International Journal of Mathematics Research.
ISSN 0976-5840 Volume 6, Number 2 (2014), pp. 191-204

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Square Difference Labeling of Graphs

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Abstract

A graph G=(V,E) with p vertices and q edges is said to admit square difference labeling, if there exists a bijection f: V(G)→{ 0,1, …., p-1} such that the induced function f*: E(G)→N given by $f^*(uv)=|[f(u)]^2-[f(v)]^2|$ for every uv ∈ E(G) are all distinct. A graph which admits square difference labeling is called square difference graph.
In this paper we prove that some classes of graph like Alternative Double Triangular Snake, Alternative Triangular Snake, Banana Tree, Umbrella Graph, $P_n(Qs_n)$ graph, $C_n(Q_{sn})$ graphs are square difference graphs.

Keywords: Square difference labeling, square difference graph.

1. Introduction

All graphs in this paper are simple finite undirected and nontrivial graph G=(V, E) with vertex set V and the edge set E. For graph theoretic terminology, we refer to Harary [2]. A dynamic survey on graph labeling is regularly updated by Gallian [3] and it is published by Electronic Journal of Combinatorics. Vast amount of literature is available on different types of graph labeling and more than 1000 research papers have been published so far in past three decades. The square sum labeling is previously defined by V. Ajitha, S. Arumugam and K. A. Germina [1]. The concept of square difference labeling was first introduced by J. Shiama proved in [6] many standard graphs like Pn, Cn, complete graphs, cycle cactus, ladder, lattice grids, quadrilateral snakes, Wheels, K_2+ m K_1, comb, star graphs, m K_3, m C_3,duplication of vertices by an edge to some star graphs and crown graphs are square difference graphs. Also proved that the path is an odd square difference graphs and star graphs are perfect square graphs. Some graphs like shadow and split graphs [4] and [5] can also be investigated for the square difference.

Definition: 1.1: Let G=(V (G), E (G)) be a graph. G is said to be square difference labeling if there exist a bijection f: V(G)→{0,1,2,….p-1} such that the induced function f*: E(G)→N given by $f^*(u\,v)=[f(u)]^2-[f(v)]^2|$ is injective.

Definition: 1.2: Let $G_1,G_2,\dots,G_n$ be a family of disjoint stars. The tree obtained by joining a new vertex a to one pendant vertex of each star Gi is called a banana tree. Let K_1 n_1, K_1 n_2, …,K_1n_k be a family of disjoint stars with the vertex-sets V $(K_1;n_i)$=fc_i, a_{i1}, ainig and $\deg(c_i)=n_i$; . 1≤i≤k. A banana tree BT(n_1, n_2,…n_k)is a tree obtained by adding a new vertex a and joining it to $a_{11,}$ a_{21},…$a_{1k.}$

Definition: 1.3: Let G=Pn (Qs_n) is a graph let V(G)={u_1,u_2, . .,un,v_1,v_2,…,vmn,w_1,w_2,…,wmn}be the vertices of the graph and E(G)={u_iu_i+1/1≤i≤n-1}U{uiv_{2i}-1,w_iv_{2i}-1/1≤i≤n}U{u_iv_{2i},w_iv_{2i}/1≤i≤n}. Let $G_1,G_2,\dots G_n$ be m copies of C_4 and P_n : u_1,u_2,…un be a path. The P_n (Qs_n) is 4mn + (n-1) copies of P_2.

Definition: 1.4: Let G=Cn (Qs_n) is a graph let V(G)={u_1,u_2, . .,un,v_1,v_2,…,vmn,w_1,w_2,…,wmn}be the vertices of the graph and E(G)={u_iu_i+1/1≤i≤n-1}U{u_iv_{2i}-1,w_iv_{2i}-1/1≤i≤n}U{u_iv_{2i},w_iv_{2i}/1≤i≤n}. Let $G_1,G_2,\dots G_n$ be m copies of C_4 and Let C_n : u_1,u_2,…un be a cycle.

2. Main Result

Theorem: 1 The Double Triangular Snake G_n is a square difference graph.

Proof

Let G be the Double Triangular Snake and Let V(G)={u_1,u_2, . . .un,v_1,v_2,…,vn-$_1$,w_1,w_2,…,wn-1} be the vertices of the graph and E(G)={u_iu_{i+1}/1≤i≤n}U{v_iu_i,w_iu_i/1≤i≤n-1} U{v_iu_i+1,w_iu_i+1/1≤i≤n-1} be the edges of the graph.

Let. Let │V(G)│=2n+2 and │E(G)│=4n-1

Define the vertex labeling f:V(G)→ {0,1,2,…p-1}

$f(u_i)=i-1,\ 1\le i\le n$

$f(v_i)=i+n-1,\ 1\le i\le n-3$

$f(w_i)=2(n-1)+i,\ 1\le i\le n-1$

and the induced edge labeling function

f: E(G)→N defined by

$f(uv)=\left|[f(u)]^2-[f(v)]^2\right|$ for every uv ∈ E(G)

is injective such that $f(e_i)\neq f(e_j)$ for every $e_i\neq e_j$

The edge sets are

E1={u_iu_{i+1}/ 1≤ i ≤ n-1}

E2={v_iu_i/ 1≤ i ≤ n-1}

E3={v_iu_{i+1}/ 1≤ i ≤ n-1}

E4={w_iu_i/1≤ i ≤ n-1}

E5={w_iu_{i+1}/ 1≤ i ≤ n-1}

and the edge labels are

In E1

$f^*(u_iu_{i+1})=\cup_{i=1}^{n} \mid f(u_i)^2 - f(u_{i+1})^2 \mid$

$=\cup_{i=1}^{n} \mid (i-1)^2 - (n-1+i)^2 \mid$

$=\cup_{i=1}^{n} \mid 1-2i \mid$

$=\{1,3,5,\ldots 1-2n\}$

In E_2

$f^*(v_i u_i)=\cup_{i=1}^{n-1} \mid f(v_i)^2 - f(u_i)^2 \mid$

$=\cup_{i=1}^{n-1} \mid k2+2i(k+1)-1 \mid$

$=\{16,24,32,\ldots,3n^2-4n\}$

In E_3

$f(v_i u_{i+1})=\cup_{i=1}^{n-1} \mid f(v_i)^2 - f(u_{i+1})^2 \mid$

$=\cup_{i=1}^{n-1} \mid k2+2ki \mid$

$=\{15,21,27,\ldots,3n^2-6n+3\}$

In E_4

$f^*(w_iu_i)=\cup_{i=1}^{n-1} \mid f(w_i)^2 - f(u_i)^2 \mid$

$=\cup_{i=1}^{n-1} \mid 4k2+2i(2k+1)-1 \mid$

$=\{49,63,77,\ldots,8n^2-14n+5\}$

In E_5

$f^*(w_iu_{i+1})=\cup_{i=1}^{n-1} \mid f(w_i)^2 - f(u_{i+1})^2 \mid$

$=\cup_{i=1}^{n-1} \mid 4k(k+i) \mid$

$=\{48,60,72,\ldots,8n^2-16n+8\}$

Here all the edge labels are distinct.

Hence the Double triangular snake graph admits a square difference labeling.

Theorem:2

The Alternative triangular snake G is a square difference graph.

Proof

Let $G=A(T_n)$ be the graph and Let $V(G)=\{u_1,u_2,\ldots u_n,v_1,v_2,\ldots,v_{n-1},w_1,w_2,\ldots,w_{n-1}\}$be the vertices of the graph and $E(G)=\{u_iu_{i+1}/1\le i\le n-1\}U\{v_iu_{2i},v_iu_{2i-1}/1\le i\le n/2\}$ $U\{v_iu_{2i-1}/1\le i\le n/2\}$ be the edges of the graph. Let $|V(G)|=2n-3$ and $|E(G)|=2n-1$.

Define the vertex labeling $f:V(G)\rightarrow \{0,1,2,\ldots p-1\}$

$f(u_i)=i-1 \quad ,1\le i \le n$

$f(v_i)=i+n-1,\ 1\le i \le n-3$

and the induced edge labeling function

$f:E(G)\rightarrow N$ defined by

$f(uv)=\left|[f(u)]^2-[f(v)]^2\right|$ for every $uv \in E(G)$

is injective such that $f(e_i)\neq f(e_j)$ for every $e_i \neq e_j$

The edge sets are

$E_{1=}\{u_iu_{i+1}/ 1\le i \le n\}$

$E_2=\{v_iu_{2i}/ 1\le i \le n/2\}$

$E_3=\{v_iu_{2i-1}/ 1\le i \le n/2\}$

and the edge labels are

In E_1

$f^*(u_iu_{i+1})=\bigcup_{i=1}^{n-1} \mid f(u_i)^2 - f(u_{i+1})^2 \mid$

$=\bigcup_{i=1}^{n-1} \mid (i-1)^2 - (n-1+i)^2 \mid$

$=\bigcup_{i=1}^{n-1} \mid 1-2i \mid$

$=\{1,3,5,\ldots 2n-3\}$

In E_2

$f^*(v_i u_{2i-1})=\bigcup_{i=1}^{n/2} \mid f(v_i)^2 - f(u_{2i-1})^2 \mid$

$=\bigcup_{i=1}^{n/2} \mid n^2 + 2n(i-1) + 3i(2-i) - 3 \mid$

$=\{36,45,48,\ldots, 5n^2/4+n-3\}$

In E_3

$f^*(v_i u_{2i})=\bigcup_{i=1}^{n/2} \mid f(v_i)^2 - f(u_{2i})^2 \mid$

$=\bigcup_{i=1}^{n/2} \mid n^2 + 2n(i-1) + i(2-3i) \mid$

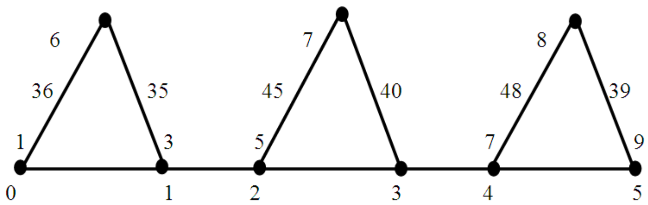
$=\{35,40,39,\ldots,5n^2/4-n\}$

Here all the edge labels are distinct.

Hence the Alternative triangular snake graph admits a square difference labeling.

Example:

The alternative triangular snake graph $A(T_6)$ is a square difference labeling.



Theorem: 3

The alternative double triangular snake G_n is a square difference graph.

Proof:

Let P: $u_1, u_2, \ldots, u_n$ be the path of the graph G. Let $V(G)=\{u_1,u_2,\ldots u_n,v_1,v_2,\ldots,v_{n-1}, w1,w2,\ldots,wn-1\}$ be the vertices of the graph and $E(G)=\{u_iu_i+1/1\leq i\leq n-1\}\cup\{v_iu_{2i-1},w_iu_{2i-1}/1\leq i\leq n/2\}\cup\{v_iu_{2i},w_iu_{2i}/1\leq i\leq n/2\}$ be the edges of the graph.

Let $|V(G)|=2n$ and $|E(G)|=3n-1$

Define the vertex labeling $f : VA(T_n)\rightarrow\{0,\ldots,p-1\}$

$f(u_i)=i-1,\ 1\leq i\leq n$

$f(v_i)=n-1+i,\ 1\leq i\leq n/2$

$f(w_i)=\frac{3n}{2}+i-1,\ 1\leq i\leq n/2$

and the induced edge labeling function

$f: EA(T_n)\rightarrow N$ defined by

$f(uv)=\left|[f(u)]^2-[f(v)]^2\right|$ for every $uv \in E(G)$

is injective .such that $f(e_i)\neq f(e_j)$ for every $e_i \neq e_j$

The edge sets are

$E_1=\{u_iu_{i+1}/\ 1\leq i \leq n-1\}$

$E_2=\{v_iu_{2i-1}/\ 1\leq i \leq n/2\}$

$E_3=\{v_iu_{2i}/\ 1\leq i \leq n/2\}$

$E_4=\{w_iu_{2i-1}/\ 1\leq i \leq n/2\}$

$E_5=\{w_iu_{2i}/\ 1\leq i \leq n/2\}$

and the edge labels are

In E_1

$f^*(u_iu_{i+1})=\bigcup_{i=1}^{n-1} | f(u_i)^2 - f(u_{i+1})^2 |$

$f^*(u_iu_{i+1})=\bigcup_{i=1}^{n-1} | 1-2i |$

$=\{1,3,5,\ldots,1-2n\}$

In E_2

$f^*(v_i u_{2i-1})=\bigcup_{i=1}^{\frac{n}{2}} | f(v_i)^2 - f(u_{2i-1})^2 |$

$=\bigcup_{i=1}^{n/2} | n^2 + 2n(i-1) + 3i(2-i) - 3 |$

$=\{16,21,\ldots,5n^2/4+n-3\}$

In E_3

$f^*(v_i u_{2i})=\bigcup_{i=1}^{n/2} | f(v_i)^2 - f(u_{2i})^2 |$

$=\bigcup_{i=1}^{n/2} | n^2 + 2n(i-1) + i(2-3i) |$

$=\{15,16,.., 5n^2/4-n\}$

In E_4

$f^*(w_iu_{2i-1})=\bigcup_{i=1}^{n/2} | f(w_i)^2 - f(u_{2i-1})^2 |$

$=\bigcup_{i=1}^{n/2} | \frac{5n^2}{4} + n(n+3i-3) - 3i(i-2) - 3 |$

$=\{36,45,..,3(n^2-1)\}$

In E_5

$f^*(w_iu_{2i})=\bigcup_{i=1}^{n/2} | f(w_i)^2 - f(u_{2i})^2 |$

$=\bigcup_{i=1}^{n/2} | 3n(\frac{3n}{4} + i - 1) + i(2-3i) |$

$=\{35,40,\ldots,3n^2-2n\}$

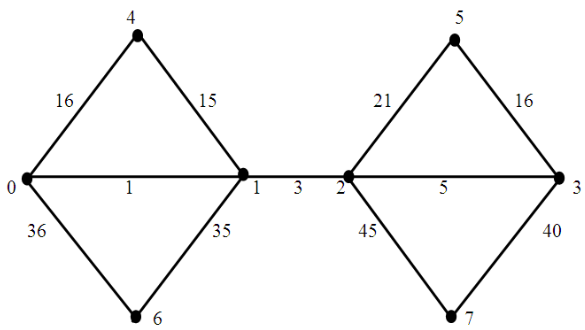
Here all the edges labels are distinct.

Hence the alternative double triangular snake admits a square difference labeling.

Example:

The alternative double triangular snake $A(T_n)$ is a square difference graph.

Solution :



Theorem: 4
The Banana tree G is a square difference labeling $G=BT(n_1,n_2,\dots n_k)$

Proof:
Let P : u_1, u_2 ,…, u_n be the path of the graph G and Let $V(G)=\{u_1,u_2, . . u_n,v_1,v_2,\dots,v_n,w_1,w_2,\dots,w_{n+1}\}$be the vertices of the graph and $E(G)=\{u_iu_{i+1}/1\leq i\leq n-1\}U\{u_iv_i/1\leq i\leq n\}U\{v_i,w_{4i-3}w_{4i-2}w_{4i-1}w_{4i}/1\leq i\leq n\}$ be the edges of the graph. Let $|V(G)|=6n$ and $|E(G)|=6n-1$.

Define the vertex labeling $f : E\rightarrow\{0,\dots,p-1\}$

$f(u_i)=i-1,\ 1\leq i \leq n$

$f(v_i)=n-1+i,\ 1\leq i \leq n$

$f(w_i)=2n+ i-1,\ 1\leq i \leq n+1$

and the induced edge labeling function

$f : E(BT(n_1,n_2,\dots n_k))\rightarrow N$ defined by

$f(uv)=|[f(u)]^2-[f(v)]^2|$ for every $uv \in E(G)$

is injective .such that $f(e_i)\neq f(e_j)$ for every $e_i \neq e_j$

The edge sets are

$E_{1=}\{u_iu_{i+1}/ 1\leq i \leq n-1\}$

$E_2=\{u_iv_i/ 1\leq i \leq n\}$

$E_3=\{w_{4i-3}vi/ 1\leq i \leq n\}$

$E_4=\{w_{4i-2}\, v_i/ 1\leq i \leq n\}$

$E_5=\{w_{4i-1}\, v_i/ 1\leq i \leq n\}$

$E_6=\{w_{4i}\, v_i/ 1\leq i \leq n\}$

and the edge labels are

In E_1

$f^*(E_1)=\cup_{i=1}^{n-1} \mid f(u_i)^2 - f(u_{i+1})^2 \mid$

$=\cup_{i=1}^{n-1} \mid 1-2i \mid$

$=\{1,3,\dots,3-2n\}$

$f^*(E_2)=\cup_{i=1}^{n} \mid f(u_i)^2 - f(v_i)^2 \mid$

$=\cup_{i=1}^{n} \mid (i-1)^2-(n+i-1)^2 \mid$

$=\cup_{i=1}^{n} \mid -n^2-2ni+2n \mid$

$=\{9,15,21,\dots,-3n^2+2n\}$

$f^*(E_3)=\cup_{i=1}^{n} \mid f(w4_{i-3})^2 - f(v_i)^2 \mid$

$=\cup_{i=1}^{n} \mid (2n+4i-4)^2-(n+i-1)^2 \mid$

$=\cup_{i=1}^{n} \mid 3n^2+15i2+14ni-14n-30i+15 \mid$

$=\{27,84,171,\dots,32n^2-44n+15\}$

$f^*(E_4)=\cup_{i=1}^{n} \mid f(w4_{i-2})^2 - f(v_i)^2 \mid$

$=\cup_{i=1}^{n} \mid (2n+4i-3)^2-(n+i-1)^2 \mid$

$=\bigcup_{i=1}^{n} \mid 3n^2 + 15i2 + 14ni - 10n - 22i + 8 \mid$

$=\{40,105,200,\ldots,8(4n^2-4n+1)\}$

$f^*(E_5)=\bigcup_{i=1}^{n} \mid f(w4_{i-1})^2 - f(v_i)^2 \mid$

$=\bigcup_{i=1}^{n} \mid (2n + 4i - 2)^2 - (n + i - 1)^2 \mid$

$=\bigcup_{i=1}^{n} \mid 3n^2 + 15i2 + 14ni - 6n - 14i + 3 \mid$

$=\{55,128,231,\ldots,32n^2-20n+3\}$

$f^*(E_6)=\bigcup_{i=1}^{n} \mid f(w4_i)^2 - f(v_i)^2 \mid$

$=\bigcup_{i=1}^{n} \mid (2n + 4i - 1)^2 - (n + i - 1)^2 \mid$

$=\bigcup_{i=1}^{n} \mid 3n2 + 15i2 + 14ni - 2n - 6i=\{72,153,264,\ldots,32n^2-8n\}$

Here all the edge labels are distinct.
Hence the Banana tree admits a square difference labeling .

Theorem:5
The Umbrella graph is a square difference labeling.

Proof:
Let U(G) be the umbrella graph. Let $V(G)=\{u_1,u_2, \ldots u_n,v_1,v_2,\ldots,v_{n+1}\}$be the vertices of the graph and $E(G)=\{u_iu_{i+1}/1\leq i\leq n-1\}\cup\{v_iv_{i+1},viun/1\leq i\leq n+1\}$ be the edges of the graph.

Let $|V(G)|=n$ and $|E(G)|=3n$.
Define the vertex labeling $f:E\rightarrow \{0,1,2,\ldots p-1\}$
$f(u_i)=i-1 \quad ,1\leq i \leq n$
$f(v_i)=i+2,\ 1\leq i \leq n+1$
and the induced edge labeling function $f:E\rightarrow N$ defined by
$f(uv)=\left|[f(u)]^2-[f(v)]^2\right|$ for every $uv \in E(G)$ are all distinct .Such that $f(e_i)\neq f(e_j)$ for every $e_i \neq e_j$

The edge sets are

$E_1=\{u_iu_{i+1}/ 1\leq i \leq n-1\}$

$E_2=\{v_iv_{i+1}/ 1\leq i \leq n\}$

$E_3=\{v_iu_n/ 1\leq i \leq n+1\}$

and the edge labels are

$f^*(E_1)=\bigcup_{i=1}^{n-1} \mid f(u_i)^2 - f(u_{i+1})^2 \mid$

$=\bigcup_{i=1}^{n-1} |1-2i|$

={1,3,…,3-2n}

$f^*(E_2)=\bigcup_{i=1}^{n} |f(v_i)^2 - f(v_{i+1})^2|$

$=\bigcup_{i=1}^{n} |-2i-5|$

={7,9,11,…,-(2n+5)}

$f^*(E_3)=\bigcup_{i=1}^{n+1} |f(v_i)^2 - f(u_n)^2|$

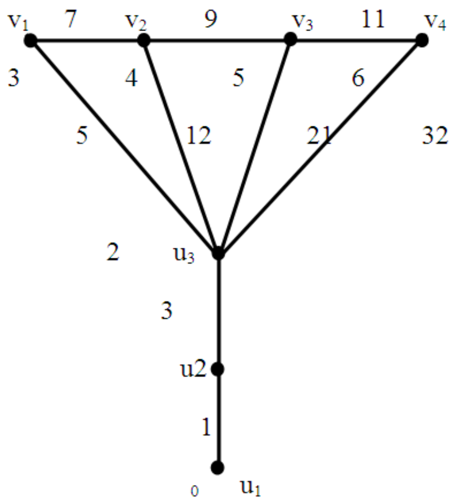
$=\bigcup_{i=1}^{n+1} |i^2+4i-n(n-2)+3|$

={5,12,21,…,8(n+1)}

Here all the edge labels are distinct.

Hence the umbrella graph admits a square difference labeling.

For example;



Theorem:6

The graph $G=P_n\ (QS_n)$ is a square difference labeling (n ≥1, m ≥ 1).

Proof:

Let $G=P_n\ (QS_n)$ is a graph . let $V(G)=\{u_1,u_2,...,u_n,v_1,v_2,…,v_{mn},w_1,w_2,…,w_{mn}\}$be the vertices of the graph and $E(G)=\{u_iu_{i+1}/1\leq i\leq n-1\}\cup\{u_iv_{2i-1},w_iv_{2i-1}/1\leq i\leq n\}\cup\{u_iv_{2i},w_iv_{2i}/1\leq i\leq n\}$.Let $G_1,G_2,…G_n$ be m copies of C_4 and P_n : $u_1,u_2,…u_n$ be a path . The $P_n\ (QS_n)$ is 4mn + (n-1) copies of P_2.

Let │V(G)│=3mn + n and │E(G)│=4mn + (n-1)

Define the vertex labeling $f:E\rightarrow\{0,1,…p-1\}$

$f(u_i)=i-1,\ 1\leq i\leq n$

$f(v_{2nk+i})=(3k+1)n+(i-1)$, $1\leq i\leq n-1$, $k=0,1,2,3,\ldots,n$

$f(w_{nk+1})=3n(k+1)+(i-1)$, $1\leq i\leq n-1$, $k=0,1,2,3,\ldots,n$

f(vi)=n+i-1 for k=0

f(wi)=3n+i-1 for k=0

and the induced edge labeling function

$f:E\rightarrow N$ defined by

$f(uv)=\left|[f(u)]^2-[f(v)]^2\right|$ for every $uv\in E(G)$

is injective .such that $f(e_i)\neq f(e_j)$ for every $e_i\neq e_j$

The edge sets are

$E_1=\{u_iu_{i+1}/\ 1\leq i\leq n-1\}$

$E_2=\{u_iv_{2i-1}/\ 1\leq i\leq n\}$

$E_3=\{u_iv_{2i}/\ 1\leq i\leq n\}$

$E_4=\{w_iv_{2i-1}/\ 1\leq i\leq n\}$

$E_5=\{w_iv_{2i}/\ 1\leq i\leq n\}$

and the edge labels are

In E_1

$f^*(u_iu_{i+1})=\cup_{i=1}^{n-1}\ |(1-i)^2-i^2|$

$=\cup_{i=1}^{n-1}|(1-2i)|$

$=\{1,3,5,\ldots,3-2n\}$

In E_2

$f^*(u_iv_{2i-1})=\cup_{i=1}^{n/2}\ |f(u_i)^2\ -f(v_{2i-1})^2|$

$=\cup_{i=1}^{n}\ |(i-1)^2\ -(n+2i-2)^2|$ /

$=\cup_{i=1}^{n}\ |3i(2-i)-n(n+4i-4)-3|$

$=\{4,15,\ldots,-(8n^2-10n+3)\}$

In E_3

$f^*(u_iv_{2i})=\cup_{i=1}^{n}\ |f(u_i)^2\ -f(v_{2i})^2|$

$=\cup_{i=1}^{n}\ |(i-1)^2\ -(n+2i-1)^2|$

$=\cup_{i=1}^{n}\ |-3i^2+2i-n^2-2n(2i-1)|$

$=\{9,24,\ldots,-(8n^2-8)\}$

In E_4

$f^*(w_iv_{2i-1})=\cup_{i=1}^{n} \mid f(w_i)^2 - f(v_{2i})^2 \mid$

$=\cup_{i=1}^{n} \mid (3n + i - 1)^2 - (n + 2i - 2)^2 \mid$

$=\cup_{i=1}^{n} \mid 8n2 - 3(i2 + 1) + 2n(i - 1) + 6i) \mid$

$=\{32,33,\ldots,7n^2+4n-3\}$

In E_5

$f^*(w_iv_{2i})=\cup_{i=1}^{n} \mid f(w_i)^2 - f(v_{2i})^2 \mid$

$=\cup_{i=1}^{n} \mid (3n + i - 1)2 - (n + 2i - 1)2 \mid$

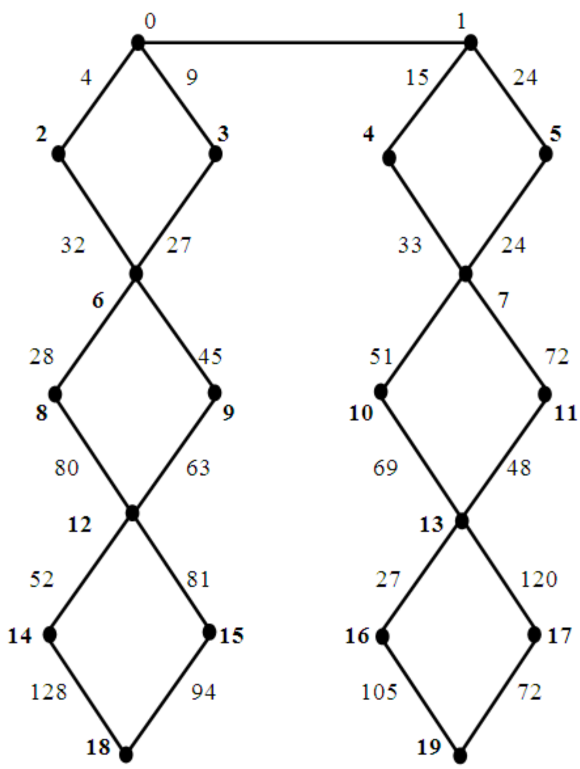
$=\cup_{i=1}^{n} \mid 8n2 - 3i2 + 2ni - 4n + 2i \mid$

$=\cup_{i=1}^{n} \mid 8n2 - i(3i - 2) + 2n(i - 2) \mid$

$=\{27,24,\ldots,7n^2-2n\}$

Here all the edge labels are distinct.
Hence the $P_n(QS_n)$ graph admits a square difference labeling.
For example;
The graph P_2 (QS_3) is a square difference labeling.

Solution:
If n $\geq$1 and m$\geq$ 1



Theorem: 10
The graph $G=C_n\ (QS_n)$ is a square difference labeling for all $m\geq1$ and for all $n\geq3$.

Proof:
Let $G=C_n\ (QS_n)$ is a graph . let $V(G)=\{u_1,u_2,...,u_n,v_1,v_2,\ldots,v_{mn},w_1,w_2,\ldots,w_{mn}\}$be the vertices of the graph and $E(G)=\{u_iu_{i+1}/1\leq i\leq n-1\}U\{u_iv_{2i-1},w_iv_{2i-1}/1\leq i\leq n\}U\{u_iv_{2i},w_iv_{2i}/1\leq i\leq n\}$. Let G denote m copies of C_n and Let C_n : $u_1,u_2,\ldots u_n\ u_1$ be a cycle . Let $|V(G)|=3mn + n$ and $|E(G)|=4mn + n$.

Define the vertex labeling f: $E\rightarrow\{0,1,\ldots p-1\}$.

$f(u_i)=i-1,\ 1\leq i\leq n$

$f(v_{2nk+i})=(3k+1)n+(i-1),\ 1\leq i\leq n-1,\ k=0,1,2,3,\ldots,n$

$f(w_{nk+1})=3n(k+1)+(i-1),\ 1\leq i\leq n-1,\ k=0,1,2,3,\ldots,n$

$f(vi)=n+i-1$ for $k=0$

$f(wi)=3n+i-1$ for $k=0$

and the induced edge labeling function

$f:E\rightarrow N$ defined by

$f(uv)=|[f(u)]^2-[f(v)]^2|$ for every $uv\in E(G)$

is injective .such that $f(e_i)\neq f(e_j)$ for every $e_i\neq e_j$

The edge sets are

$E_{1=}\{u_iu_{i+1}/\ 1\leq i\leq n-1\}$

$E_2=\{u_iv_{2i-1}/\ 1\leq i\leq n\}$

$E_3=\{u_iv_{2i}/\ 1\leq i\leq n\}$

$E_4=\{w_iv_{2i-1}/\ 1\leq i\leq n\}$

$E_5=\{w_iv_{2i}/\ 1\leq i\leq n\}$
and the edge labels are

In E_1

$f^*(u_iu_{i+1})=\bigcup_{i=1}^{n-1} |(1-i)^2-i^2|$

$=\bigcup_{i=1}^{n-1}(1-2i)|$

$=\{1,3,5,\ldots,3-2n\}$

In E_2

$f^*(u_iv_{2i-1})=\bigcup_{i=1}^{n} |f(u_i)^2-f(v_{2i-1})^2|$

$=\bigcup_{i=1}^{n} |(i-1)^2 - (n+2i-2)^2|$

$=\bigcup_{i=1}^{n} |3i(2-i) - n^2 + 4n(1-i) - 3|$

={9,24,45,…,-8n2+10n-3}

In E_3

$f^*(u_iv_{2i}) = \bigcup_{i=1}^{n} |f(u_i)^2 - f(v_{2i})^2|$

$=\bigcup_{i=1}^{n} |(i-1)^2 - (n+2i-1)^2|$

$=\bigcup_{i=1}^{n} |-3i^2 + 2i - n^2 - 2n(2i-1)|$

={16,35,60,…,-8n2+4n}

In E_4

$f^*(w_iv_{2i-1}) = \bigcup_{i=1}^{n} |f(w_i)^2 - f(v_{2i})^2|$

$=\bigcup_{i=1}^{n} |(3n+i-1)^2 - (n+2i-2)^2|$

$=\bigcup_{i=1}^{n} |8n2 - 3(i2+1) + 2n(i-1) + 6i)|$

$=\{72,75,72,\ldots,7n^2+4n-3\}$

In E_5

$f^*(w_iv_{2i}) = \bigcup_{i=1}^{n} |f(w_i)^2 - f(v_{2i})^2|$

$=\bigcup_{i=1}^{n} |(3n+i-1)2 - (n+2i-1)2|$

$=\bigcup_{i=1}^{n} |8n2 - 3i2 + 2ni - 4n + 2i|$

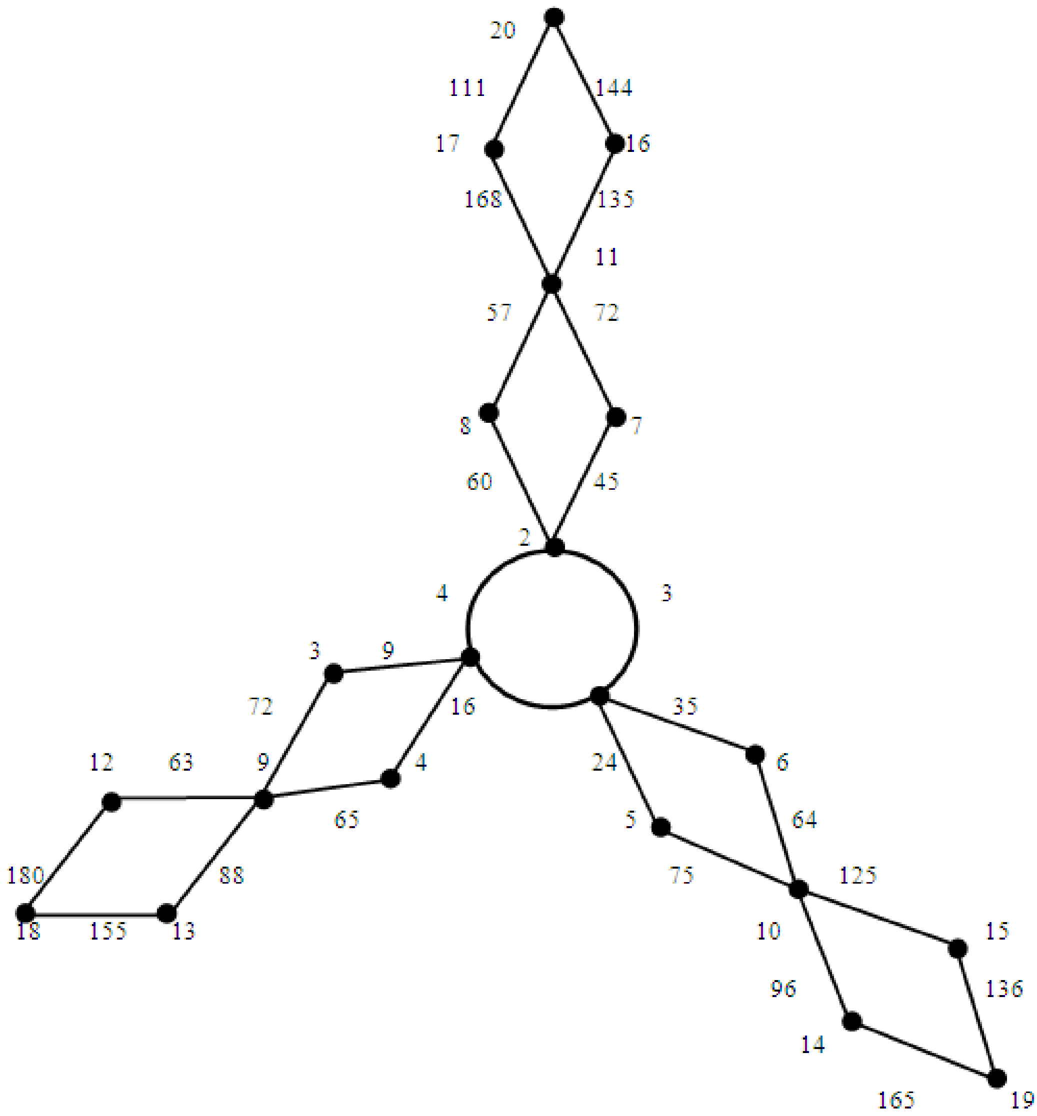
$=\bigcup_{i=1}^{n} |8n2 - i(3i-2) + 2n(i-2)|$

$=\{65,64,57,\ldots,7n^2-2n\}$

Here all the edge labels are distinct.
Hence the $C_n(QS_n)$ graph admits a square difference labeling.

Example;
The graph $C_n(QS_n)$ is a square difference labeling. ($n \geq 3$, $m \geq 1$).

Solution:
If $n \geq 3$ and $m \geq 1$



3. Acknowledgement

The author thanks to Mannonmaniam Sundaranar University for providing facilities. I wish to express my deep sense of gratitude to Dr. M. Karuppaiyan my husband for their immense help in my studies.

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