Root Square Mean Labeling of Subdivision of Some More Graphs

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Abstract

A graph $G = (V,E)$ with $p$ vertices and $q$ edges is called a Root Square Mean graph if it is possible to label the vertices $x \in V$ with distinct labels $f(x)$ from $1, 2, \ldots, q + 1$ in such a way that when each edge $e = uv$ is labeled with $f(e) = uv = \sqrt{\frac{f(u)^2 + f(v)^2}{2}}$ or $\sqrt{\frac{f(u)^2 + f(v)^2}{2}}$, then the edge labels are distinct. In this case $f$ is called Root Square Mean Labeling of $G$. In this paper we prove that subdivision of some graphs are Root Square Mean graphs.

Key Words: Graph, Root Square Mean graph, Triangular Snake, Quadrilateral Snake, Alternate Triangular Snake, Alternate Quadrilateral Snake.

1. Introduction

All graphs in this paper are finite, simple, and undirected graph $G = (V,E)$ with $p$ vertices and $q$ edges. For all detailed survey of graph labeling we refer to Gallian [1]. For all other standard terminology and notations we follow Harary [2]. Root Square Mean labeling was introduced by S.S.Sandhya, S.Somasundaram, and S.Anusa in [3] and studied their behavior in [4], [5], [6], [7], [8], [9], [10]. In this paper we prove that subdivision of some graphs are Root Square Mean graphs. The following definitions and theorems are necessary for our present study.
**Definition 1.1:** A graph $G = (V, E)$ with $p$ vertices and $q$ edges is called a Root Square Mean graph if it is possible to label the vertices $x \in V$ with distinct labels $f(x)$ from $1, 2, \ldots, q + 1$ in such a way that when each edge $e = uv$ is labeled with $f(e = uv) = \left[ \sqrt{\frac{f(u)^2 + f(v)^2}{2}} \right]$ or $\left[ \sqrt{\frac{f(u)^2 + f(v)^2}{2}} \right]$, then the edge labels are distinct. In this case $f$ is called Root Square Mean Labeling of $G$.

**Definition 1.2:** A Triangular Snake $T_n$ is obtained from a path $u_1u_2 \cdots u_n$ by joining $u_i$ and $u_{i+1}$ to a new vertex $v_i$, $1 \leq i \leq n - 1$.

**Definition 1.3:** An Alternate Triangular Snake $A(T_n)$ is obtained from a path $u_1u_2 \cdots u_n$ by joining $u_i$ and $u_{i+1}$ (Alternatively) to a new vertex $v_i$.

**Definition 1.4:** A Quadrilateral Snake $Q_n$ is obtained from a path $u_1u_2 \cdots u_n$ by joining $u_i$ and $u_{i+1}$ to two new vertices $v_i$ and $w_i$, $1 \leq i \leq n - 1$ respectively and then joining $v_i$ and $w_i$.

**Definition 1.5:** An Alternate Quadrilateral Snake $A(Q_n)$ is obtained from a path $u_1u_2 \cdots u_n$ by joining $u_i$ and $u_{i+1}$ (Alternatively) to new vertices $v_i$ and $w_i$.

**Theorem 1.6:** A Triangular snake $T_n$ is a Root Square Mean graph.

**Theorem 1.7:** Alternate Triangular Snake $A(T_n)$ is a Root Square Mean graph.

**Theorem 1.8:** A Quadrilateral Snake $Q_n$ is a Root Square Mean graph.

**Theorem 1.9:** Alternate Quadrilateral Snake $A(Q_n)$ is a Root Square Mean graph.

### 2. Main Results

**Theorem 2.1:** $S(T_n)$ is a Root Square Mean graph.

**Proof:** Let $u_1u_2 \cdots u_n$ be a path of length $n$. Let $T_n$ be the triangular snake obtained by joining $u_i$ and $u_{i+1}$ to a new vertex $v_i$, $1 \leq i \leq n - 1$. Let us subdivide the edges of $T_n$. Here we consider the following cases.

**Case(1):** $G$ is obtained by subdividing each edge of the path.

Let $t_1, t_2, \ldots, t_{n-1}$ be the vertices which subdivide the edge $u_iu_{i+1}$.

Define a function $f: V(G) \rightarrow \{1, 2, \ldots, q + 1\}$ by

- $f(u_i) = 4i - 3, 1 \leq i \leq n$
- $f(v_i) = 4i - 2, 1 \leq i \leq n - 1$
- $f(t_i) = 4i, 1 \leq i \leq n - 1$
Then the edges are labeled as
\[ f(u_iv_i) = 4i - 3, 1 \leq i \leq n - 1 \]
\[ f(u_it_i) = 4i - 2, 1 \leq i \leq n - 1 \]
\[ f(t_iu_{i+1}) = 4i, 1 \leq i \leq n - 1 \]
\[ f(v_iu_{i+1}) = 4i - 1, 1 \leq i \leq n - 1 \]

Then we get distinct edge labels. Hence \( f \) is a Root Square Mean labeling of \( G \).

The labeling pattern of \( S(T_5) \) is shown below.

![Figure 1](image1)

**Case(2):** \( G \) is obtained by subdividing the edges \( u_iv_i \) and \( u_{i+1}v_i \).
Let \( x_i \) and \( y_i \) be the two vertices which subdivide the edges \( u_iv_i \) and \( u_{i+1}v_i, 1 \leq i \leq n - 1 \) respectively. Define a function \( f:V(G) \rightarrow \{1,2,\ldots,q + 1\} \) by
\[ f(u_i) = 5i - 4, 1 \leq i \leq n \]
\[ f(v_i) = 5i - 2, 1 \leq i \leq n - 1 \]
\[ f(x_i) = 5i - 3, 1 \leq i \leq n - 1 \]
\[ f(y_i) = 5i - 1, 1 \leq i \leq n - 1 \]

Then the edges are labeled as
\[ f(u_ix_i) = 5i - 4, 1 \leq i \leq n - 1 \]
\[ f(x_iv_i) = 5i - 3, 1 \leq i \leq n - 1 \]
\[ f(v_iy_i) = 5i - 2, 1 \leq i \leq n - 1 \]
\[ f(y_iu_{i+1}) = 5i, 1 \leq i \leq n - 1 \]
\[ f(u_iu_{i+1}) = 5i - 1, 1 \leq i \leq n - 1 \]

Then we get distinct edge labels. Hence \( f \) is a Root Square Mean labeling of \( G \).

The labeling pattern of \( S(T_5) \) is shown below.

![Figure 2](image2)
**Case (3):** \( G \) is obtained by subdividing all the edges of \( T_n \).
Let \( x_i, y_i \) and \( t_i \) be the vertices which subdivide the edges \( u_i v_i, v_i u_{i+1} \) and \( u_i u_{i+1} \) respectively. Define a function \( f: V(G) \to \{1, 2, \ldots, q + 1\} \) by
\[
\begin{align*}
f(u_i) &= 6i - 5, 1 \leq i \leq n \\
f(v_i) &= 6i - 3, 1 \leq i \leq n - 1 \\
f(x_i) &= 6i - 4, 1 \leq i \leq n - 1 \\
f(y_i) &= 6i - 1, 1 \leq i \leq n - 1 \\
f(t_i) &= 6i - 2, 1 \leq i \leq n - 1
\end{align*}
\]

Then the edges are labeled as
\[
\begin{align*}
f(u_i t_i) &= 6i - 3, 1 \leq i \leq n - 1 \\
f(u_i x_i) &= 6i - 5, 1 \leq i \leq n - 1 \\
f(x_i v_i) &= 6i - 4, 1 \leq i \leq n - 1 \\
f(t_i u_{i+1}) &= 6i - 1, 1 \leq i \leq n - 1 \\
f(v_i y_i) &= 6i - 2, 1 \leq i \leq n - 1 \\
f(y_i u_{i+1}) &= 6i - 1, 1 \leq i \leq n - 1
\end{align*}
\]

Then we get distinct edge labels. Hence \( f \) is a Root Square Mean labeling of \( G \).

The labeling pattern of \( S(T_5) \) is shown below.

**Theorem 2.2:** \( S(Q_n) \) is a Root Square Mean graph.

**Proof:** Let \( u_1 u_2 \cdots u_n \) be a path \( P_n \). Join \( u_i \) and \( u_{i+1} \) to new vertices \( v_i \) and \( w_i \), \( 1 \leq i \leq n - 1 \) respectively and then joining \( v_i \) and \( w_i \). The resulting graph is a Quadrilateral snake \( Q_n \). Let \( G \) be the graph obtained by subdividing the edges of \( Q_n \). Here we consider the following cases.

**Case (1):** \( G \) is obtained by subdividing the edges of the path.
Let \( t_1, t_2, \cdots, t_{n-1} \) be the vertices which subdivide the edge \( u_i u_{i+1}, 1 \leq i \leq n - 1 \).
Define a function \( f: V(G) \to \{1, 2, \ldots, q + 1\} \) by
\[
\begin{align*}
f(u_i) &= 5i - 4, 1 \leq i \leq n \\
f(v_i) &= 5i - 3, 1 \leq i \leq n - 1 \\
f(w_i) &= 5i - 2, 1 \leq i \leq n - 1 \\
f(t_i) &= 5i - 1, 1 \leq i \leq n - 1
\end{align*}
\]

Then the edges are labeled as
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\[ f(u_i v_i) = 5i - 4, 1 \leq i \leq n - 1 \]
\[ f(v_i w_i) = 5i - 3, 1 \leq i \leq n - 1 \]
\[ f(w_i u_{i+1}) = 5i - 1, 1 \leq i \leq n - 1 \]
\[ f(u_i t_i) = 5i - 2, 1 \leq i \leq n - 1 \]
\[ f(t_i u_{i+1}) = 5i, 1 \leq i \leq n - 1 \]

Then the edge labels are distinct. Hence \( G \) is a Root Square Mean graph.

The labeling pattern of \( S(Q_5) \) is shown below.

Case(2): \( G \) is obtained by subdividing all the edges of \( Q_n \).

Let \( t_i, x_i, s_i, y_i \) be the vertices which subdivide the edges \( u_i u_{i+1}, u_i v_i, v_i w_i \) and \( w_i u_{i+1} \) respectively. Define a function \( f: V(G) \to \{1, 2, \ldots, q + 1\} \) by

\[ f(u_i) = 8i - 7, 1 \leq i \leq n \]
\[ f(x_i) = 8i - 6, 1 \leq i \leq n - 1 \]
\[ f(v_i) = 8i - 5, 1 \leq i \leq n - 1 \]
\[ f(s_i) = 8i - 4, 1 \leq i \leq n - 1 \]
\[ f(w_i) = 8i - 3, 1 \leq i \leq n - 1 \]
\[ f(y_i) = 8i - 2, 1 \leq i \leq n - 1 \]
\[ f(t_i) = 8i, 1 \leq i \leq n - 1 \]

Then the edges are labeled as

\[ f(u_i x_i) = 8i - 7, 1 \leq i \leq n - 1 \]
\[ f(x_i v_i) = 8i - 6, 1 \leq i \leq n - 1 \]
\[ f(v_i s_i) = 8i - 5, 1 \leq i \leq n - 1 \]
\[ f(s_i w_i) = 8i - 4, 1 \leq i \leq n - 1 \]
\[ f(w_i y_i) = 8i - 3, 1 \leq i \leq n - 1 \]
\[ f(y_i u_{i+1}) = 8i - 2, 1 \leq i \leq n - 1 \]
\[ f(u_{i+1} t_i) = 8i, 1 \leq i \leq n - 1 \]
\[ f(t_i u_i) = 8i - 3, 1 \leq i \leq n - 1 \]

Then the edge labels are distinct. Hence \( G \) is a Root Square Mean graph.

The labeling pattern of \( S(Q_4) \) is shown below.
From case(1), case(2), it is clear that, $S(Q_n)$ is a Root Square Mean graph.

**Theorem 2.3:** $S(A(T_n))$ is a Root Square Mean graph.

**Proof:** Let $u_1u_2\cdots u_n$ be the path. Let $A(T_n)$ be the alternate triangular snake obtained by joining $u_i$ and $u_{i+1}$ (Alternatively) to a new vertex $v_i, 1 \leq i \leq n - 1$. Let $G$ be the graph obtained by subdividing the edges of $A(T_n)$. Here we consider two cases

**Case(1):** If the triangle starts from $u_1$.
Let $t_i, x_i, y_i$ be the vertices which subdivide the edges $u_iu_{i+1}, u_iv_i, v_iu_{i+1}$ respectively.
Here we have to consider two sub cases

**Sub case(1.a):** If $n$ is odd
Define a function $f: V(G) \rightarrow \{1, 2, \ldots, q + 1\}$ by
$$f(u_i) = \begin{cases} 
4i - 3, & i = 1, 3, 5, \ldots, n \\
4i - 1, & i = 2, 4, 6, \ldots, n - 1
\end{cases}$$
$$f(v_i) = 8i - 5, 1 \leq i \leq \frac{n - 1}{2}$$
$$f(x_i) = 8i - 6, 1 \leq i \leq \frac{n - 1}{2}$$
$$f(y_i) = 8i - 4, 1 \leq i \leq \frac{n - 1}{2}$$
$$f(t_i) = \begin{cases} 
4i + 2, & i = 1, 3, 5, \ldots, n - 2 \\
4i, & i = 2, 4, 6, \ldots, n - 1
\end{cases}$$

Then the edges are labeled as
$$f(u_it_i) = \begin{cases} 
4i, & i = 1, 3, 5, \ldots, n - 2 \\
4i - 1, & i = 2, 4, 6, \ldots, n - 1
\end{cases}$$
$$f(t_iu_{i+1}) = \begin{cases} 
4i + 2, & i = 1, 3, 5, \ldots, n - 2 \\
4i, & i = 2, 4, 6, \ldots, n - 1
\end{cases}$$
$$f(u_{2i-1}x_i) = 8i - 7, 1 \leq i \leq \frac{n - 1}{2}$$
$$f(x_iv_i) = 8i - 6, 1 \leq i \leq \frac{n - 1}{2}$$
Sub Case (1.b): If \( n \) is even
Define a function \( f: V(G) \rightarrow \{1,2,\ldots,q+1\} \) by

\[
\begin{align*}
\text{If } n \text{ is even, then define } f & : V(G) \rightarrow \{1,2,\ldots,q+1\} \text{ by } \\
f(u_i) &= \begin{cases} 
4i - 3, & i = 1,3,5,\ldots,n-1 \\
4i - 1, & i = 2,4,6,\ldots,n \n\end{cases} \\
f(v_i) &= 8i - 5, 1 \leq i \leq \frac{n-1}{2} \\
f(x_i) &= 8i - 6, 1 \leq i \leq \frac{n}{2} \\
f(y_i) &= 8i - 4, 1 \leq i \leq \frac{n}{2} \\
f(t_i) &= \begin{cases} 
4i + 2, & i = 1,3,5,\ldots,n-1 \\
4i, & i = 2,4,6,\ldots,n-2 \n\end{cases}
\end{align*}
\]

Then the edges are labeled as

\[
\begin{align*}
\text{If } n \text{ is even, then define } f & : V(G) \rightarrow \{1,2,\ldots,q+1\} \text{ by } \\
f(u_iu_i+1) &= \begin{cases} 
4i - 1, & i = 2,4,6,\ldots,n-2 \\
4i, & i = 2,4,6,\ldots,n-2 \n\end{cases} \\
f(u_{2i-1}x_i) &= 8i - 7, 1 \leq i \leq \frac{n}{2} \\
f(x_iu_i) &= 8i - 6, 1 \leq i \leq \frac{n}{2} \\
f(v_iu_i) &= 8i - 5, 1 \leq i \leq \frac{n}{2} \\
f(y_iu_{2i}) &= 8i - 3, 1 \leq i \leq \frac{n}{2}
\end{align*}
\]

Then the edge labels are distinct. Hence \( G \) is a Root Square Mean graph. The labeling pattern of \( S(A(T_6)) \) is shown below.
Figure 7

Case(2) If the triangle starts from $u_2$.
Let $t_i, x_i, y_i$ be the vertices which subdivide the edges $u_iu_{i+1}, u_iv_i, v_iu_i$ respectively.
Here we have to consider two sub cases

Sub case(2.a): If $n$ is odd
Define a function $f: V(G) \to \{1, 2, \ldots, q + 1\}$ by
\[
\begin{align*}
 f(u_i) &= \begin{cases} 
 4i - 3, & i = 1, 3, 5, \ldots, n \\
 4i - 5, & i = 2, 4, 6, \ldots, n - 1 
\end{cases} \\
 f(v_i) &= 8i - 3, 1 \leq i \leq \frac{n-1}{2} \\
 f(x_i) &= 8i - 4, 1 \leq i \leq \frac{n-1}{2} \\
 f(y_i) &= 8i - 2, 1 \leq i \leq \frac{n-1}{2} \\
 f(t_i) &= \begin{cases} 
 4i - 2, & i = 1, 3, 5, \ldots, n - 2 \\
 4i, & i = 2, 4, 6, \ldots, n - 1 
\end{cases}
\end{align*}
\]

Then the edges are labeled as
\[
\begin{align*}
 f(u_i t_i) &= \begin{cases} 
 4i - 3, & i = 1, 3, 5, \ldots, n - 2 \\
 4i - 2, & i = 2, 4, 6, \ldots, n - 1 
\end{cases} \\
 f(t_i u_{i+1}) &= \begin{cases} 
 4i - 2, & i = 1, 3, 5, \ldots, n - 2 \\
 4i, & i = 2, 4, 6, \ldots, n - 1 
\end{cases} \\
 f(u_{2i} x_i) &= 8i - 5, 1 \leq i \leq \frac{n-1}{2} \\
 f(x_i v_i) &= 8i - 4, 1 \leq i \leq \frac{n-1}{2} \\
 f(v_i y_i) &= 8i - 3, 1 \leq i \leq \frac{n-1}{2} \\
 f(y_i u_{2i+1}) &= 8i - 1, 1 \leq i \leq \frac{n-1}{2}
\end{align*}
\]

Then the edge labels are distinct. Hence $G$ is a Root Square Mean graph.
The labeling pattern of $S(A(T_5))$ is shown below.
Sub case(2.b): If \( n \) is even

Define a function \( f: V(G) \to \{1,2,...,q+1\} \) by

\[
f(u_i) = \begin{cases} 
4i - 3, & i = 1,3,5,..., n-1 \\
4i - 5, & i = 2,4,6,..., n 
\end{cases}
\]

\[
f(v_i) = 8i - 3, 1 \leq i \leq \frac{n - 2}{2}
\]

\[
f(x_i) = 8i - 4, 1 \leq i \leq \frac{n - 2}{2}
\]

\[
f(y_i) = 8i - 2, 1 \leq i \leq \frac{n - 2}{2}
\]

\[
f(t_i) = \begin{cases} 
4i - 2, & i = 1,3,5,..., n-1 \\
4i, & i = 2,4,6,..., n-2 
\end{cases}
\]

Then the edges are labeled as

\[
f(u_i t_i) = \begin{cases} 
4i - 3, & i = 1,3,5,..., n-1 \\
4i - 2, & i = 2,4,6,..., n-2 
\end{cases}
\]

\[
f(t_i u_{i+1}) = \begin{cases} 
4i - 2, & i = 1,3,5,..., n-1 \\
4i, & i = 2,4,6,..., n-2 
\end{cases}
\]

\[
f(u_{2i} x_i) = 8i - 5, 1 \leq i \leq \frac{n - 2}{2}
\]

\[
f(x_i v_i) = 8i - 4, 1 \leq i \leq \frac{n - 2}{2}
\]

\[
f(v_i y_i) = 8i - 3, 1 \leq i \leq \frac{n - 2}{2}
\]

\[
f(y_1 u_{2i+1}) = 8i - 1, 1 \leq i \leq \frac{n - 2}{2}
\]

Then the edge labels are distinct. Hence \( G \) is a Root Square Mean graph.

The labeling pattern of \( S(A(T_6)) \) is shown below.
From case(1) and case(2), $S(A(T_n))$ is a Root Square Mean graph.

**Theorem 2.4:** $S(A(Q_n))$ is a Root Square Mean graph.

**Proof:** Let $u_1u_2\cdots u_n$ be the path. $A(Q_n)$ is obtained by joining $u_i$ and $u_{i+1}$ (Alternatively) to two new vertices $v_i$ and $w_i$ respectively and then joining $v_i$ and $w_i$. Let $G$ be the graph obtained by subdividing the edges of $A(Q_n)$. Here we consider two cases.

**Case(1):** If the Quadrilateral snake starts from $u_1$.
Let $t_i,x_i,y_i,s_i$ be the vertices which subdivide the edges $u_iu_{i+1},u_iv_i,w_iu_{i+1},v_iw_i$ respectively.

Here we have to consider two sub cases.

**Sub case(1.a):** If $n$ is odd.
Define a function $f: V(G) \rightarrow \{1,2,\ldots,q+1\}$ by

$$f(u_i) = \begin{cases} 
5i - 4, & i = 1,3,5,\ldots,n \\
5i - 1, & i = 2,4,6,\ldots,n-1
\end{cases}$$

$$f(v_i) = 10i - 7, 1 \leq i \leq \frac{n-1}{2}$$

$$f(w_i) = 10i - 5, 1 \leq i \leq \frac{n-1}{2}$$

$$f(x_i) = 10i - 8, 1 \leq i \leq \frac{n-1}{2}$$

$$f(y_i) = 10i - 4, 1 \leq i \leq \frac{n-1}{2}$$

$$f(s_i) = 10i - 6, 1 \leq i \leq \frac{n-1}{2}$$

$$f(t_i) = \begin{cases} 
5i + 2, & i = 1,3,5,\ldots,n-2 \\
5i, & i = 2,4,6,\ldots,n-1
\end{cases}$$

Then the edges are labeled as

$$f(u_it_i) = \begin{cases} 
5i, & i = 1,3,5,\ldots,n-2 \\
5i - 1, & i = 2,4,6,\ldots,n-1
\end{cases}$$

$$f(t_iu_{i+1}) = \begin{cases} 
5i + 3, & i = 1,3,5,\ldots,n-2 \\
5i, & i = 2,4,6,\ldots,n-1
\end{cases}$$

$$f(u_{2i-1}x_i) = 10i - 9, 1 \leq i \leq \frac{n-1}{2}$$

$$f(x_iv_i) = 10i - 8, 1 \leq i \leq \frac{n-1}{2}$$

$$f(v_is_i) = 10i - 7, 1 \leq i \leq \frac{n-1}{2}$$

$$f(s_iw_i) = 10i - 6, 1 \leq i \leq \frac{n-1}{2}$$

$$f(w_iy_i) = 10i - 4, 1 \leq i \leq \frac{n-1}{2}$$
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\[ f(y_i u_{2i}) = 10i - 3, 1 \leq i \leq \frac{n - 1}{2} \]

Then the edge labels are distinct. Hence \( G \) is a Root Square Mean graph. The labeling pattern of \( S(A(Q_5)) \) is shown below.

**Figure 10**

**Sub case (1.b)** If \( n \) is even

Define a function \( f: V(G) \rightarrow \{1, 2, \ldots, q + 1\} \) by

\[
\begin{align*}
  f(u_i) &= \begin{cases} 
    5i - 4, & i = 1, 3, 5, \ldots, n - 1 \\
    5i - 1, & i = 2, 4, 6, \ldots, n 
  \end{cases} \\
  f(v_i) &= 10i - 7, 1 \leq i \leq \frac{n}{2} \\
  f(w_i) &= 10i - 5, 1 \leq i \leq \frac{n}{2} \\
  f(x_i) &= 10i - 8, 1 \leq i \leq \frac{n}{2} \\
  f(y_i) &= 10i - 4, 1 \leq i \leq \frac{n}{2} \\
  f(s_i) &= 10i - 6, 1 \leq i \leq \frac{n}{2} \\
  f(t_i) &= \begin{cases} 
    5i + 2, & i = 1, 3, 5, \ldots, n - 1 \\
    5i, & i = 2, 4, 6, \ldots, n - 2 
  \end{cases}
\]

Then the edges are labeled as

\[
\begin{align*}
  f(u_i t_i) &= \begin{cases} 
    5i, & i = 1, 3, 5, \ldots, n - 1 \\
    5i - 1, & i = 2, 4, 6, \ldots, n - 2 
  \end{cases} \\
  f(t_i u_{i+1}) &= \begin{cases} 
    5i + 3, & i = 1, 3, 5, \ldots, n - 1 \\
    5i, & i = 2, 4, 6, \ldots, n - 2 
  \end{cases} \\
  f(u_{2i-1} x_i) &= 10i - 9, 1 \leq i \leq \frac{n}{2} \\
  f(x_i v_i) &= 10i - 8, 1 \leq i \leq \frac{n}{2} \\
  f(v_i s_i) &= 10i - 7, 1 \leq i \leq \frac{n}{2} \\
  f(s_i w_i) &= 10i - 6, 1 \leq i \leq \frac{n}{2} \\
  f(w_i y_i) &= 10i - 4, 1 \leq i \leq \frac{n}{2}
\end{align*}
\]
$f(y_iu_{2i}) = 10i - 3, 1 \leq i \leq \frac{n}{2}$

Then the edge labels are distinct. Hence $G$ is a Root Square Mean graph.

The labeling pattern of $S(A(Q_6))$ is shown below.

**Figure 11**

**Case(2):** If the triangle starts from $u_2$.

Let $t_i, x_i, y_i, s_i$ be the vertices which subdivide the edges $u_iu_{i+1}, u_iv_i, w_iu_{i+1}, v_iw_i$ respectively.

Here we consider two sub cases

**Sub case(1.a):** If $n$ is odd.

Define a function $f: V(G) \rightarrow \{1,2,...,q+1\}$ by

$f(u_i) = \begin{cases} 5i - 4, & i = 1,3,5,...,n \\ 5i - 7, & i = 2,4,6,...,n-1 \end{cases}$

$f(v_i) = 10i - 5, 1 \leq i \leq \frac{n-1}{2}$

$f(w_i) = 10i - 3, 1 \leq i \leq \frac{n-1}{2}$

$f(x_i) = 10i - 6, 1 \leq i \leq \frac{n-1}{2}$

$f(y_i) = 10i - 2, 1 \leq i \leq \frac{n-1}{2}$

$f(s_i) = 10i - 4, 1 \leq i \leq \frac{n-1}{2}$

$f(t_i) = \begin{cases} 5i - 3, & i = 1,3,5,...,n-2 \\ 5i - 1, & i = 2,4,6,...,n-1 \end{cases}$

Then the edges are labeled as

$f(t_iu_{i+1}) = \{5i - 3, i = 1,3,5,...,n-2 \}$

$f(u_it_i) = 5i - 4, 1 \leq i \leq n-1$

$f(u_{2i}x_i) = 10i - 7, 1 \leq i \leq \frac{n-1}{2}$

$f(x_iv_i) = 10i - 6, 1 \leq i \leq \frac{n-1}{2}$

$f(v_is_i) = 10i - 5, 1 \leq i \leq \frac{n-1}{2}$
Root Square Mean Labeling of Subdivision of Some More Graphs

\[\begin{align*}
f(s_iw_i) &= 10i - 3, 1 \leq i \leq \frac{n - 1}{2} \\
f(w_iy_i) &= 10i - 2, 1 \leq i \leq \frac{n - 1}{2} \\
f(y_iu_{2i+1}) &= 10i - 1, 1 \leq i \leq \frac{n - 1}{2}
\end{align*}\]

Then the edge labels are distinct. Hence \(G\) is a Root Square Mean graph.

The labeling pattern of \(S(A(Q_5))\) is shown below.

![Figure 12](image.png)

**Sub case(1.a):** If \(n\) is even.

Define a function \(f: V(G) \rightarrow \{1,2,\ldots,q+1\}\) by

\[\begin{align*}
f(u_i) &= \begin{cases} 
5i - 4, & i = 1,3,5,\ldots,n-1 \\
5i - 7, & i = 2,4,6,\ldots,n
\end{cases} \\
f(v_i) &= 10i - 5, 1 \leq i \leq \frac{n - 2}{2} \\
f(w_i) &= 10i - 3, 1 \leq i \leq \frac{n - 2}{2} \\
f(x_i) &= 10i - 6, 1 \leq i \leq \frac{n - 2}{2} \\
f(y_i) &= 10i - 2, 1 \leq i \leq \frac{n - 2}{2} \\
f(s_i) &= 10i - 4, 1 \leq i \leq \frac{n - 2}{2} \\
f(t_i) &= \begin{cases} 
5i - 3, & i = 1,3,5,\ldots,n-1 \\
5i - 1, & i = 2,4,6,\ldots,n-2
\end{cases}
\end{align*}\]

Then the edges are labeled as

\[\begin{align*}
f(t_iu_{i+1}) &= \begin{cases} 
5i - 3, & i = 1,3,5,\ldots,n-1 \\
5i, & i = 2,4,6,\ldots,n-2
\end{cases} \\
f(u_it_i) &= 5i - 4, 1 \leq i \leq n - 1 \\
f(u_{2i}x_i) &= 10i - 7, 1 \leq i \leq \frac{n - 2}{2} \\
f(x_iv_i) &= 10i - 6, 1 \leq i \leq \frac{n - 2}{2} \\
f(v_is_i) &= 10i - 5, 1 \leq i \leq \frac{n - 2}{2}
\end{align*}\]
\[ f(s_iw_i) = 10i - 3, 1 \leq i \leq \frac{n-2}{2} \]
\[ f(w_iy_i) = 10i - 2, 1 \leq i \leq \frac{n-2}{2} \]
\[ f(y_iw_{2i+1}) = 10i - 1, 1 \leq i \leq \frac{n-2}{2} \]

Then the edge labels are distinct. Hence \( G \) is a Root Square Mean graph.

The labeling pattern of \( S(A(Q_6)) \) is shown below.

From case(1) and case(2), \( S(A(Q_n)) \) is a Root Square Mean graph.

Reference:


