Strong Edge Graceful Labeling of Windmill Graphs

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Abstract

A \((p, q)\) graph \(G\) is said to have strong edge graceful labeling if there exists an injection \(f\) from the edge set to \(\left\{1, 2, \ldots, \left\lfloor \frac{3q}{2} \right\rfloor \right\}\) so that the induced mapping \(f^\ast\) defined on the vertex set given by

\[
f^\ast(x) = \sum \{f(xy) : xy \in E(G)\} \pmod{2p}
\]

are distinct. A graph \(G\) is said to be strong edge graceful if it admits a strong edge graceful labeling. In this paper we investigate strong edge graceful labeling of Windmill graph.

Definition: The windmill graphs \(K_m^{(n)} (n \geq 3)\) to be the family of graphs consisting of \(n\) copies of \(K_m\) with a vertex in common.

Theorem: 1. The windmill graph \(K_4^{(n)}\) is strong edge graceful for all \(n \geq 3\) when \(n\) is even.

Proof: Let \(\{v_1, v_2, v_3, \ldots, v_{3n}\}\) be the vertices of \(K_4^{(n)}\) and \(\{e_1, e_2, e_3, \ldots, e_{3n}\}\) be the edges of \(K_4^{(n)}\) which are denoted as in the following Fig. 1.
Fig. 1: $K_4^{(n)}$ with ordinary labeling

We first label the edges of $K_4^{(n)}$ as follows:

- $f(e_i) = i$ \hspace{1cm} \(1 \leq i \leq \frac{3n}{2}\)
- $f(f_i) = 3n+1 + i$ \hspace{1cm} \(\frac{3n}{2} + 1 \leq i \leq 3n\)
- $f(e_i) = 3n+1 - i$ \hspace{1cm} \(1 \leq i \leq \frac{3n}{2}\)
- $f(e_i) = 6n+2 - i$ \hspace{1cm} \(\frac{3n}{2} + 1 \leq i \leq 3n\)

Then the induced vertex labels are:

- $f^+(v_0) = 0$
- $f^+(v_i) = 6n+2 - i$ \(1 \leq i \leq \frac{3n}{2}\)$
- $f^+(v_i) = 3n+1 - i$ \(\frac{3n}{2} + 1 \leq i \leq 3n\)$
Clearly, the vertex labels are all distinct. Hence The windmill graph $K_d^{(n)}$ is strong edge graceful for all $n \geq 3$ when $n$ is even.

The SEGL of $K_d^{(4)}$, $K_d^{(8)}$ are illustrated in Fig.2, Fig.3, respectively.

![Diagram](image)  

Fig.2. $K_d^{(d)}$ with SEGL
Theorem: 2. The windmill graph $K_4^{(n)}$ is strong edge graceful for all $n \geq 3$ when $n \equiv 1 \pmod{4}$.

Proof: Let \{v_1, v_2, v_3, ..., v_{3n}\} be the vertices of $K_4^{(n)}$ and \{e_1, e_2, e_3, ..., e_{3n-1}, f_1, f_2, f_3, ..., f_{3n-1}, f_{3n}\} be the edges of $K_4^{(n)}$ which are denoted as in the above Fig. 1.

We first label the edges of $K_4^{(n)}$ as follows:

\[ f(e_i) = 6n- i \quad 1 \leq i \leq 3n \]
\[ f(f_i) = i \quad 1 \leq i \leq 3n-1 \]
Then the induced vertex labels are:

\[
\begin{align*}
    f^+(v_0) &= \frac{3n-1}{2} \\
    f^+(v_i) &= 6n - 2 - i \quad 1 \leq i \leq 3n-3 \\
    f^+(v_{3n-2}) &= 6n \\
    f^+(v_i) &= 6n - 2 - i \quad 3n - 1 \leq i \leq 3n
\end{align*}
\]

Clearly, the vertex labels are all distinct. Hence, the windmill graph \(K_4^{(n)}\) is strong edge graceful for all \(n \geq 3\) when \(n \equiv 1 \pmod{4}\).

The SEGL of \(K_4^{(5)}\), \(K_4^{(9)}\) are illustrated in Fig.4, Fig.5, respectively.

Fig.4. \(K_4^{(5)}\) with SEGL
Theorem: 3. The windmill graph $K_4^{(n)}$ is strong edge graceful for all $n \geq 3$ when $n \equiv 3 \pmod{4}$.

Proof: Let $\{v_1, v_2, v_3, \ldots, v_{3n}\}$ be the vertices of $K_4^{(n)}$ and $\{e_1, e_2, e_3, \ldots, e_{3n-1}, f_1, f_2, f_3, \ldots, f_{3n}\}$ be the edges of $K_4^{(n)}$ which are denoted as in the above Fig. 1.

We first label the edges of $K_4^{(n)}$ as follows:

$$f\left(f_i\right) = i \quad \quad 1 \leq i \leq 3n$$

$$f\left(e_i\right) = 6n+1 - i \quad \quad 1 \leq i \leq 3n$$
Then the induced vertex labels are:

\[ f^+(v_0) = \frac{3n+1}{2} \]

\[ f^+(v_i) = 6n - i \leq i \leq 3n \]

Clearly, the vertex labels are all distinct. Hence the windmill graph \( K_{d(n)} \) is strong edge graceful for all \( n \geq 3 \) when \( n \equiv 3 \) (mod 4).

The SEGL of \( K_d(3) \), \( K_d(7) \) are illustrated in Fig.6, Fig.7, respectively.

![Fig.6. \( K_d(3) \) with SEGL](image-url)
REFERENCES: