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The Effect of Magnetic Field on a Developing Flow and Flow Reversal in a Vertical Channel with Asymmetric Wall Temperatures

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Abstract

This paper deals with the effect of magnetic field on a developing flow and flow reversal in a vertical channel with asymmetric wall temperatures is considered under the influence of transverse magnetic field. In the developing region, the flow problem is described by means of parabolic partial differential equations and solutions are obtained by an implicit finite difference technique. Boundary conditions of uniform wall temperatures are considered. The effect of magnetic field on velocity and temperature for fixed Prandtl number Pr and for the different values of the ratio Gr/Re and the ratio of wall temperature r_T is studied numerically. It is observed that the velocity decreases with the increase in magnetic field parameter M for fixed r_T. It is noticed that the increase in magnetic field parameter M for fixed Gr/Re, centre line velocity decreases. A skewness in the velocity profile also appears as the fluid moves toward hot wall(Y=1) for fixed M. It is observed that the increasing M for fixed Gr/Re at r_T=0.5 for different X values the temperature is decreases.

Keywords: Mixed Convection, Flow Reversal, Asymmetric Wall Temperature & MHD.

Introduction

Recent technological implications have given rise to increased interest in mixed convection problems in vertical channels. The physical situations involve both buoyancy-aided and opposed cases, for laminar and turbulent flows. Consequently, the number of technical papers and technical sessions at professional society meetings that deal with combined free and forced convection is on the rise.

Existing literature for the parallel-plate vertical channel deals mostly with the limiting case of free and forced convection, little information is available for mixed convection. Consider the situation in which the channel walls are cooled by forced flow in the upward direction at a prescribed coolant flow rate at the duct entrance. Assume that the wall heating is sufficiently intense that free convection effects are significant. Such a mixed convection problem has not been fully treated in the literature.

Aung and Worku [2] presented the numerical results for the effects of buoyancy on the hydrodynamic and thermal parameters in the laminar vertically upward flow of a viscous fluid in a parallel plate channel. Mixed convection effects on fully developed flow (FDF) in a parallel plate vertical channel with asymmetric wall temperatures was studied by Aung and Worku [3].

One of the earliest study on laminar, fully developed mixed convection in a vertical channel with uniform wall temperature was by Tao [22]. Recently Cheng et.al. [11] and Hamadah and Wirtz [15] have studied the mixed convection in a vertical channel with symmetric and asymmetric heating of the walls. These authors reported that the buoyancy force can cause flow reversal both for upward flow and for downward flow. More recently Barletta [4] has studied the fully developed combined free and forced convection flow in a vertical channel with viscous dissipation. An analytical solution is found by a perturbation method and in particular, forced convection flow with viscous heating is treated as the base heat transfer process while the effect of buoyancy is accounted for by expressing the fluid velocity and temperature as power series in the ratio between the Grashof number and Reynolds number. Analysis of flow reversal for laminar mixed convection in a vertical rectangular duct with one or more isothermal walls is studied by Barletta [6].

The analysis of magneto-hydrodynamic flow through ducts has received considerable attention. This class of flow has many applications in the design of MHD generators, cross-field accelerators, shock tubes, pumps and flow meters. In many cases the flow in these devices will be accompanied by heat either that dissipated internally through viscous or Joule heating or that produced by electric currents in the walls. The use of electrically conducting fluids under the influence of magnetic fields in various industries has led to a renewed interest in investigating hydro-magnetic flow and heat transfer in different geometrics. Sparrow and Cess [20] considered the effect of a magnetic field on the free convection heat transfer from a surface. Chamka [10] studied on laminar hydro-magnetic mixed convection flow in a vertical channel with symmetric and asymmetric wall heating conditions. Umavathi and Malashetty [23] analyzed combined free and forced convective magneto- hydrodynamic flow in a vertical channel. The channel walls are maintained at equal or at different constant temperatures. Reddy [18] obtained the closed form solutions for the effect of magnetic field on a fully developed combined convection flows in a vertical channel. Fully developed MHD free convection flow of a viscous electrically conducting fluid in a vertical parallel plate channel was studied by Hughes and Young [16].

Nomenclature

A Surface area

b Spacing between plates
C_p Specific heat at constant pressure
g Acceleration due to gravity
Gr Grashof number, $g \beta(T_2 - T_0) b3 / \upsilon 2$
h Heat transfer coefficient
H_o Applied magnetic field
J Current density vector
k Thermal conductivity
M Magnetic parameter
Nu Nusselt number
p Pressure difference, $p' - p''$
p' Static pressure
p'' Hydrostatic pressure
p Dimensionless pressure difference, $(p' - p'') / \rho_u 2$
Pr Prandtl number
$\bar{q}$ Velocity vector, (u, v, 0)
r_T Ratio of wall temperature difference, $(T_1 - T_0) / (T_2 - T_0)$
Re Reynolds number, $u b / \upsilon$
T Temperature
u Axial velocity
u_0 Average fluid velocity
v Transverse velocity
U Dimensionless stream wise velocity, u / u_o
V Dimensionless transverse velocity, $v b / \upsilon$
x Stream wise distance from channel entrance
y Transverse coordinate (measured from cool wall)
X Dimensionless stream wise distance from Channel entrance, x/(b Re)
Y Dimensionless transverse coordinate, y / b
β Thermal expansion coefficient
μ Dynamic viscosity
μe Magnetic permeability
υ Kinematic viscosity
ρ Density
ρ_0 Fluid density at ambient temperature
σ Electrical conductivity
θ Dimensionless temperature difference, $(T - T_0) / (T_2 - T_0)$

Subscripts
0 Value at channel entrance (at x = 0)
1 Cool wall (i.e. value at y = 0)
2 Hot wall (i.e. value at y = b)
b Bulk value
c Value at center line
m Mean value

Formulation of the Problem

We consider the effect of magnetic field on developing flow and flow reversal in a vertical channel with asymmetric wall. The fluid assumed to be two dimensional and steady and the fluid properties are constant except for the variation of density in the buoyancy term of the momentum equation. The physical model of the problem is given in Figure.1. The distance between the plates is 'b'. The fluid has a uniform vertically upward stream wise velocity distribution at the channel entrance. The walls are heated at uniform wall temperature (UWT) but the temperatures on the two walls may be different, resulting in an asymmetric heating. A uniform transverse magnetic field of strength H_0 is applied perpendicular to the walls (i.e. in the Y-direction).

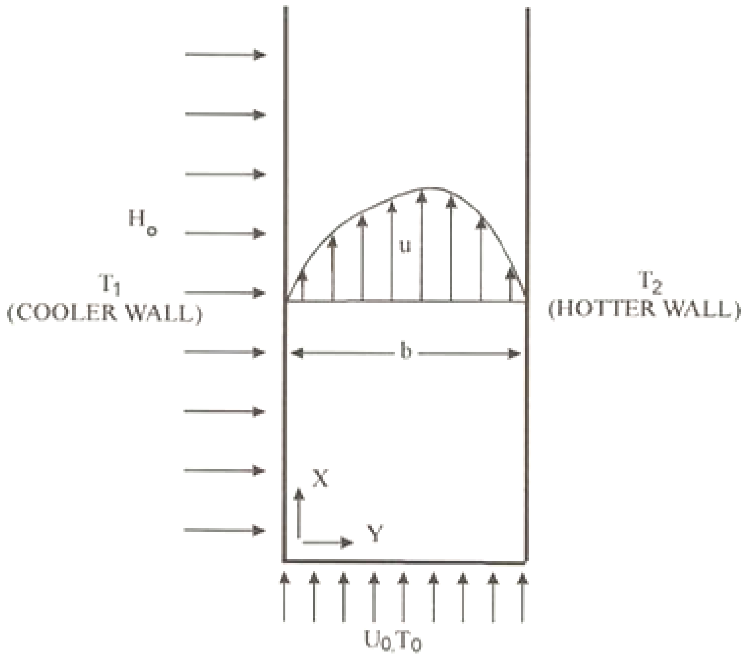

Figure 1: Two Dimensional Channel

The governing equations for the steady viscous flow of an electrically conducting fluid in the presence of an external magnetic field with the following assumptions are made:

The flow is steady, viscous, incompressible, and developed.

The flow is assumed to be two-dimensional steady, and the fluid properties are constant except for the variation of density in the buoyancy term of the momentum equation.

The electric field E, and induced magnetic field are neglected [19, 21].

Energy dissipation is neglected.

Applying the above assumptions, the boundary layer equations appropriate for this problem are

Continuity

$$\frac{\partial U}{\partial X}+\frac{\partial V}{\partial Y}=0 \tag{1}$$

X - momentum

$$U\frac{\partial U}{\partial X}+V\frac{\partial U}{\partial Y}=-\frac{\partial P}{\partial X}+\frac{Gr}{\mathrm{Re}}\theta+\frac{\partial^2 U}{\partial Y^2}-M^2U \tag{2}$$

Y - momentum $$\frac{\partial P}{\partial Y}=0 \tag{3}$$

Energy

$$U\frac{\partial \theta}{\partial X}+V\frac{\partial \theta}{\partial Y}=\frac{1}{\mathrm{Pr}}\frac{\partial^2 \theta}{\partial Y^2} \tag{4}$$

where

M2 = σ $\mu 2_e$ $H_0 2$ b2 / μ

In equation (2), use has been made of the Boussinisq equation of state, $\rho - \rho_0 = -\rho\ \beta\ (T-T_0)$, and of the definition $p = p' - p''$, where p'' is the pressure at any stream-wise position if the temperature were T_0 everywhere. The latter definition gives $dp''/dx = -\rho_0 g$. Hence

$$-\frac{dp'}{dx}-\rho\ g=-\frac{dp}{dx}+\rho g\beta(T-T_0)$$

It is noted that the dimensionless pressure is $P = (p' - p'') / \rho\ u_0{}^2$, if the channel were horizontal, we would have $P = p' /\rho\ u_0{}^2$, the convectional definition in pure forced flow.

The boundary conditions are

At X = 0, $0 \le Y \le 1$: U = 1, V = 0, θ = 0, P = 0
At X > 0, Y = 0 : U = 0, V = 0, $\theta = r_T$ (5)
At X >0, Y = 1 : U = 0, V = 0, θ = 1

The above, dimensionless parameters have been depend as:

$U = u / u_0$, $V = vb/\upsilon$, $X = x / (b\ \mathrm{Re})$, $Y = y / b$
$P = (p' - p'') /\rho\ u_0{}^2$, $\mathrm{Pr} = \mu C_p / k$, $\mathrm{Re} = b\ u_0 /\upsilon$ (6)
$Gr = g\beta\ (T_2 - T_0)\ b^3/\upsilon^2$, $\theta = (T - T_0) / (T_2 - T_0)$

To obtain a solution of the mixed convection problem formulated above, an additional equation expressing the global conservation of mass at any cross section in the channel is also required.

This becomes

$$\int_0^1 U\,dY = 1 \tag{7}$$

The systems of non-linear equations (1) to (3) are solved by a numerical method based on finite difference approximations. An implicit difference technique is employed whereby the differential equations are transformed into a set of simultaneous linear algebraic equations.

Numerical Solution

The solution of the governing equations for developing flow is discussed in this section. Considering the finite difference grid net work of figure.2, equations (2) and (4) are replaced by the following difference equations which were also used in [7].

$$U(i,j)\frac{U(i+1,j)-U(i,j)}{\Delta X}+V(i,j)\frac{U(i+1,j+1)-U(i+1,j-1)}{2\Delta Y}$$
$$=\frac{U(i+1,j+1)-2U(i+1,j)+U(i+1,j-1)}{(\Delta Y)^2}-M^2U(i+1,j) \tag{8}$$
$$-\frac{P(i+1)-P(i)}{\Delta X}+\frac{Gr}{\mathrm{Re}}\theta(i+1,j)$$

$$U(i,j)\frac{\theta(i+1,j)-\theta(i,j)}{\Delta X}+V(i,j)\frac{\theta(i+1,j+1)-\theta(i+1,j-1)}{2\Delta Y}$$
$$=\frac{1}{\mathrm{Pr}}\frac{\theta(i+1,j+1)-2\theta(i+1,j)+\theta(i+1,j-1)}{(\Delta Y)^2} \tag{9}$$

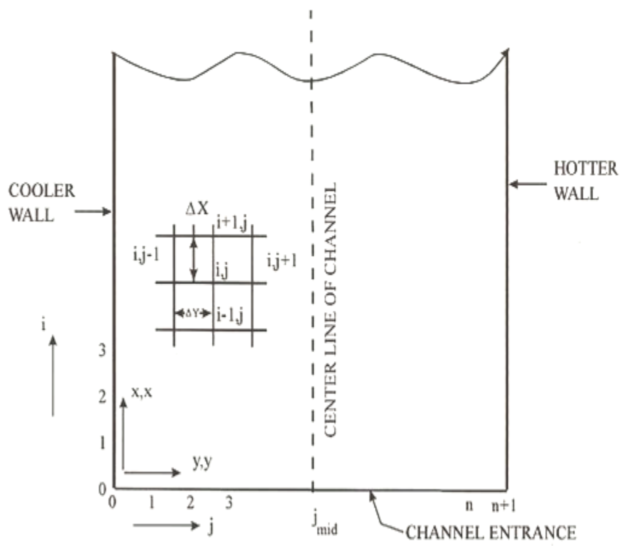

Figure 2: Finite Difference Network

To calculate V, the transverse component of velocity, the centerline value of the index is denoted (figure 2) j_{mid} = (n+3)/2, where (n+2) is the total number of nodes along Y and is an odd integer.

For $j < j_{mid}$, the continuity equation (1) can be written

$$V(i+1, j) = V(i+1, j+1) - \frac{\Delta Y}{2\Delta X}(U(i+1, j) + U(i+1, j-1) - U(i, j) - U(i, j-1)),\ j = 1,2,\ldots, j_{mid} - 1 \tag{10a}$$

For $j > j_{mid}$, the above equation is modified and the following form is used:

$$V(i+1, j) = V(i+1, j+1) + \frac{\Delta Y}{2\Delta X}(U(i+1, j) + U(i+1, j+1) - U(i, j) - U(i, j+1)),\ j = n, n-1, \ldots, j_{mid} + 1 \tag{10b}$$

In principle, either equation (10a) or equation (10b) could be used to evaluate the centerline velocity V(i+1, j_{mid}). However, since both equations employ one-sided differences, a different value of V(i+1, j_{mid}) could result depending on which equation is considered. Consequently, the transverse velocity at the centerline is calculated by fitting a third order polynomial through the values on the two immediate mesh points on both sides of the centerline.

By means of Simpson's rule, we write equation (7) as

$$4U(i+1,1)+2U(i+1,2) + 4U(i+1, 3) +\ldots+ 4U(i+1, n) = 3(n+1) \tag{11}$$

A set of finite-difference equations written about each mesh point in a column for the equation (8) as shown:

$$\beta_1 U(i+1,1) + \gamma_1 U(i+1,2) + \xi P(i+1) + \frac{Gr}{\mathrm{Re}}\theta(i+1,1) = \phi_1$$

$$\alpha_2 U(i+1,1) + \beta_2 U(i+1,2) + \gamma_2 U(i+1,3) + \xi P(i+1) + \frac{Gr}{\mathrm{Re}}\theta(i+1,2) = \phi_2$$

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$$\alpha_n U(i+1,n-1) + \beta_n U(i+1,n) + \xi P(i+1) + \frac{Gr}{\mathrm{Re}}\theta(i+1,n) = \phi_n$$

where

$$\alpha_k = \frac{1}{(\Delta Y)^2} + \frac{V(i,j)}{2\Delta Y},\quad \beta_k = -\left[\frac{2}{(\Delta Y)^2} + M^2 + \frac{U(i,j)}{\Delta X}\right]$$

$$\gamma_k = \frac{1}{(\Delta Y)^2} - \frac{V(i,j)}{2\Delta Y},\quad \xi = \frac{-1}{\Delta X},$$

$$\phi_k = -\left[\frac{P(i)+U^2(i,j)}{\Delta X}\right]$$

for k = 1,2… n

A set of finite-difference equations written about each mesh point in a column for the equation (9) as shown:

$$\bar{\beta}_1\theta(i+1,1)+\bar{\gamma}_1\theta(i+1,2)+-------------- = \bar{\phi}_1 - r_T\bar{\alpha}_1,$$
$$\bar{\alpha}_2\theta(i+1,1)+\bar{\beta}_2\theta(i+1,2)+\bar{\gamma}_2\theta(i+1,3)+------------ = \bar{\phi}_2$$
$$---------------------------------------$$
$$---------------------------------------$$
$$\bar{\alpha}_n\theta(i+1,n-1)+\bar{\beta}_n\theta(i+1,n) = \bar{\phi}_n - \bar{\gamma}_n$$

where

$$\bar{\alpha}_k = \frac{1}{\Pr(\Delta Y)^2}+\frac{V(i,j)}{2\Delta Y},\ \bar{\beta}_k = -\left[\frac{2}{\Pr(\Delta Y)^2}+\frac{U(i,j)}{\Delta X}\right]$$
$$\bar{\gamma}_k = \frac{1}{\Pr(\Delta Y)^2}-\frac{V(i,j)}{2\Delta Y},\ \bar{\phi}_k = -\frac{U(i,j)\theta(i,j)}{\Delta X}$$

for k = 1,2… n

The solutions of the difference equations are obtained by first selecting values for Pr, Gr/Re, M and r_T and then by means of a marching procedure the variables U, V, θ and P for each row beginning at row (i+1) = 2 are obtained using the values at the previous row ‘i’. Thus, by applying equations (8), (9) and (11) to the points 1, 2, ..., n on row i, 2n+1 algebraic equations with the 2n+1 unknowns U(i+1,1),U(i+1,2),

, U(i+1,n), P(i+1), θ(i+1,1), θ(i+1,2), ... ,θ(i+1, n) are obtained. This system of equations is then solved by a matrix reduction technique. Equations (10) are then used to calculate V(i+1,1),V(i+1,2), ..., V(i+1,n).

