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Digging for the Best Models for Distribution of Repeaters

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Abstract

In modern cities, repeaters are used to send and receive signals of mobile phones. In this paper, we dig for the best models to determine the minimum number of repeaters to cover a circle area.

We consider two cases. The simpler one assumes that the circle area is flat and the more complicated one is used to analyze a rugged area with mountains in it. In both cases, we assume that simultaneous users follow the random distribution. Based on Voronoi Diagram and Min-max problem, we build two models for both cases respectively. Afterwards, we analyze a real case in Nanjing city. The result shows that the distribution given by our model is very close to the distribution shown in the satellite pictures.

Keywords: Repeaters' distribution Voronoi Diagram Min-max Problem

Introduction

A repeater is an electronic device that receives signals and retransmits them to a higher level so that the signals can cover a longer distance. In order to avoid the interference between different repeaters, a "continuous tone-coded squelch system" (or called CTCSS) is often used to mitigate interference problems.

CTCSS is a circuit that is used to reduce the annoyance of listening to other users on a shared two-way radio communications channel. With this system, two nearby repeaters can share the same frequency pair for receiving and transmitting signals. In our paper, we assume there are 54 different CTCSS available.

Our paper is also based on Voronoi Diagram. Its precise definition is as follows:

Let P be a discrete set of points in Euclidean Space .Two or more points are non-collinear and four or more points are not concyclic. For every point P_i (also called Voronoi cell) in the Euclidean space, the region called $V_{(i)}$the point P_i's Voronoi Polygon.

Where $V_{(i)} = \{x \in E^2 \mid d(x, p_i) \le d(x, p_j), j = 1, 2, \cdots, n, j \ne i\}$

The Voronoi Diagram (As shown in the Figure 1) consists of every point's Voronoi polygon.

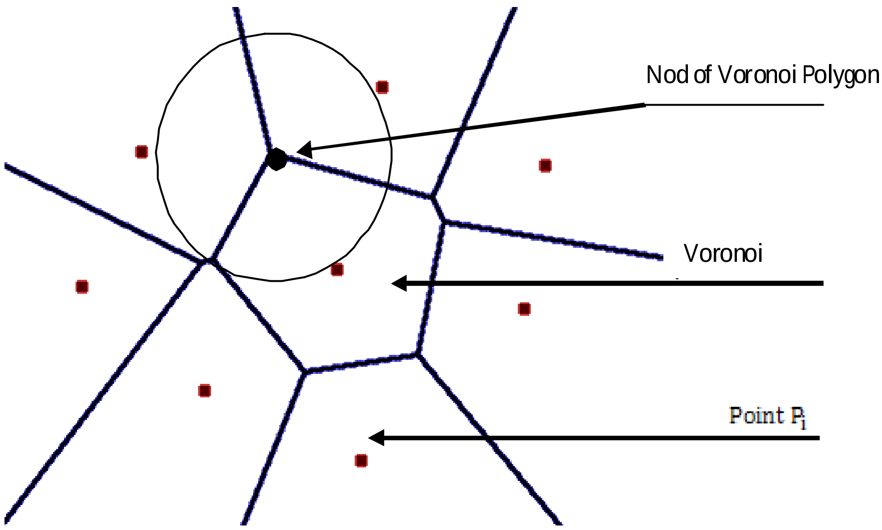

Figure 1 Voronoi Diagram and the Properties in Euclidean Space

The Analysis of the Problem

Analysis.

To solve the problem, our basic idea is: regard the given area as a large circle and the repeater's span of control as a small circle; our purpose is to cover the large circle with users in it by small ones.

We choose the distance in which the radio waves will be distorted as the radius of repeater's control circle.

To obtain the "distance", we use the Okumura-Hata model.

As the O-H model says, radio wave transmission loss formula in open areas is

$$LM(dB) = 69.55 + 26.16 \lg F - 13.82 \lg h_b - \alpha(h_m) + (44.9 - 6.55 \lg h_b) \lg d$$

$$\alpha(h_m) = (1.1 \lg F - 0.7) h_m - 1.56 \lg F + 0.8$$

Terms, definitions and symbols.

F	transmission spectrum (MHz)
h_b	the repeater antenna's effective height (m)
h_m	the mobile device's height (m)
d	the distance between sending and receiving antennas (km)
α	a correction factor
s_i	the location of the ith repeater
u_j	the jth user
w_{ij}	the weight of u_j

Model assumptions.

1. The number of simultaneous users a repeater can accommodate has no limitation.
2. The signals sent by repeaters can reach as far as possible.

Models

Model building.

Assuming that we place m repeaters in the target area and there are n simultaneous users. We place the m repeaters in the way that they can cover all the users but every repeater's control circle is as small as possible. That is a max-min problem.

Let repeater's Voronoi polygon be its maximum cover area in order to find the best place for each repeater in its Voronoi polygon.

We build the model below:

$$\min\{\max_{u_j \in V_i}(w_{ij} \cdot d(s_i, u_j)), i = 1, \cdots, m\}$$

Algorithm and Solution.

1. Estimate the number of repeaters and give it an initial value m.
2. Based on actual situation, put all the simultaneous users on target area in the form of scatters, and let repeaters evenly distributed on target area.
3. Regard repeaters as scatters P_i, and divide the target area into m Voronoi areas (As shown in the Figure 1).
4. Calculate the total distance between the repeater and all the users in its Voronoi polygon D.

5. Use " Shamos Algorithm" (it is explained below) to obtain the minimum cover circle of all the users in one Voronoi polygon.
6. Remove the repeater to center of its Voronoi polygon's minimum cover circle (As shown in the Figure 7).
7. Rebuild the new Voronoi polygons based on the new positions of repeaters and calculate the total distance between the repeater and all the users in its Voronoi polygon D'.
8. If D-D' is small enough, go to the Step 9; else, go back to Step 6.
9. Assign radius of the repeater whose control circle is the largest to r.
10. If r>10km, conclude that m repeaters can't accommodate n users; else, conclude that m repeaters can accommodate n users.

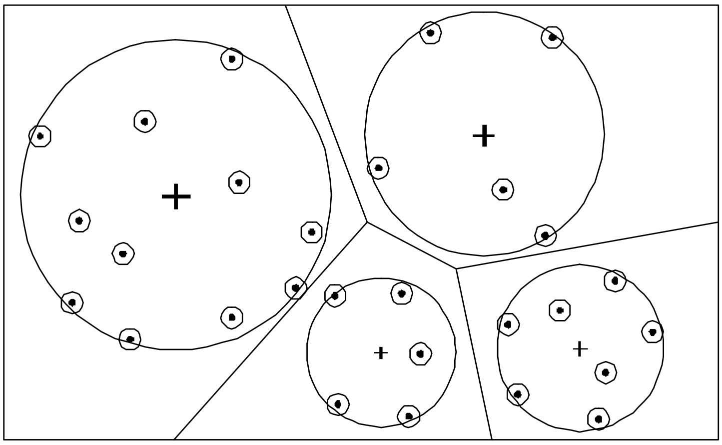

Figure 7 Voronoi Polygon Minimum Cover Circle

Shamos Algorithm.

Aiming at solving the selection of each facility's (such as repeater) location, Shamos designed an algorithm, it take only O(nlogn) time, and the description of this algorithm is as follows:

1. Create the convex hulls *CH(Loc)* offacilities location set $Loc = \{Loc_1, Loc_2, ..., Loc_n\}$.
2. Calculate the diameter of *CH(Loc)*, set it as $\overline{p_i p_j}$. Then, construct the circle *C* with diameter equal to $\overline{p_i p_j}$. If the circle *C* covers all the points of facilities location, *C* is the smallest circle we are looking for. Otherwise, skip to Step 3.
3. Draw the Voronoi Diagram of facilities location set Loc, name it as $Vor_n(Loc)$.
4. Suppose *v* is one of the Voronoi vertex among $Vor_n(Loc)$, draw a circle with center at *v* and radius equal to the distance from *v* to the most remote point among the Loc, this circle is the most smallest covering circle, and its center

is the most suitable location to build the repeater.

If m repeaters are enough, assign m/2 to m; else, assign 2m to m.
Take an example below:
If the simultaneous users (1000p) follow uniform distribution as follows:

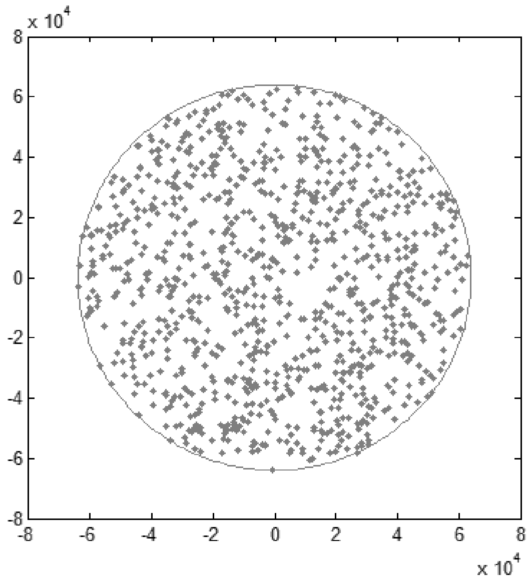

Figure 8 Uniform Distribution of 1000 Simultaneous Users

We get the result as follows:

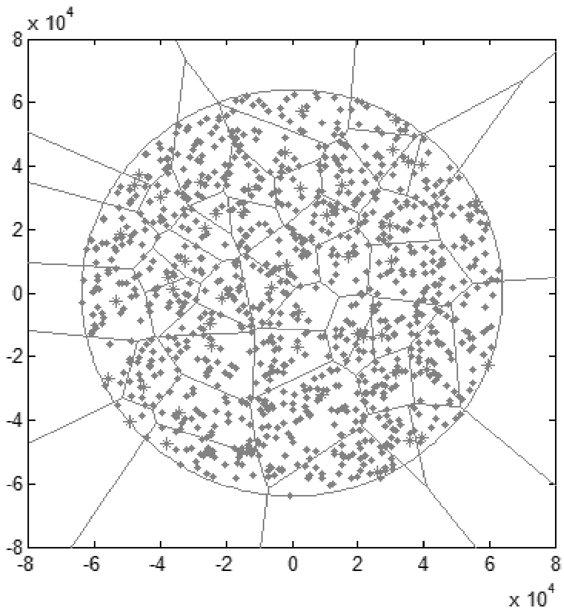

Figure 9 Voronoi Tessellation Based on the Repeaters

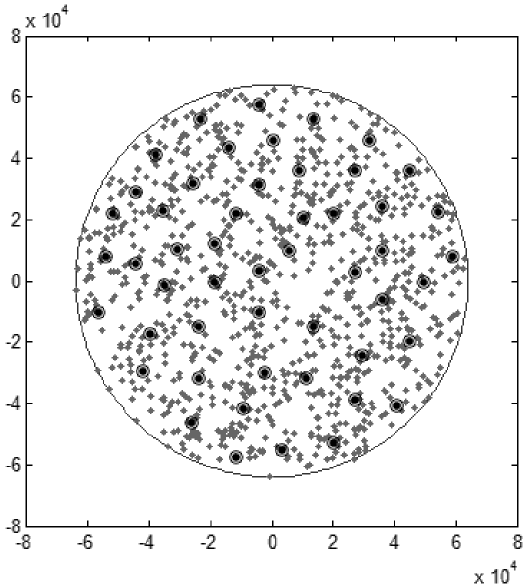

Figure 10 Repeaters' Positions in Target Area

The minimum number of repeaters needed is 48. And their positions are shown by the circles above.

And if there are 10000 users in the area and they follow uniform distribution, we can also use our model to get the result. The result is 51.

Model Test.

We take Nanjing, China as a real example to check the accuracy of our model. Using the O-H formula, we can obtain a repeater's cover radius.

Specifically, we deal with

$$F = 145\text{MHz}$$
$$h_b = 30\text{m}$$
$$h_m = 1.5\text{m}$$
$$TP = 36\text{dBm}$$
$$RS = -116\text{dBm}$$

Take all the data to O-H model formula and we can obtain the distance

$$d \approx 10km$$

Then, we calculate the minimum number of repeaters needed in Nanjing according to our model. The result is 10. In fact, there are 9 repeaters in Nanjing. And the positions of the repeaters are similar. Hence we think that our model is good enough

Model Improvement.

If there are defects in line-of-sight propagation caused by mountainous areas, there are some differences.

1. Mountains stop the signal transmission. So if there is no repeater on the mountain, two users from different sides of the mountain cannot communicate with each other.
2. Based on the O-H model, repeater on the mountain has a large control area than that on the flat ground.

By taking all these factors into account, we give out our solution below:

1. Calculate the span of control of the repeaters on the mountains.
2. Each intersection of mountains gets a repeater.
3. Place repeaters along the mountains until all of the mountains are covered.
4. After that, there are some separate areas divided by mountains.
5. Regard each area a separate system and use the improved model to calculate the needed repeaters in each of the systems.
6. Add all the repeaters both on mountains and flat areas together so that we can obtain the minimum number of repeaters.

Example:

If there are 1000 simultaneous users in the target area, and as figure 11 shows, the target area has a mountain across it. Assuming that every point on the mountain has the same height 200m, we can take all the data to the algorithm and obtain the result as follows:

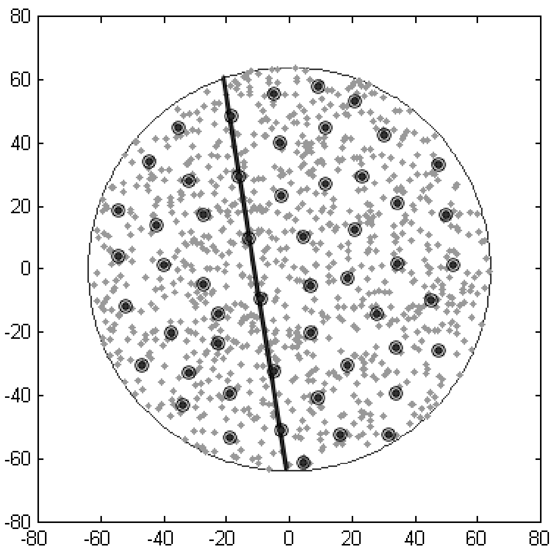

Figure 11 Repeaters' Positions in Target Mountainous Area

The result is that to accommodate all the simultaneous users in this area, there should be at least 53 repeaters and the repeaters positions are shown in figure 11.

Conclusions.
By using the two models we build, we have obtained a mature program to solve the given problem. What's more, for any certain area, if only we know the distribution of users and topographies, we can get the best plan of how to place repeaters in the target area.

Taking the given area for example, we assume users in it follow uniform distribution. If the area is a flat circle, the minimum number of repeaters accommodating 1000 simultaneous users is about 48 and that accommodating 10000 simultaneous users is about 51. And if it is a mountainous area, the minimum number of repeaters is larger. For instance, if there is a straight mountains as long as 70 miles across the circle area, the minimum number for 1000 users is about 53.

Strength and Weakness

- Strength: The model is not just limited to a certain problem. It can be applied to many other situations as well, no matter how the users' distribution changes. So it is very practical. Moreover it is more sensitive to the number of users so we can say that it confirms to reality better.
- Weakness: The algorithm used in it is not fast enough.

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