1.10[3]**
Comb is a Super Harmonic mean graph.

**Remarks 1.11**
In a Super Geometric mean labeling, the labels of edges and vertices are together from 1, 2, ....p+q.

**2. Main Results**

**Theorem 2.1**
Let $G$ be a graph obtained from a Ladder $L_n$, $n \geq 2$ by joining a pendant vertex with a vertex of degree two on both sides of upper and lower Path of the Ladder. Then $G$ is Super Geometric mean Graph.

**Proof:**
Let $L_n = P_n \times P_2$ be a Ladder.

Let $G$ be a graph obtained from a Ladder by joining pendant vertices $u, w, x, z$ with $v_1, v_m, u_1, u_n$ (vertices of degree 2) respectively on both sides of upper and lower Path of the Ladder.

Define a function $f: V(G) \rightarrow \{1, 2, ..., p+q\}$ by,

$f(u) = 1$

$f(v_1) = 5$

$f(v_i) = 5i-1, 2 \leq i \leq n$

$f(w) = 5n+5$
\[ f(x) = 3 \]
\[ f(u_i) = 5i+3, \ 1 \leq i \leq n \]
\[ f(z) = 5n+6 \]

Edges are labeled with,
\[ f(v_i v_{i+1}) = 5i+2, \ 1 \leq i \leq n-1 \]
\[ f(uv_1) = 2 \]
\[ f(v_n w) = 5n+2 \]
\[ f(xu_1) = 4 \]
\[ f(u_i u_{i+1}) = 5i+5, \ 1 \leq i \leq n-1 \]
\[ f(u_n z) = 5n+4 \]
\[ f(v_i u_i) = 5i+1, \ 1 \leq i \leq n \]

In view of the above labeling pattern, \( f \) provides a Super Geometric Mean labeling of \( G \).
Hence \( G \) is a Super Geometric mean Graph.

**Example: 2.2**
A Super Geometric mean labeling of \( G \) when \( n=5 \) is shown below.

![Figure : 2](image)

**Theorem 2.3:**
Let \( G \) be a graph obtained by joining a pendant vertex with a vertex, of degree two of a Comb graph. Then \( G \) is a Super Geometric mean graph.

**Proof:**
Comb \( (P_n \odot K_1) \) is a graph obtained from a Path \( P_n = v_1 v_2 v_3 \ldots v_n \) by joining a vertex \( u_i \) to \( v_i, \ 1 \leq i \leq n \).
Let \( G \) be a graph obtained by joining a pendant vertex \( w \) to \( v_n \) (a vertex of degree 2)
Define a function \( f: V(G) \rightarrow \{1,2,3,\ldots,p+q\} \) by,
\[ f(v_i) = 4i-1, \ 1 \leq i \leq n \]
\[ f(w) = 4n+1 \]
\[ f(u_i) = 4i-3, 1 \leq i \leq n \]

Edges are labeled with,
\[ f(v_i v_{i+1}) = 4i, \ 1 \leq i \leq n-1 \]
\[ f(v_n w) = 4n \]
\[ f(v_i u_i) = 4i-2, \ 1 \leq i \leq n \]
Then the edge labels are distinct.
Hence G is a Super Geometric mean Graph.

**Example:2.4**

A Super Geometric mean labeling of G when n=6 is given below.

\[\begin{align*}
\text{Figure: 3}
\end{align*}\]

**Theorem 2.5**

Let G be a graph obtained by joining a pendant vertex with a vertex of degree two on both sides of a Comb graph. Then G is a Super Geometric mean Graph.

**Proof:**

Comb \((P_n \odot K_1)\) is a graph obtained from a Path \(P_n = v_1v_2v_3\ldots v_n\) by joining a vertex \(u_i\) to \(v_i\), \(1 \leq i \leq n\).

Let G be a graph obtained by joining pendant vertices w and z to \(v_1\) and \(v_n\) respectively.

Define a function \(f: V(G) \rightarrow \{1, 2, 3, \ldots, p+q\}\) by,

- \(f(w) = 1\)
- \(f(v_1) = 3\)
- \(f(v_i) = 4i+1, \ 2 \leq i \leq n\)
- \(f(z) = 4n+3\)
- \(f(u_1) = 5\)
- \(f(u_i) = 4i-1, \ 2 \leq i \leq n\)

Edges are labeled with,

- \(f(wv_1) = 2\)
- \(f(v_1v_{i+1}) = 4i+2, \ 1 \leq i \leq n-1\)
- \(f(v_nv) = 4n+2\)
- \(f(v_iu_i) = 4i, \ 1 \leq i \leq n\)

\(\therefore f(V(G)) \cup \{f(e): e \in E(G)\} = \{1, 2, \ldots, p+q\}\)

Thus f is a Super Geometric mean Labeling of G.
Hence G is a Super Geometric mean Graph.

**Example:2.6**

A Super Geometric mean labeling of G when n=5 is given below.
Theorem 2.7

Let \( P_n \) be a Path and \( G \) be the graph obtained from \( P_n \) by attaching \( C_3 \) in both end edges of \( P_n \). Then \( G \) is a Super Geometric mean graph.

Proof:
Let \( P_n \) be a Path \( u_1 u_2 \ldots u_n \) and \( v_1 u_1 u_2, v_2 u_{n-1} u_n \) be the triangles at the end edges of \( P_n \). Define a function \( f: V(G) \rightarrow \{1, 2, 3, \ldots, p+q\} \) by,

\[
  \begin{align*}
  f(v_1) &= 4 \\
  f(u_1) &= 1 \\
  f(u_i) &= 2i + 2, \ 2 \leq i \leq n-1 \\
  f(u_n) &= 2n + 5 \\
  f(v_2) &= 2n + 2 \\
  f(v_1 u_1) &= 2 \\
  f(v_1 u_2) &= 5 \\
  f(u_1 u_2) &= 3 \\
  f(u_i u_{i+1}) &= 2i + 3, \ 2 \leq i \leq n-2 \\
  f(u_{n-1} u_n) &= 2n + 3 \\
  f(v_2 u_{n-1}) &= 2n + 1 \\
  f(v_2 u_n) &= 2n + 4
  \end{align*}
\]

Edges are labeled with,

Thus both vertices and edges together get distinct labels from \( \{1, 2, 3, \ldots, p+q\} \).

Hence \( G \) is a Super Geometric mean Graph.

Example: 2.8
A Super Geometric mean labeling of \( G \) obtained from \( P_8 \) is given below.

\[\text{Figure: 5}\]
References:


