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Fuzzy Almost Contra rw-Continuous Functions in Topological Spaces

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Abstract

In this paper, we introduce fuzzy almost contra rw-continuous functions and to investigate properties and relationships of fuzzy functions.

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Keywords: fuzzy almost contra rw-continuous, fuzzy rw-convergent, fuzzy rw-connected, fuzzy rw-normal, fuzzy strongly rw-normal, fuzzy rw-T_2, fuzzy rw-T_1, fuzzy weakly almost contra rw-continuous.

1. Introduction

The concept of fuzzy sets was introduced by Zedeh [13] in his classical paper. Dontchev [3] introduced the notion of contra continuous mappings. Thangaraj [9], Ekici and Kerre [4] introduced the concept of fuzzy contra mappings. Recently, A. Vadivel et al. [10] introduced the concept of fuzzy contra rw-continuous functions in fuzzy topological spaces. The purpose of this paper is to introduce the forms of fuzzy almost contra rw-continuous functions and to investigate properties and relationships of fuzzy functions.

The class of fuzzy sets on a universe X will be denoted by I^X and fuzzy sets on X will be denoted by Greek letters as μ, ρ, η, etc.

A family τ of fuzzy sets in X is called a fuzzy topology for X if and only if (i) $0, 1 \in \tau$, (ii) $\mu \wedge \rho \in \tau$ whenever $\mu, \rho \in \tau$. (iii) If $\mu_i \in \tau$ for each $i \in I$, then $\vee\mu_i \in \tau$. Moreover, the pair (X, τ) is called a fuzzy topological space. Every member of τ is called a fuzzy open set [7].

In this paper, X and Y are fuzzy topological spaces. Let μ be a fuzzy set in X. We denote the interior and the closure of a fuzzy set μ by $int(\mu)$ and $cl(\mu)$, respectively.

A fuzzy set in X is called a fuzzy singleton if and only if it takes the value 0 for all $y \in X$ except one, say, $x \in X$. If its value at x is ϵ $(0 < \epsilon \leq 1)$ we denote this fuzzy singleton by x_ϵ, where the point x_ϵ is called its support [7]. For any fuzzy singleton x_ϵ and any fuzzy set μ, we write $x_\epsilon \in \mu$ if and only if $\epsilon \leq \mu(x)$.

A fuzzy singleton x_ϵ is called quasi-coincident with a fuzzy set ρ, denoted by $x_\epsilon q \rho$, iff $\epsilon + \rho(x) > 1$. A fuzzy set μ is called quasi-coincident with a fuzzy set ρ, denoted by $\mu q \rho$, if and only if there exists a $x \in X$ such that $\mu(x) + \rho(x) > 1$.

Let $f : X \to Y$ a fuzzy function from a fuzzy topological space X to a fuzzy topological space Y. Then the function $g : X \to X \times Y$ defined by $g(x_\epsilon) = (x_\epsilon, f(x_\epsilon))$ is called the fuzzy graph function of f [1].

Recall that for a fuzzy function $f : X \to Y$, the subset $(x_\epsilon, f(x_\epsilon)) : x_\epsilon \in X\} \leq X \times Y$ is called the fuzzy graph of f and is denoted by $G(f)$.

A fuzzy set μ of a fuzzy space X is said to be fuzzy regular open (respectively fuzzy regular closed) if $\mu = int(cl(\mu))$ (respectively $\mu = cl(int(\mu)))$ [1].

Definition 1.1. [14] Let (X, τ) be a fuzzy topological space. A fuzzy set α of a fts X is said to be fuzzy regular semiopen set in fts if there exists a fuzzy regular open set σ in X such that $\sigma \leq \alpha \leq cl(\sigma)$.

Definition 1.2. [12] Let (X, τ) be a fuzzy topological space. A fuzzy set α of a fts X is said to be fuzzy regular w-closed (briefly,fuzzy rw-closed) if $cl(\lambda) \leq \sigma$ whenever $\lambda \leq \sigma$ and σ is fuzzy regular semiopen in fts X.

Definition 1.3. [12] Let X and Y be fuzzy topological spaces. A map $f : X \to Y$ is said to be fuzzy rw-continuous if the inverse image of every fuzzy closed set in Y is fuzzy rw-closed in X.

Definition 1.4. [10] Let X and Y be fuzzy topological spaces. A map $f : X \to Y$ is said to be fuzzy contra rw-continuous if the inverse image of every fuzzy open set in Y is fuzzy rw-closed in X.

2. Fuzzy almost contra rw-continuous functions

In this section, the notion of fuzzy almost contra rw-continuous functions is introduced.

Definition 2.1. Let X and Y be fuzzy topological spaces. A fuzzy function $f : X \to Y$ is said to be fuzzy almost contra rw-continuous if inverse image of each fuzzy regular open set in Y is fuzzy rw-closed in X.

Theorem 2.2. For a fuzzy function $f : X \to Y$, the following statements are equivalent:

(1) f is fuzzy almost contra rw-continuous,

(2) for every fuzzy regular closed set μ in Y, $f^{-1}(\mu)$ is fuzzy rw-open,

(3) for any fuzzy regular closed set $\mu \leq Y$ and for any $x_\epsilon \in X$ if $f(x_\epsilon)q\mu$, then $x_\epsilon q rw\text{-}int(f^{-1}(\mu))$,

(4) for any fuzzy regular closed set $\mu \leq Y$ and for any $x_\epsilon \in X$ if $f(x_\epsilon)q\mu$, then there exists a fuzzy rw-open set η such that $x_\epsilon q\eta$ and $f(\eta) \leq \mu$,

(5) $f^{-1}(int(cl(\mu)))$ is fuzzy rw-closed for every fuzzy open set,

(6) $f^{-1}(cl(int(\rho)))$ is fuzzy rw-open for every fuzzy closed subset ρ,

(7) for each fuzzy singleton $x_\epsilon \in X$ and each fuzzy regular closed set η in Y containing $f(x_\epsilon)$, there exists a fuzzy rw-open set μ in X containing x_ϵ such that $f(\mu) \leq \eta$.

Proof. (1) $\Leftrightarrow$ (2): Let ρ be a fuzzy regular open set in Y. Then, $Y\backslash\rho$ is fuzzy regular closed. By (2), $f^{-1}(Y\backslash\rho) = X\backslash f^{-1}(\rho)$ is fuzzy rw-open. Thus, $f^{-1}(\rho)$ is fuzzy rw-closed. Converse is similar.
(2) $\Leftrightarrow$ (3): Let $\mu \leq Y$ be a fuzzy regular closed set and $f(x_\epsilon)q\mu$. Then $x_\epsilon qf^{-1}(\mu)$ and from (2), $f^{-1}(\mu) \leq rw\text{-}int(f^{-1}(\mu))$. From here $x_\epsilon qrw\text{-}int(f^{-1}(\mu))$. Thus, (3) holds. The reverse is obvious.
(3) $\Rightarrow$ (4): Let $\mu \leq Y$ be any fuzzy regular closed set and let $f(x_\epsilon)q\mu$. Then $x_\epsilon qrw\text{-}int(f^{-1}(\mu))$. Take $\eta = rw\text{-}int(f^{-1}(\mu))$, then $f(\eta) = f(rw\text{-}int(f^{-1}(\mu))) \leq f(f^{-1}(\mu)) \leq \mu$.
(4) $\Rightarrow$ (3): Let $\mu \leq Y$ be any fuzzy regular closed set and let $f(x_\epsilon)q\mu$. From (4), there exists fuzzy rw-open set η such that $x_\epsilon q\eta$ and $f(\eta) \leq \mu$. From here $\eta \leq f^{-1}(\mu)$ and then $x_\epsilon qrw\text{-}int(f^{-1}(\mu))$.
(1) $\Leftrightarrow$ (5): Let μ be a fuzzy open set. Since $int(cl(\mu))$ is fuzzy regular open, then by (1), it follows that $f^{-1}(int(cl(\mu)))$ is rw-closed. The converse can be shown easily.
(2) $\Leftrightarrow$ (6): It can be obtained similar as (1) $\Leftrightarrow$ (5).
(2) $\Leftrightarrow$ (7) : Obvious. ■

Theorem 2.3. Let $f : X \to Y$ be a fuzzy function and let $g : X \to X \times Y$ be the fuzzy graph function of f, defined by $g(x_\epsilon) = (x_\epsilon, f(x_\epsilon))$ for every $x_\epsilon \in X$. If g is fuzzy almost contra-rw-continuous, then f is fuzzy almost contra-rw-continuous.

Proof. Let η be a fuzzy regular closed set in Y, then $X \times \eta$ is a fuzzy regular closed set in $X \times Y$. Since g is fuzzy almost contra-rw-continuous, then $f^{-1}(\eta) = g^{-1}(X \times \eta)$ is fuzzy rw-open in X. Thus, f is fuzzy almost contra-rw-continuous. ■

Definition 2.4. A fuzzy filter base Λ is said to be fuzzy rw-convergent to a fuzzy singleton x_ϵ in X if for any fuzzy rw-open set η in X containing x_ϵ, there exists a fuzzy set $\mu \in \Lambda$ such that $\mu \leq \eta$.

Definition 2.5. A fuzzy filter base Λ is said to be fuzzy rc-convergent [5] to a fuzzy singleton x_ϵ in X if for any fuzzy regular closed set η in X containing x_ϵ, there exists a fuzzy set $\mu \in \Lambda$ such that $\mu \leq \eta$.

Theorem 2.6. If a fuzzy function $f : X \to Y$ is fuzzy almost contra-rw-continuous, then for each fuzzy singleton $x_\epsilon \in X$ and each fuzzy filter base Λ in X rw-converging to x_ϵ, the fuzzy filter base $f(\Lambda)$ is fuzzy rc-convergent to $f(x_\epsilon)$.

Proof. Let $x_\epsilon \in X$ and Λ be any fuzzy filter base in X rw-converging to x_ϵ. Since f is fuzzy almost contra-rw-continuous, then for any fuzzy regular closed set λ in Y containing $f(x_\epsilon)$, there exists a fuzzy rw-open set μ in X containing x_ϵ such that $f(\mu) \leq \lambda$. Since Λ is fuzzy rw-converging to x_ϵ, there exists a $\xi \in \Lambda$ such that $\xi \leq \mu$. This means that $f(\xi) \leq \lambda$ and therefore the fuzzy filter base $f(\Lambda)$ is fuzzy rc-convergent to $f(x_\epsilon)$. ■

Definition 2.7. A space X is called fuzzy rw-connected if X is not the union of two disjoint nonempty fuzzy rw-open sets.

Definition 2.8. A space X is called fuzzy connected [8] if X is not the union of two disjoint nonempty fuzzy open sets.

Theorem 2.9. If $f : X \to Y$ is a fuzzy almost contra-rw-continuous surjection and X is fuzzy rw-connected, then Y is fuzzy connected.

Proof. Suppose that Y is not a fuzzy connected space. There exist nonempty disjoint fuzzy open sets η_1 and η_2 such that $Y = \eta_1 \vee \eta_2$. Therefore, η_1 and η_2 are fuzzy clopen in Y. Since f is fuzzy almost contra-rw-continuous, $f^{-1}(\eta_1)$ and $f^{-1}(\eta_2)$ are fuzzy rw-open in X. Moreover, $f^{-1}(\eta_1)$ and $f^{-1}(\eta_2)$ are nonempty disjoint and $X = f^{-1}(\eta_1) \vee f^{-1}(\eta_2)$. This shows that X is not fuzzy rw-connected. This contradicts that Y is not fuzzy connected assumed. Hence, Y is fuzzy connected. ■

Definition 2.10. A fuzzy space X is said to be fuzzy rw-normal if every pair of nonempty disjoint fuzzy closed sets can be separeted by disjoint fuzzy rw-open sets.

Definition 2.11. A fuzzy space X is said to be fuzzy strongly rw-normal if for every pair of nonempty disjoint fuzzy closed sets μ and η there exist disjoint fuzzy rw-open sets ρ and ξ such that $\mu \leq \rho, \eta \leq \xi$ and $cl(\rho) \wedge cl(\xi) = \phi$.

Theorem 2.12. If Y is fuzzy strongly rw-normal and $f : X \to Y$ is fuzzy almost contra-rw-continuous closed injection, then X is fuzzy rw-normal.

Proof. Let η and ρ be disjoint nonempty fuzzy closed sets of X. Since f is injective and closed, $f(\eta)$ and $f(\rho)$ are disjoint fuzzy closed sets. Since Y is fuzzy strongly rw-normal, there exist fuzzy rw-open sets μ and ξ such that $f(\eta) \leq \mu$ and $f(\rho) \leq \xi$ and $cl(\mu) \wedge cl(\xi) = \phi$. Then, since $cl(\mu)$ and $cl(\xi)$ are fuzzy regular closed and f is fuzzy almost contra-rw-continuous, $f^{-1}(cl(\mu))$ and $f^{-1}(cl(\xi))$ are fuzzy rw-open set. Sinc $\eta \leq f^{-1}(cl(\mu))$, $\rho \leq f^{-1}(cl(\xi))$, and $f^{-1}(cl(\mu))$ and $f^{-1}(cl(\xi))$ are disjoint, X is fuzzy rw-normal. ■

Definition 2.13. A space X is said to be fuzzy weakly T_2 [5] if each element of X is an intersection of fuzzy regular closed sets.

Definition 2.14. A space X is said to be fuzzy rw-T_2 if for each pair of distinct points x_ϵ and y_ν in X, there exist fuzzy rw-open set μ containing x_ϵ and fuzzy rw-open set η containing y_ν such that $\mu \vee \eta = \phi$.

Definition 2.15. A space X is said to be fuzzy rw-T_1 if for each pair of distinct fuzzy singletons x_ϵ and y_ν in X, there exist fuzzy rw-open sets μ and η containing x_ϵ and y_ν, respectively, such that $y_\nu \notin \mu$ and $x_\epsilon \notin \eta$.

Theorem 2.16. If $f : X \to Y$ is a fuzzy almost contra rw-continuous injection and Y is fuzzy Urysohn, then X is fuzzy rw-T_2.

Proof. Suppose that Y is fuzzy Urysohn. By the injectivity of f, it follows that $f(x_\epsilon) \neq f(y_\nu)$ for any distinct fuzzy singletons $x_\epsilon x$ and y_ν in X. Since Y is fuzzy Urysohn, there exist fuzzy open sets η and ρ such that $f(x_\epsilon) \in \eta,\ f(y_\nu) \in \rho$ and $cl(\eta) \wedge cl(\rho) = \phi$. Since f is fuzzy almost contra rw-continuous, there exist fuzzy open sets μ and ξ in X containing x_ϵ and y_ν, respectively, such that $f(\mu) \leq cl(\eta)$ and $f(\xi) \leq cl(\rho)$. Hence $\mu \wedge \xi = \phi$. This shows that X is fuzzy $\bar{}rw$-T_2. ■

Theorem 2.17. If $f : X \to Y$ is a fuzzy almost contra rw-continuous injection and Y is fuzzy weakly T_2, then X is fuzzy rw-T_1.

Proof. Suppose that Y is fuzzy weakly T_2. For any distinct points x_ϵ and y_ν in X, there exist fuzzy regular closed sets η, ρ in Y such that $f(x_\epsilon) \in \eta,\ f(y_\nu) \notin \eta,\ f(x_\epsilon) \notin \rho$ and $f(y_\nu) \in \rho$. Since f is fuzzy almost contra rw-continuous, by Theorem 2.2, (2) $f^{-1}(\eta)$ and $f^{-1}(\rho)$ are fuzzy $\bar{}rw$-open subsets of X such that $x_\epsilon \in f^{-1}(\eta),\ y_\nu \notin f^{-1}(\eta)$, $x_\epsilon \notin f^{-1}(\rho)$ and $y_\nu \in f^{-1}(\rho)$. This shows that X is fuzzy rw-T_1. ■

Theorem 2.18. Let (X_i, τ_i) be fuzzy topological space for all $i \in I$ and I be finite. Suppose that $(\prod_{i\in I} X_i, \sigma)$ is a product space and $f : (X, \tau) \to (\prod_{i\in I} X_i, \sigma)$ is any fuzzy function. If f fuzzy almost contra rw-continuous, then $pr_i \circ f$ is fuzzy almost contra rw-continuous where pr_i is projection function for each $i \in I$.

Proof. Let $x_\epsilon \in X$ and $(pr_i \circ f)(x_\epsilon) \in \rho_i$ and ρ_i be a fuzzy regular closed set in (X_i, τ_i). Then $f(x_\epsilon) \in pr_i^{-1}(\rho_i) = \rho_i \times \prod_{j\neq i} X_j$ a fuzzy regular closed set in $(\prod_{i\in I} X_i, \sigma)$. Since f is fuzzy almost contra rw-continuous, there exists a fuzzy rw-open set μ containing x_ϵ such that $f(\mu) \leq \cdot\rho_i \times \prod_{j\neq i} X_j = pr_i^{-1}(\rho_i)$ and hence $\mu \leq (pr_i \circ f)^{-1}(\rho_i)$ and we obtain that $pr_i \circ f$ is fuzzy almost contra rw-continuous for each $i \in I$. ■

Definition 2.19. The fuzzy graph $G(f)$ of a fuzzy function $f : X \to Y$ is said to be fuzzy strongly contra-rw-closed if for each $(x_\epsilon, y_\nu) \in (X \times Y)\backslash G(f)$, there exist a fuzzy rw-open set μ in X containing x_ϵ and a fuzzy regular closed set η in Y containing y_ν

such that $(\mu \times \eta) \wedge G(f) = \phi$.

Lemma 2.20. The following properties are equivalent for the fuzzy graph $G(f)$ of a fuzzy function f:

(1) $G(f)$ is fuzzy strongly contra-rw-closed;

(2) for each $(x_\epsilon, y_\nu) \in (X \times Y)\backslash G(f)$, there exist a fuzzy rw-open set μ in X containing x_ϵ and a fuzzy regular closed set η containing y_ν such that $f(\mu) \wedge \eta = \phi$.

Theorem 2.21. If $f : X \to Y$ is fuzzy almost contra-rw-continuous and Y is fuzzy Urysohn, $G(f)$ is fuzzy strongly contra-rw-closed in $X \times Y$.

Proof. Suppose that Y is fuzzy Urysohn. Let $(x_\epsilon, y_\nu) \in (X \times Y)\backslash G(f)$. It follows that $f(x_\epsilon) \neq y_\nu$. Since Y is fuzzy Urysohn, there exist fuzzy open sets η and ρ such that $f(x_\epsilon) \in \eta$, $y_\nu \in \rho$ and $cl(\eta) \wedge cl(\rho) = \phi$. Since f is fuzzy almost contra-rw-continuous, there exists a fuzzy rw-open set μ in X containing x_ϵ such that $f(\mu) \leq cl(\eta)$. Therefore, $f(\mu) \wedge cl(\rho) = \phi$ and $G(f)$ is fuzzy strongly contra-rw-closed in $X \times Y$. ∎

Theorem 2.22. Let $f : X \to Y$ have a fuzzy strongly contra-rw-closed graph. If f is injective, then X is fuzzy rw-T_1.

Proof. Let x_ϵ and y_ν be any two distinct points of X. Then, we have $(x_\epsilon, f(y_\nu)) \in (X \times Y)\backslash G(f)$. By Lemma 2.20, there exist a fuzzy rw-open set μ in X containing x_ϵ and a fuzzy regular closed set ρ in Y containing $f(y_\nu)$ such that $f(\mu) \wedge \rho = \phi$; hence $\mu \wedge f^{-1}(\rho) = \phi$. Therefore, we have $y_\nu \notin \mu$. This implies that X is fuzzy rw-T_1. ∎

3. The relationships

In this section, the relationships between fuzzy almost contra-rw-continuous functions and the other forms are investigated.

Definition 3.1. A function $f : X \to Y$ is called fuzzy weakly almost contra-rw-continuous if for each $x \in X$ and each fuzzy regular closed set η of Y containing $f(x_\epsilon)$, there exists a fuzzy rw-open set μ in X containing x_ϵ such that $int(f(\mu)) \leq \eta$.

Definition 3.2. A function $f : X \to Y$ is called fuzzy (rw, s)-open if the image of each fuzzy rw-open set is fuzzy semi-open.

Theorem 3.3. If a function $f : X \to Y$ is fuzzy weakly almost contra-rw-continuous and fuzzy (rw, s)-open, then f is fuzzy almost contra-rw-continuous.

Proof. Let $x_\epsilon \in X$ and η be a fuzzy regular closed set containing $f(x_\epsilon)$. Since f is fuzzy weakly almost contra-rw-continuous, there exists a fuzzy rw-open set μ in X containing x_ϵ such that $int(f(\mu)) \leq \eta$. Since f is fuzzy (rw, s)-open, $f(\mu)$ is a semi-open set in Y and $f(\mu) \leq cl(int(f(\mu))) \leq \eta$. This shows that f is fuzzy almost contra-rw-continuous. ∎

Definition 3.4. [5] A fuzzy space is said to be fuzzy P_Σ if for any fuzzy open set μ of X and each $x_\Sigma \in \mu$, there exists fuzzy regular closed set ρ containing x_ϵ such that $x_\epsilon \in \rho \leq \mu$.

Definition 3.5. A fuzzy function $f : X \to Y$ is said to be fuzzy rw-continuous [12] if $f^{-1}(\mu)$ is fuzzy rw-open in X for every fuzzy open set μ in Y.

Theorem 3.6. Let $f : X \to Y$ be a fuzzy function. Then, if f is fuzzy almost contra-rw-continuous and Y is fuzzy P_Σ, then f is fuzzy rw-continuous.

Proof. Let μ be any fuzzy open set in Y. Since Y is fuzzy P_Σ, there exists a family Ψ whose members are fuzzy regular closed set of Y such that $\mu = \vee\{\rho : \rho \in \Psi\}$. Since f is fuzzy almost contra-rw-continuous, $f^{-1}(\rho)$ is fuzzy rw-open in X for each $\rho \in \Psi$ and $f^{-1}(\mu)$ is fuzzy rw-open in X. Therefore, f is fuzzy almost contra-rw-continuous. ■

Definition 3.7. [5] A space is said to be fuzzy weakly P_Σ if for any fuzzy regular open set μ and each $x_\epsilon \in \mu$, there exists a fuzzy regular closed set ρ containing x_ϵ such that $x_\epsilon \in \rho \leq \mu$.

Definition 3.8. A fuzzy function $f : X \to Y$ is said to be fuzzy almost rw-continuous at $x_\epsilon \in X$ if for each fuzzy open set η containing $f(x_\epsilon)$, there exists a fuzzy rw-open set μ containing x_ϵ such that $f(\mu) \leq int(cl(\eta))$.

Theorem 3.9. Let $f : X \to Y$ be a fuzzy almost contra-rw-continuous function. If Y is fuzzy weakly P_Σ, then f is fuzzy almost rw-continuous.

Proof. Let μ be any fuzzy regular open set of Y. Since Y is fuzzy weakly P_Σ, there exists a family Ψ whose members are fuzzy regular closed set of Y such that $\mu = \vee\{\rho : \rho \in \Psi\}$. Since f is fuzzy almost contra-rw-continuous, $f^{-1}(\rho)$ is fuzzy rw-open in X for each $\rho \in \Psi$ and $f^{-1}(\mu)$ is fuzzy rw-open in X. Hence, f is fuzzy almost rw-continuous. ■

Definition 3.10. [12] A fuzzy function $f : X \to Y$ is called fuzzy rw-irresolute if inverse image of each fuzzy rw-open set is fuzzy rw-open.

Theorem 3.11. Let X, Y, Z be fuzzy topological spaces and let $f : X \to Y$ and $g : Y \to Z$ be fuzzy functions. If f is fuzzy rw-irresolute and g is fuzzy almost contra-rw-continuous, then $g \circ f : X \to Z$ is a fuzzy almost contra-rw-continuous function.

Proof. Let $\mu \leq Z$ be any fuzzy regular closed set and let $(g \circ f)(x_\epsilon) \in \mu$. Then $g(f(x_\epsilon)) \in \mu$. Since g is fuzzy almost contra-rw-continuous function, it follows that there exists a fuzzy rw-open set ρ containing $f(x_\epsilon)$ such that $g(\rho) \leq \cdot\mu$. Since f is fuzzy rw-irresolute function, it follows that there exists a fuzzy rw-open set η containing x_ϵ such that $f(\eta) \leq \cdot\rho$. From here we obtain that $(g \circ f)(\eta) = g(f(\eta)) \leq \cdot g(\rho) \leq \mu$. Thus, we show that $g \circ f$ is a fuzzy almost contra-rw-continuous function. ■

Definition 3.12. A fuzzy function $f : X \to Y$ is called fuzzy rw-open [12] if image of each fuzzy rw-open set is fuzzy rw-open.

Theorem 3.13. If $f : X \to Y$ is a surjective fuzzy rw-open function and $g : Y \to Z$ is a fuzzy function such that $g \circ f : X \to Z$ is fuzzy almost contra-rw-continuous, then g is fuzzy almost contra-rw-continuous.

Proof. Suppose that x_ϵ is a fuzzy singleton in X. Let η be a regular closed set in Z containing $(g \circ f)(x_\epsilon)$. Then there exists a fuzzy rw-open set μ in X containing x_ϵ such that $g(f(\mu)) \leq \eta$. Since f is fuzzy rw-open, $f(\mu)$ is a fuzzy rw-open set in Y containing $f(x_\epsilon)$ such that $g(f(\mu)) \leq \eta$. This implies that g is fuzzy almost contra-rw-continuous. ■

Corollary 3.14. Let $f : X \to Y$ be a surjective fuzzy rw-irresolute and fuzzy rw-open function and let $g : Y \to Z$ be a fuzzy function. Then, $g \circ f : X \to Z$ is fuzzy almost contra-rw-continuous if and only if g is fuzzy almost contra-rw-continuous.

Proof. It can be obtained from Theorem 3.11 and Theorem 3.13 ■

Definition 3.15. A space X is said to be fuzzy rw-compact [11] (fuzzy S-closed [2]) if every fuzzy rw-open (respectively fuzzy regular closed) cover of X has a finite subcover.

Theorem 3.16. The fuzzy almost contra-rw-continuous images of fuzzy rw-compact spaces are S-closed.

Proof. Suppose that $f : X \to Y$ is a fuzzy almost contra-rw-continuous surjection. Let $\{\eta_i : i \in I\}$ be any fuzzy regular closed cover of Y. Since f is fuzzy almost contra-rw-continuous, then $\{f^{-1}(\eta_i) : i \in I\}$ is a fuzzy rw-open cover of X and hence there exists a finite subset I_0 of I such that $X = \vee\{f^{-1}(\eta_i) : i \in I_0\}$. Therefore, we have $Y = \vee\{\eta_i : i \in I_0\}$ and Y is fuzzy S-closed. ■

Definition 3.17. A space X is said to be

(1) fuzzy rw-closed-compact if every fuzzy rw-closed cover of X has a finite subcover,

(2) fuzzy nearly compact [6] if every fuzzy regular open cover of X has a finite subcover.

Theorem 3.18. The fuzzy almost contra-rw-continuous images of fuzzy rw-closed-compact spaces are fuzzy nearly compact.

Proof. Suppose that $f : X \to Y$ is a fuzzy almost contra-rw-continuous surjection. Let $\{\eta_i : i \in I\}$ be any fuzzy regular open cover of Y. Since f is fuzzy almost contra-rw-continuous, then $\{f^{-1}(\eta_i) : i \in I\}$ is a fuzzy rw-closed cover of X. Since X is fuzzy rw-closed-compact, there exists a finite subset I_0 of I such that $X = \vee\{f^{-1}(\eta_i) : i \in I_0\}$. Thus, we have $Y = \vee\{\eta_i : i \in I_0\}$ and Y is fuzzy nearly compact. ■

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