Results and Discussion

In the present study, quantitative information on the effects of buoyancy and asymmetric heating have been obtained for pr= 0.72 at Gr/Re = 0, 50, 100, 250 and 500 for different values of magnetic field parameter M.

The present results show that at small Gr/Re, the velocity profile, specified as U=1 at the channel entrance, remains positive through out at all X for different values of magnetic parameter M. At a sufficiently high value of Gr/Re for a fixed r_T, the stream wise velocity is everywhere positive up to a certain X, then a separation point (i.e., $\partial U/\partial Y$ = 0) develops on the cool wall when $r_T < 1$ for fixed values of magnetic field parameter M.

The present numerical approach yielded stable solution in one particular case involving reversed flow. The calculation was carried out for Gr/Re = 250 and r_T = 0.5. The results are reported in [18] to afford comparison with the analytical solution for FDF, and point to the need for additional clarification of the usefulness and limitations of the present approach for situation involving flow recirculation. Also in need of further investigation is the concept of fully developed flow in the presence of by directional flow.

An additional discussion of flow reversal in the context of FDF is given in [18] where, for example it is shown that even in the presence of flow reversal, the centre line velocity is always positive at any r_T and has a numerical value of 1.49, the same as when buoyancy is absent. It should be recognized that flow separation as a fundamental fluid flow phenomena is still poorly understood, even in laminar flow.

For a channel with symmetric heating at UWT (r_T=1), the stream wise variation of the centre line velocity is indicated in figures 3(a) to 3(d). It can be seen that buoyancy effects are felt very close to the channel entrance (X=0). Buoyancy causes increased mass flow close to the walls, and since the global mass is fixed, the fluid velocities near the center line decrease for fixed magnetic field parameter M. At sufficiently high value of Gr/Re (larger than 100), the centre line velocity undergoes a minimum, then once again increases monotonically. It is seen that increasing the magnetic field parameter M for fixed Gr/Re, centerline velocity decreases.

Since buoyancy leads to increased velocities near the walls, the velocity profile attains a concave shape near the centre and the concavity becomes more severe as Gr/Re increases. However, for r_T=1 at all values are Gr/Re, the concavity eventually disappears and the profile develops in to the fully developed shape predicted by the fully developed flow theory given in [18]. This effect is illustrated in figures 5(a) to 5(b). For asymmetric wall temperatures (r_T < 1), the concavity never completely disappears, as the FDF theory also predicts [18]. A skeweness in the velocity profile also appears as the fluid moves toward hot wall (Y=1) for fixed M. The smaller r_T, the greater is the skeweness. The distortion of the profile is, however, reduced at increased X. On the other hand, increased buoyancy introduces a more severe distortion as illustrated in figure 4(a) to 4(d).

The development of the temperature field is exemplified by figures 7(a) to 7(d). The FDF temperature distribution is a function only of r_T and not of Gr/Re. The effect of the latter parameter is felt in the developing region, where the buoyancy deceases the temperature in the region adjacent to the hot wall while increasing the temperature else where in the flow. The phenomenon is evident in figures 6(a) to 6(d). Thus, buoyancy tense to equalize the temperature in the fluid. It is seen that increasing the magnetic parameter M for fixed Gr/Re at r_T=0.5 for different X values the temperature is decreases.

Figures 8(a) to 8(b) shows the variation of the dimensionless pressure parameter P for r_T=1. The figures indicate the steam-wise variation of the parameter at different Gr/Re for fixed magnetic parameter M. At some point along the channel, for Gr/Re values of 50, 100, 250 and 500 the pressures attains a minimum (i.e., -P achieves a maximum) and starts increasing. In the upper range of the Gr/Re values (Gr/re>250), the maximum pressure occurs at about the point where buoyancy effects begin to be felt and the center line velocity starts to decrease, for fixed magnetic parameter M. In the same range, it is also observed that P becomes positive when the center line velocity attains a value of less than that of the entry velocity, i.e., U=1.

The axial variation the bulk temperature for r_T=1 at different Gr/Re for fixed magnetic parameter M is displayed in figures 11(a) to 11(d). The bulk temperature is defined as

$$\theta_b = \frac{\int_0^1 U\theta\, dY}{\int_0^1 U\, dY} \tag{12}$$

It may be noted that buoyancy effects are noticeable through a long segment of the channel, but not for small or large X. At large X all the curves converge to the value 1. The value of bulk temperature when $r_T < 1$ is shown, in the asymptotic limit of large X to increase with buoyancy for fixed magnetic field parameter M.

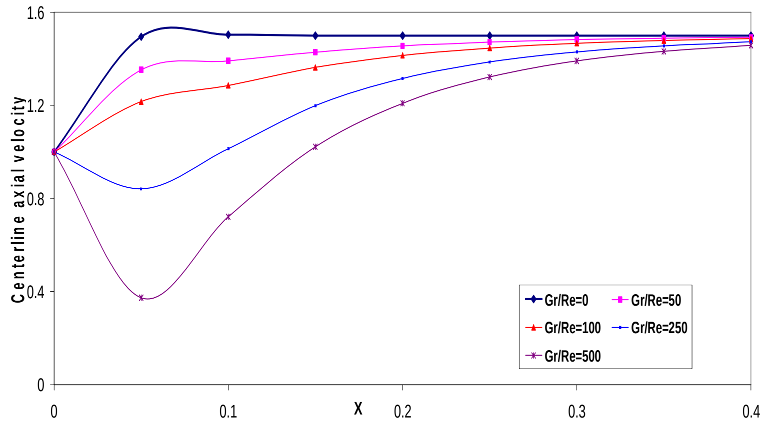

Fig. 3(a) : Centeriline axial velocity values for fixed r_T=1 and M=0

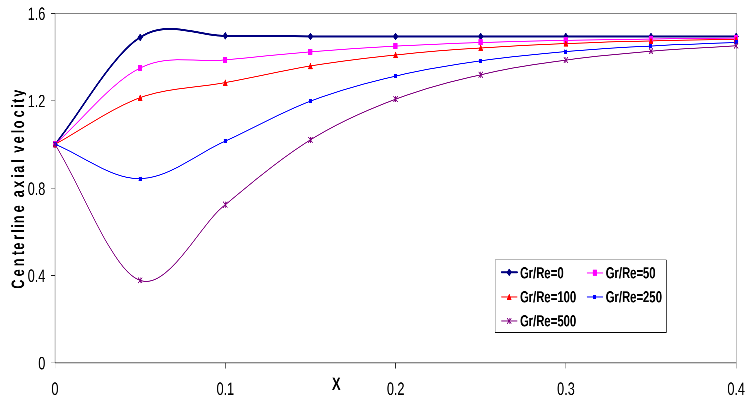

Fig. 3(b): Centerline axial velocity values for fixed r_T=1 and M =1

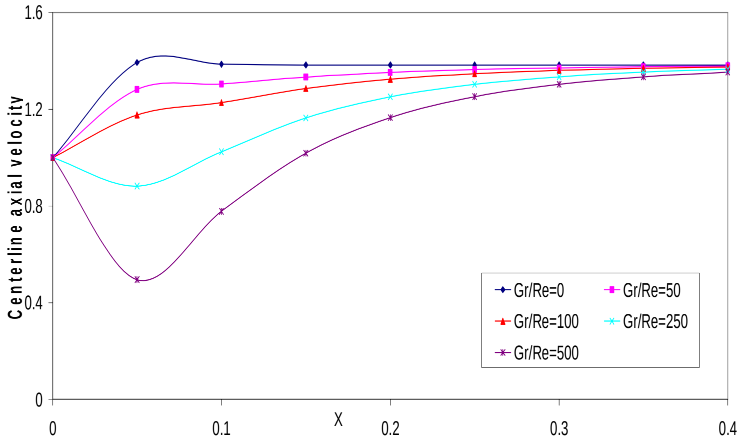

Fig.3(c): Centerline axial velocity values for fixed r_T=1 and M =5

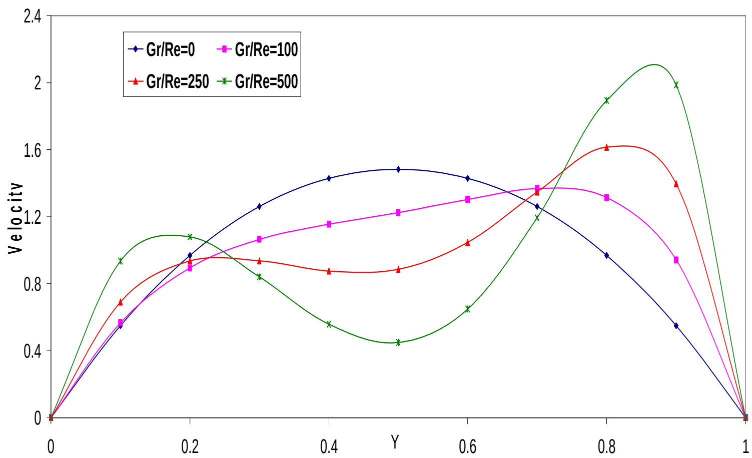

Fig. 4(a): Velocity value for fixed r_T=0.5, X=0.04 and M=0

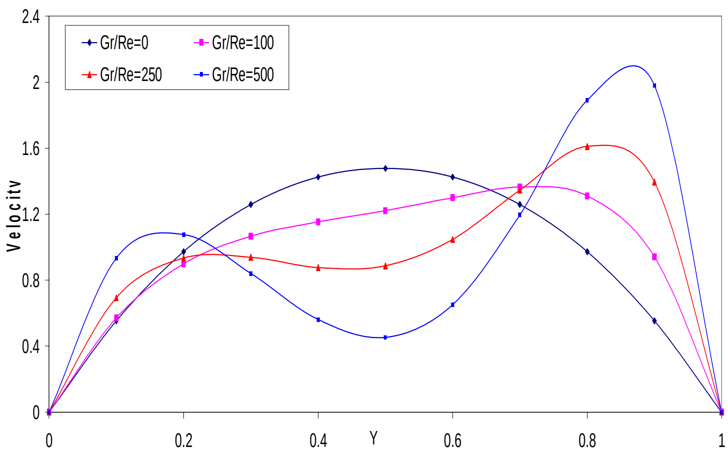

Fig. 4(b): Velocity value for fixed r_T=0.5, X=0.04 and M=1

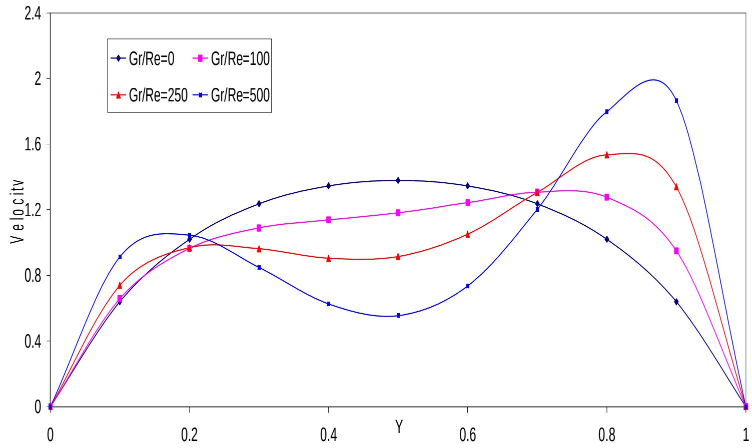

Fig. 4(c) : Velocity value for fixed rT=0.5, X=0.04 and M=5

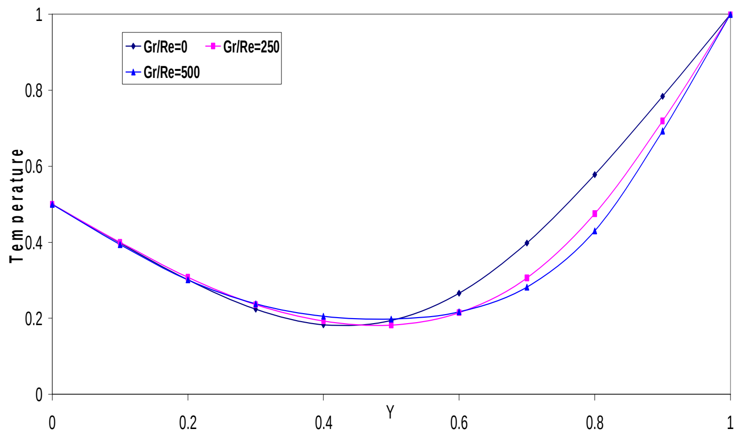

Fig.5(a): Temperature value for fixed r_T=0.5, X=0.04 and M =0

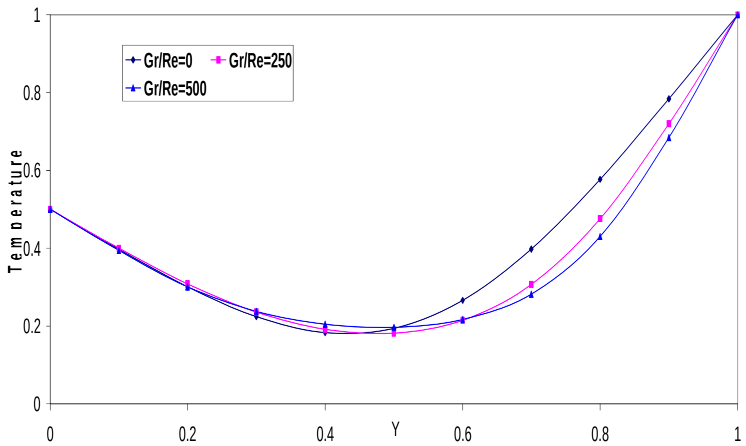

Fig. 5(b): Temperature value for fixed r_T=0.5, X=0.04 and M =1

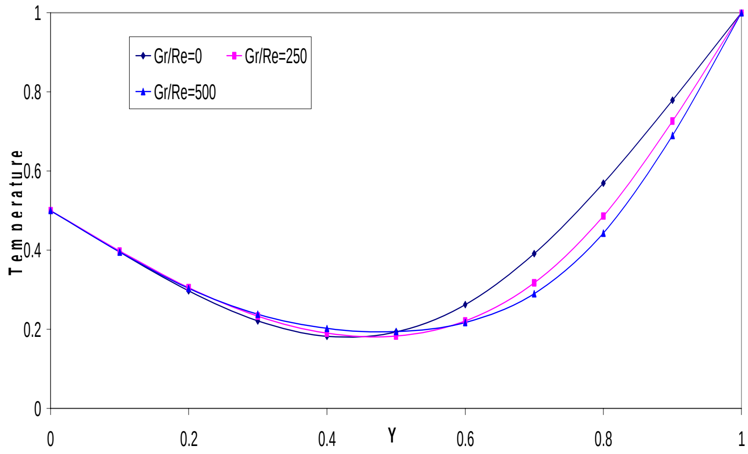

Fig.5(c):Temperature value for fixed r_T=0.5, X=0.04 and M =5

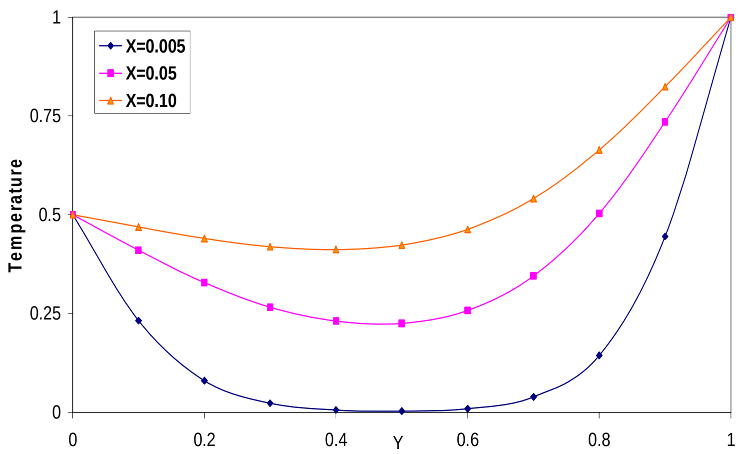

Fig. 6(a): Dimensionless temperature distribution at r_T=0.5, Gr/Re=250 and M=0

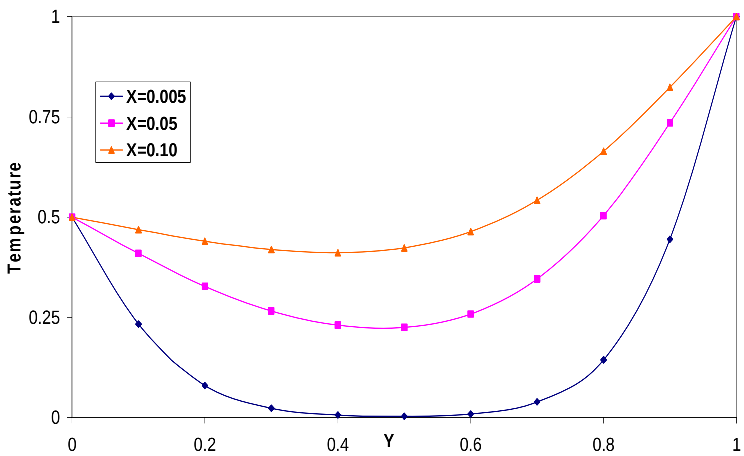

Fig.6(b): Dimensionless temperature distribution at r_T=0.5, Gr/Re=250 and M=1

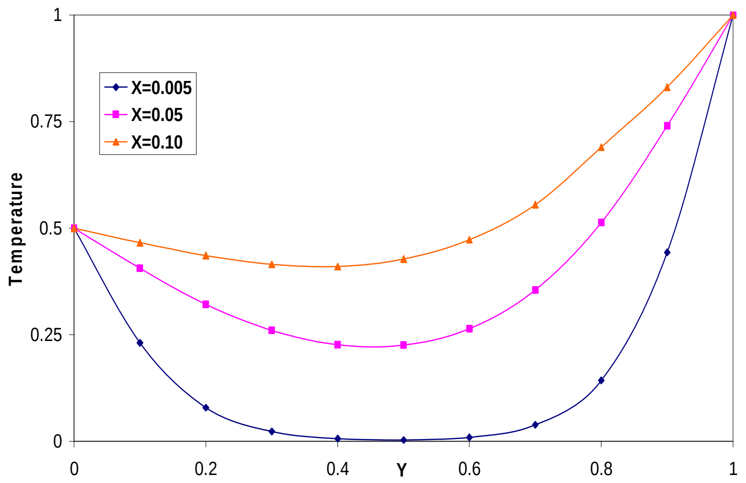

Fig.6(c): Dimensionless temperature distribution at r_T=0, Gr/Re=250 and M=5

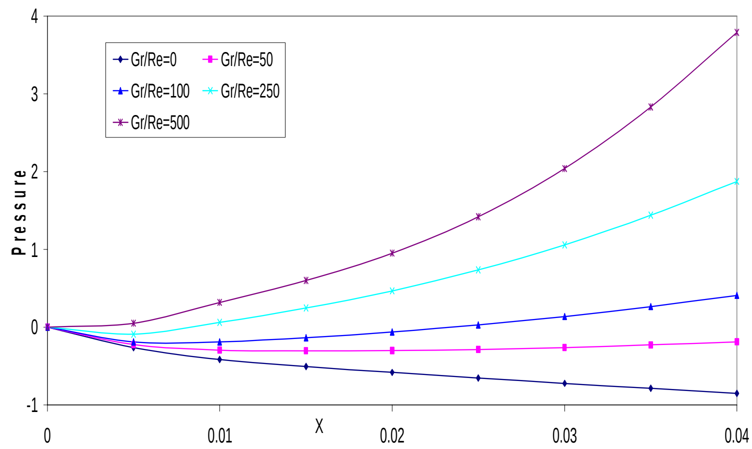

Fig.7(a): Pressure values for fixed r_T = 1.0 and M = 0

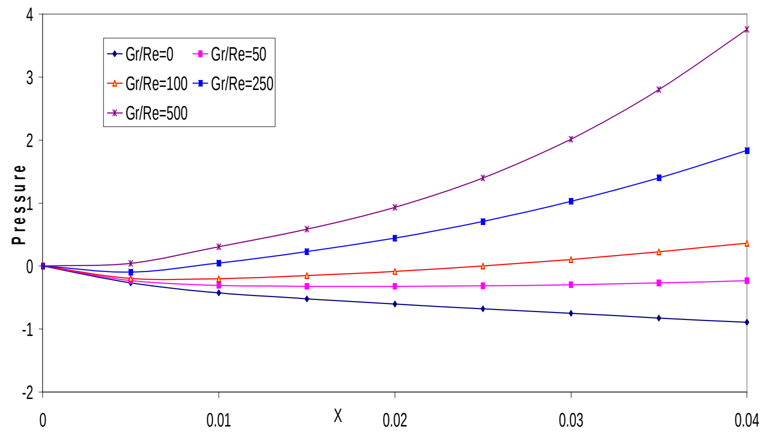

Fig.7(b): Pressure values for fixed r_T = 1.0 and M=1

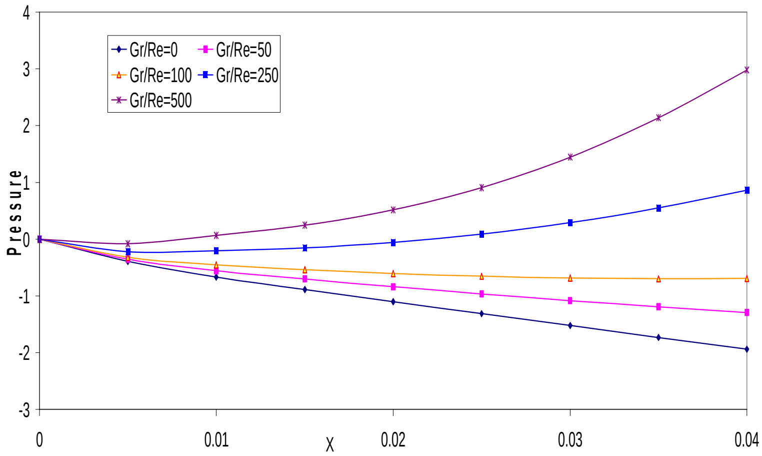

Fig.7(c): Pressure values for fixed $r_T = 1.0$ and M=5

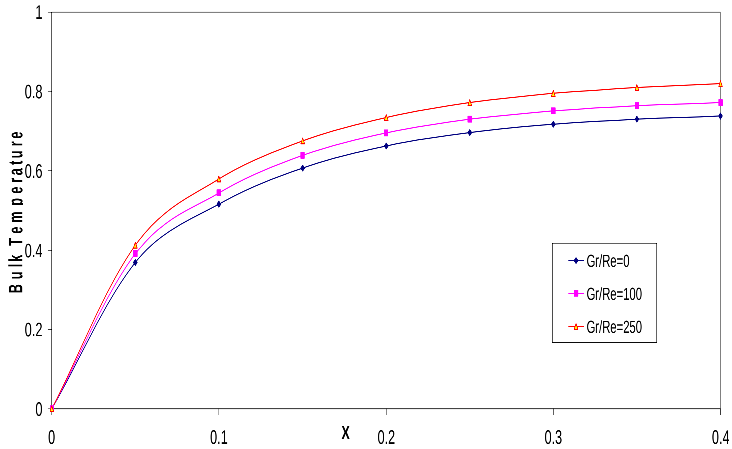

Fig. 8(a) : Bulk Temperture (θ_b) for fixed r_T=0.5 and M=0

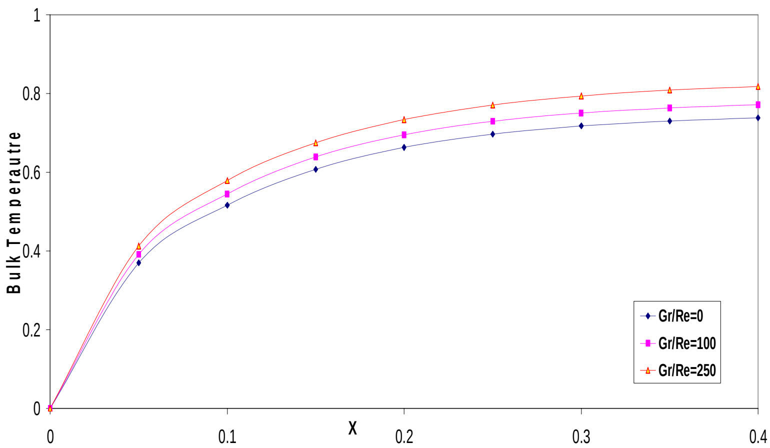

Fig. 8(b): Bulk Temperture(θ_b) for fixed r_T=0.5 and M=1

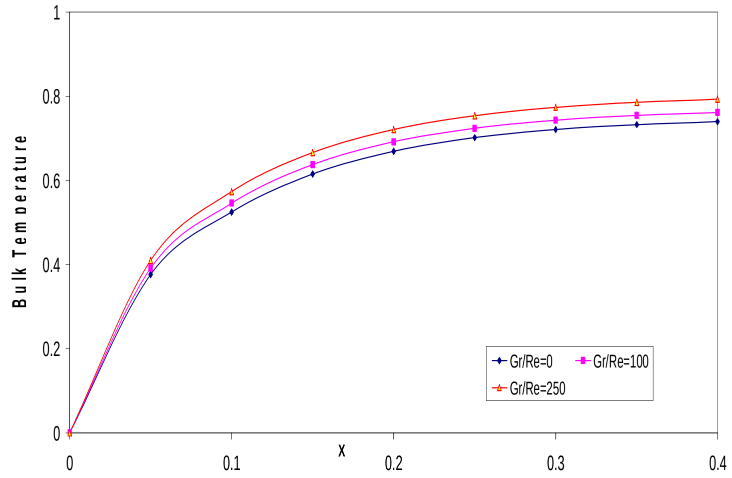

Fig. 8(c) : Bulk Temperature (θ_b) for fixed r_T=0.5 and M=5

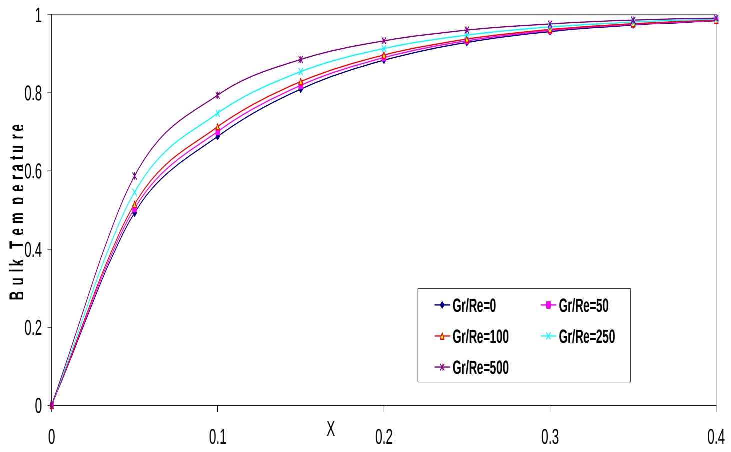

Fig. 9(a) : Bulk Temperature(θ_b) for fixed r_T=1 and M=0

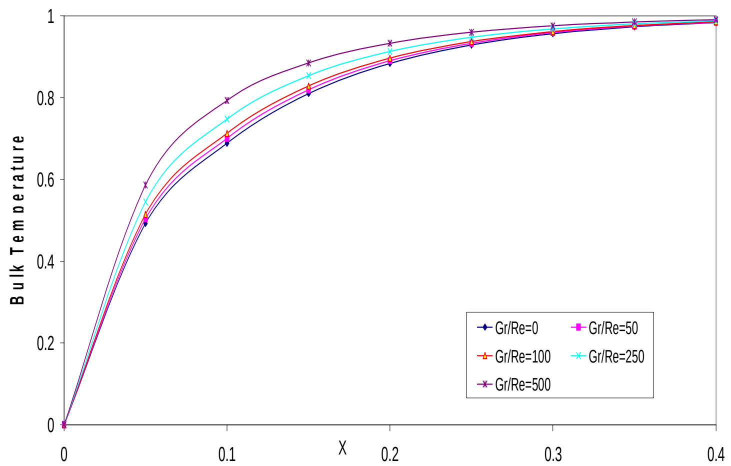

Fig. 9(b) : Bulk Temperature (θ_b) for fixed r_T=1 and M=1

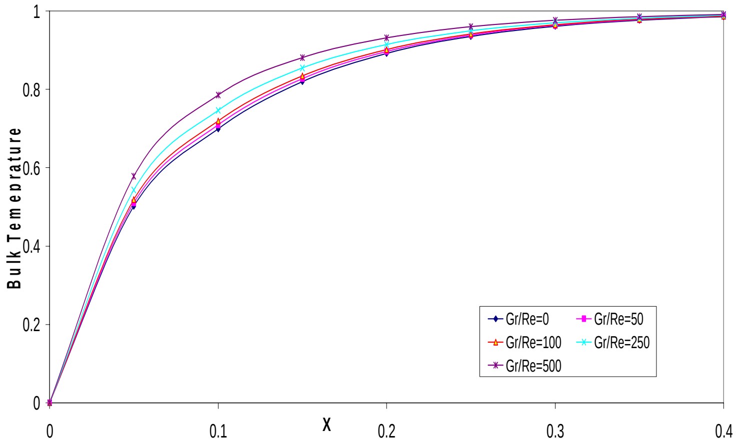

Fig. 9(c) : Bulk Temperature (θ_b) for fixed r_T=1 and M=5

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