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Effects of Radiation on Flow Past an Impulsively Started Infinite Vertical Plate with Uniform Heat and Mass Flux

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Abstract

Numerical technique is employed to derive a solution to the transient natural convection flow of an incompressible viscous fluid past an impulsively started infinite vertical plate with uniform heat and mass flux in the presence of thermal radiation. Heat and mass transfer effects are taken into account and the governing equations are solved using implicit finite-difference method. The effect of velocity and temperature for different parameters like thermal radiation, thermal Grashof number and mass Grashof number are studied. It is observed that the velocity decreases in the presence of thermal radiation.

Key Words: radiation, vertical plate, finite-difference.

Nomenclature

a^*- absorption coefficient

C' - concentration

C - dimensionless concentration

D- mass diffusion coefficient

g- accelaration due to gravity

Gr- thermal Grashof number

Gc- mass Grashof number

j'' - mass flux per unit area at the plate

k- thermal conductivity of the fluid

Pr- Prandtl number

q- heat flux per unit area at the plate

R- radiation parameter

Sc- Schmidt number

T' - temperature

T- dimensionless temperature

t' - time

t- dimensionless time

u_0- velocity of the plate

u- velocity components in x-directions respectively

U- dimensionless velocity components in X-directions respectively

x- spatial coordinate along the plate

X- dimensionless spatial coordinate along the plate

y- spatial coordinate normal to the plate

Y- dimensionless spatial coordinate normal to the plate

Greek symbols

α- thermal diffusivity

β- coefficient of volume expansion

β^*- volumetric coefficient of expansion with concentration

μ- coefficient of viscosity

ν- kinematic viscosity

σ- Stefan-Boltzmann constant

Subscripts

w- conditions at the wall

∞- conditions in the free stream

i- grid point along the X-direction

j- grid point along the Y-direction

1. INTRODUCTION

Processes involving coupled heat and mass transfer occur frequently in nature. It occurs not only due to temperature difference, but also due to concentration difference or the combination of these two. The influence of magnetic field on viscous incompressible flow of electrically conducting fluid has its importance in many applications such as extrusion of plastics in the manufacture of Rayon and Nylon, purification of crude oil, pulp, paper industry, textile industry and in different geophysical cases etc. In many process industries, the cooling of threads or sheets of some polymer materials is of importance in the production line. The rate of cooling can be controlled effectively to achieve final products of desired characteristics by drawing threads, etc. in the presence of an electrically conducting fluid subject to a magnetic field.

Radiative convective flows are encountered in countless industrial and Environment processes e.g. heating and cooling chambers, fossil fuel combustion Energy processes, evaporation from large open water reservoirs, astrophysical Flows, solar power technology and space vehicle re-entry. Radiative heat and Mass transfer play an important role in manufacturing industries for the design of reliable equipment. Nuclear power plants, gas turbines and various propulsion device for aircraft, missiles, satellites and space vehicles are examples of such engineering applications.

England and Emery [2] have studied the thermal radiation effects of a optically thin gray gas bounded by a stationary vertical plate. Soundalgekar and Takhar [7] have considered the radiative free convective flow of an optically thin gray-gas past a semi-infinite vertical plate. Radiation effect on mixed convection along a isothermal vertical plate were studied by Hossain and Takhar [3]. In all above studies, the stationary vertical plate is considered. Raptis and Perdikis [5] have studied the effects of thermal radiation and free convection flow past a moving infinite vertical plate.

Boundary layer flow on moving horizontal surfaces was studied by Sakiadis [6]. Kumari and Nath [4] studied the development of the asymmetric flow of a viscous electrically conducting fluid in the forward stagnation point region of a two-dimensional body and over a stretching surface was set into impulsive motion from the rest.

The problem of unsteady natural convection flow past an impulsively started infinite vertical plate in the presence of thermal radiation has not received attention of any researcher. Hence, the present study is to investigate the unsteady flow past an impulsively started infinite vertical plate with uniform heat and mass flux in the presence of thermal radiation by an implicit finite-difference scheme of Crank-Nicolson type.

2. MATHEMATICAL FORMULATION

A transient, laminar, unsteady natural convection flow of a viscous incom-pressible fluid past an impulsively started infinite vertical plate with uniform heat and mass flux is considered. Here, the x-axis is taken along the plate in the vertically upward direction and the y-axis is taken normal to the plate. Initially, it is assumed that the plate and the fluid are of the same temperature and concentration. At time $t' > 0$, the plate starts moving impulsively in the vertical direction with constant velocity u_0 against gravitational filed. At the same time, the heat is supplied from the plate to the fluid at a uniform rate and the concentration level near the plate is also raised at an uniform rate. Then, under the usual Boussinesq's approximation, the unsteady flow is governed by the following equations:

$$\frac{\partial u}{\partial t'} = g\beta(T' - T'_\infty) + g\beta^*(C' - C'_\infty) + \nu\frac{\partial^2 u'}{\partial y^2} \tag{1}$$

$$\rho C_p \frac{\partial T'}{\partial t'} = \frac{\partial^2 T'}{\partial y^2} - \frac{\partial q_r}{\partial y} \tag{2}$$

$$\frac{\partial C'}{\partial t'} = D\frac{\partial^2 C'}{\partial y^2} \tag{3}$$

The initial and boundary conditions are

$$\begin{aligned}
&t' \le 0: \; u = 0, \quad T' = T'_\infty, \quad C' = C'_\infty \\
&t' > 0: \; u = u_0, \; \frac{\partial T'}{\partial y} = -\frac{q}{k}, \; \frac{\partial C'}{\partial y} = -\frac{j''}{D} \quad at\; y = 0 \\
&\qquad u = 0, \quad T' = T'_\infty, \quad C' = C'_\infty \quad at\; x = 0 \\
&\qquad u \to 0, \; T' \to T'_\infty, \quad C' \to C'_\infty \quad as\; y \to \infty
\end{aligned} \tag{4}$$

For the case of an optically thin gray gas the local radiant absorption is expressed by

$$\frac{\partial q_r}{\partial y} = -4a^*\sigma(T_\infty'^4 - T'^4) \tag{5}$$

We assume that the temperature differences within the flow are sufficiently small such that T^4 may be expressed as a linear function of the temperature. This is accomplished by expanding T^4 in a Taylor series about T_∞ and neglecting higher-order terms, thus

$$T'^4 \cong 4T_\infty'^3 T' - 3T_\infty'^4 \tag{6}$$

By using equations (5) and (6), equation (2) reduces to

$$\rho C_p \frac{\partial T'}{\partial t'} = k\frac{\partial^2 T'}{\partial y^2} - 16a\sigma T_\infty'^3 (T' - T_\infty') \tag{7}$$

On introducing the following non-dimensional quantities

$$X = \frac{xu_0}{\nu},\ Y = \frac{yu_0}{\nu},\ U = \frac{u}{u_0},\ t = \frac{t'u_0^2}{\nu},$$

$$T = \frac{T' - T_\infty'}{\left(q\nu/ku_0\right)},\ Gr = \frac{g\beta q\nu^2}{ku_0^4},\ C = \frac{C' - C_\infty'}{j''\nu/(Du_0)},\ Gc = \frac{g\beta^*\nu^2 j''}{Du_0^4}, \tag{8}$$

$$\Pr = \frac{\nu}{\alpha},\ Sc = \frac{\nu}{D},\ R = \frac{16a^*\nu^2\sigma T_\infty^3}{ku_0^2}$$

Equations (1) to (3) are reduced to the following non-dimensional form

$$\frac{\partial U}{\partial t} = Gr\,T + Gc\,C + \frac{\partial^2 U}{\partial Y^2} - Mu \tag{9}$$

$$\frac{\partial T}{\partial t} = \frac{1}{\Pr}\frac{\partial^2 T}{\partial Y^2} - \frac{R}{\Pr}T \tag{10}$$

$$\frac{\partial C}{\partial t} = \frac{1}{Sc}\frac{\partial^2 C}{\partial Y^2} \tag{11}$$

The corresponding initial and boundary conditions in non-dimensional quantities are

$t \le 0$: U = 0, T = 0, C=0

$$t > 0:\ U = 1,\ \frac{\partial T}{\partial Y} = -1,\ \frac{\partial C}{\partial Y} = -1 \quad \text{at } Y = 0 \tag{12}$$

U = 0, T = 0, C = 0 at X = 0

$U \to 0$, $T \to 0$, $C \to 0$ as $Y \to \infty$

3. NUMERICAL TECHNIQUE

In order to solve the unsteady, non-linear coupled equations (9) to (11) under the conditions (12), an implicit finite difference scheme of Crank-Nicolson type has been employed. The finite difference equations corresponding to equations (9) to (11) are as follows.

$$\frac{\left[U_{i,j}^{n+1}-U_{i,j}^{n}\right]}{\Delta t}=\frac{Gr}{2}\left[T_{i,j}^{n+1}+T_{i,j}^{n}\right]+\frac{Gc}{2}\left[C_{i,j}^{n+1}+C_{i,j}^{n}\right]+\frac{\left[U_{i,j-1}^{n+1}-2U_{i,j}^{n+1}+U_{i,j+1}^{n+1}+U_{i,j-1}^{n}-2U_{i,j}^{n}+U_{i,j+1}^{n}\right]}{2(\Delta Y)^{2}} \tag{13}$$

$$\frac{\left[T_{i,j}^{n+1}-T_{i,j}^{n}\right]}{\Delta t}=\frac{1}{\Pr}\frac{\left[T_{i,j-1}^{n+1}-2T_{i,j}^{n+1}+T_{i,j+1}^{n+1}+T_{i,j-1}^{n}-2T_{i,j}^{n}+T_{i,j+1}^{n}\right]}{2(\Delta Y)^{2}}-\frac{R\left(T_{i,j}^{n+1}+T_{i,j}^{n}\right)}{2\Pr} \tag{14}$$

The thermal boundary condition at Y = 0 in the finite difference form is

$$\frac{1}{2}\frac{\left[T_{i,1}^{n+1}+T_{i,1}^{n}-T_{i,-1}^{n+1}-T_{i,-1}^{n}\right]}{2\Delta Y}=-1 \tag{15}$$

At Y = 0 (i.e., j = 0), equation (14) becomes

$$\frac{\left[T_{i,0}^{n+1}-T_{i,0}^{n}\right]}{\Delta t}=\frac{1}{\Pr}\frac{\left[T_{i,-1}^{n+1}-2T_{i,0}^{n+1}+T_{i,1}^{n+1}+T_{i,-1}^{n}-2T_{i,0}^{n}+T_{i,1}^{n}\right]}{2\left(\Delta Y\right)^{2}}-\frac{R\left(T_{i,0}^{n+1}+T_{i,0}^{n}\right)}{2\Pr} \tag{16}$$

After eliminating $T_{i,-1}^{n+1}+T_{i,-1}^{n}$ using equation (15), equation (16) reduces to the form

$$\frac{\left[T_{i,0}^{n+1}-T_{i,0}^{n}\right]}{\Delta t}=\frac{1}{\Pr}\frac{\left[T_{i,1}^{n+1}-T_{i,0}^{n+1}+T_{i,1}^{n}-T_{i,0}^{n}+2\Delta Y\right]}{\left(\Delta Y\right)^{2}}-\frac{R\left(T_{i,0}^{n+1}+T_{i,0}^{n}\right)}{2\Pr} \tag{17}$$

$$\frac{\left[C_{i,j}^{n+1}-C_{i,j}^{n}\right]}{\Delta t}=\frac{1}{Sc}\frac{\left[C_{i,j-1}^{n+1}-2C_{i,j}^{n+1}+C_{i,j+1}^{n+1}+C_{i,j-1}^{n}-2C_{i,j}^{n}+C_{i,j+1}^{n}\right]}{2\,(\Delta Y)^{2}} \tag{18}$$

The boundary condition at *Y = 0* for the concentration in the finite difference form is

$$\frac{1}{2}\frac{\left[C_{i,1}^{n+1}+C_{i,1}^{n}-C_{i,-1}^{n+1}-C_{i,-1}^{n}\right]}{2\Delta Y}=-1 \tag{19}$$

At $Y = 0$ (*i.e.*, $j = 0$), Equation (18) becomes

$$\frac{\left[C_{i,0}^{n+1}-C_{i,0}^{n}\right]}{\Delta t}=\frac{1}{Sc}\frac{\left[C_{i,-1}^{n+1}-2C_{i,0}^{n+1}+C_{i,1}^{n+1}+C_{i,-1}^{n}-2C_{i,0}^{n}+C_{i,1}^{n}\right]}{2\left(\Delta Y\right)^{2}} \tag{20}$$

After eliminating $C_{i,-1}^{n+1}+C_{i,-1}^{n}$ using equation (19), equation (20) reduces to the form

$$\frac{\left[C_{i,0}^{n+1}-C_{i,0}^{n}\right]}{\Delta t}=\frac{1}{Sc}\frac{\left[C_{i,1}^{n+1}-C_{i,0}^{n+1}+C_{i,1}^{n}-C_{i,0}^{n}+2\Delta Y\right]}{\left(\Delta Y\right)^{2}} \tag{21}$$

Here the region of integration is considered as a rectangle with sides X_{max} (= 1) and Y_{max} (=14), where Y_{max} corresponds to $Y = \infty$ which lies very well outside both the momentum and energy boundary layers. The maximum of Y was chosen as 14 after some preliminary investigations so that the last two of the boundary conditions (14) are satisfied with in the tolerance limit 10^{-5}.

After experimenting with a few set of mesh sizes have been fixed at the level $\Delta X = 0.05$, $\Delta Y = 0.25$, with time step $\Delta t = 0.01$. In this case, the spatial mesh sizes are reduced by 50% in one direction, and later in both directions, and the results are compared. It is observed that, when the mesh size is reduced by 50% in the *Y*-direction, the results differ in the fifth decimal place while the mesh sizes are reduced by 50% in *X*-direction or in both directions, the results are comparable to three decimal places. Hence, the above mesh sizes have been considered as appropriate for calculation. The coefficient $U_{i,j}^{n}$ appearing in the finite difference equation are treated as constants at any one time step. Here *i*-designates the grid point along the *X*-direction, *j* along the *Y*-direction and *k* to the *t*-time. The values of *U*, and *T* are known at all grid points at $t = 0$ from the initial conditions.

The computations of *U, T* and *C* at time level (n +1) using the values at previous time level (n) are carried out as follows: The finite-difference equations (17) at every internal nodal point on a particular *i*-level constitute a tridiagonal system of equations. Such a system of equations are solved by using Thomas algorithm as discusses in Carnahan et al [1]. Thus, the values of T are found at every nodal point for a particular *i* at $(n+1)^{th}$ time level. Similarly, the values of C are calculated from equation (21).Using the values of *C* and *T* at $(n+1)^{th}$ time level in the equation (13), the values of *U* at $(n+1)^{th}$ time level are found in a similar manner. Thus, the values of

C, T and *U* are known on a particular *i*-level. This process is repeated for various *i*-levels. Thus the values of *C, T* and *U* are known, at all grid points in the rectangle region at $(n+1)^{th}$ time level.

4. RESULTS AND DISCUSSION

The effect of velocity for different radiation parameter (R = 0, 2, 5), Gr = 2, Gc = 5, Pr = 0.71 and Sc = 0.6 are shown in figure 1. It is observed that the velocity increases with decreasing radiation parameter. This shows that velocity decreases in the presence of high thermal radiation.

In figure 2, the velocity profiles for different thermal Grashof number and mass Grashof number are shown graphically. This shows that the velocity increases with increasing thermal Grashof number or mass Grashof number. As thermal Grashof number or mass Grashof number increases, the buoyancy effect becomes more significant, as expected, it implies that, more fluid is entrained from the free stream due to the strong buoyancy effects as Gr or Gc incrases.

The temperature profiles for different values of the thermal radiation parameter are shown in figure 3. It is observed that the temperature increases with decreasing R. This shows that the buoyancy effect on the temperature distribution is very significant in air (Pr = 0.71). It is known that the radiation parameter and Prandtl number plays an important role in flow phenomena because, it is a measure of the relative magnitude of viscous boundary layer thickness to the thermal boundary layer thickness.

The effect of the Schmidt number is very important for concentration profiles. The concentration profiles for different values of Schmidt number are shown in figure 4. There is a fall in concentration due to increasing the values of the Schmidt number.

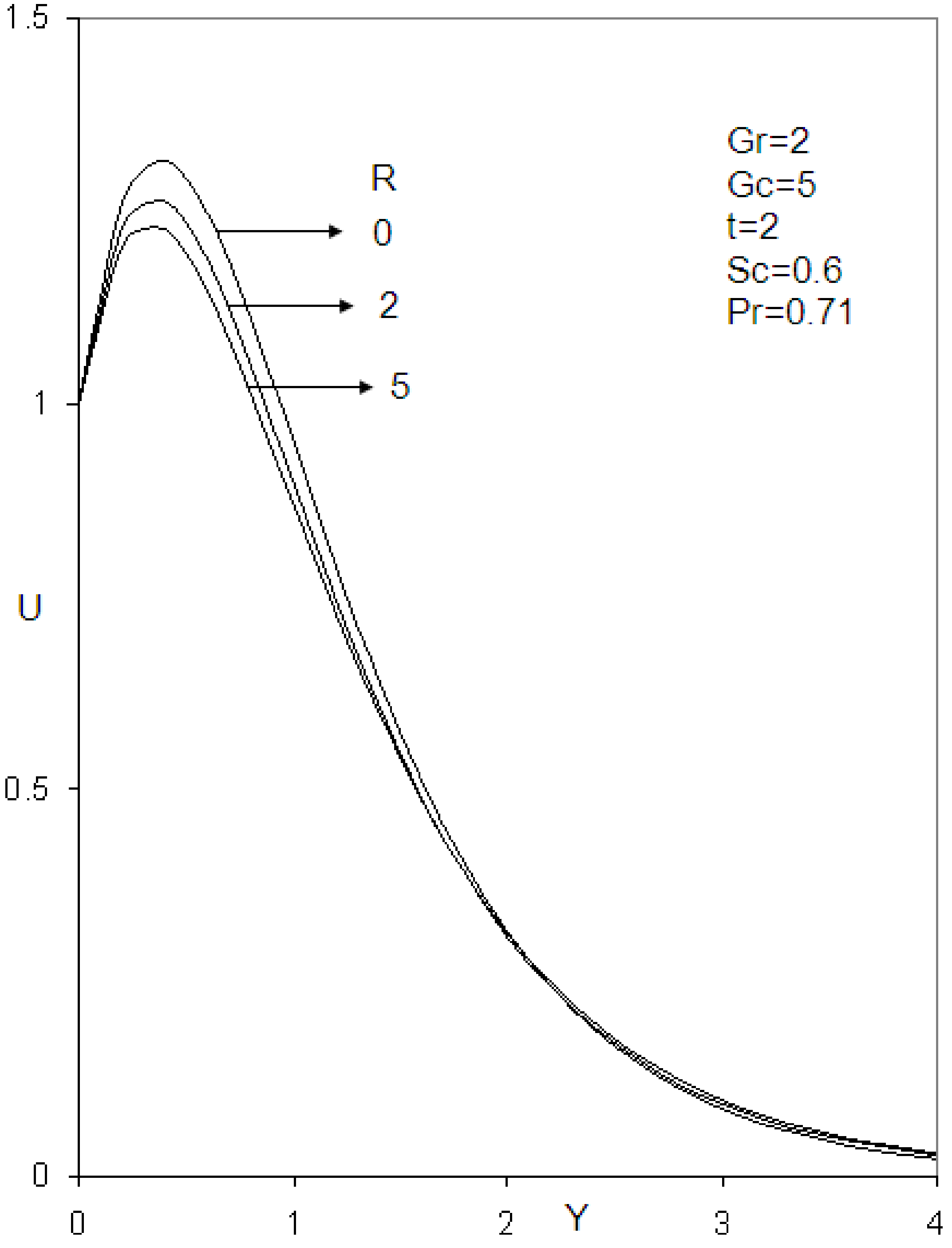

Fig.1. Velocity profiles for different R

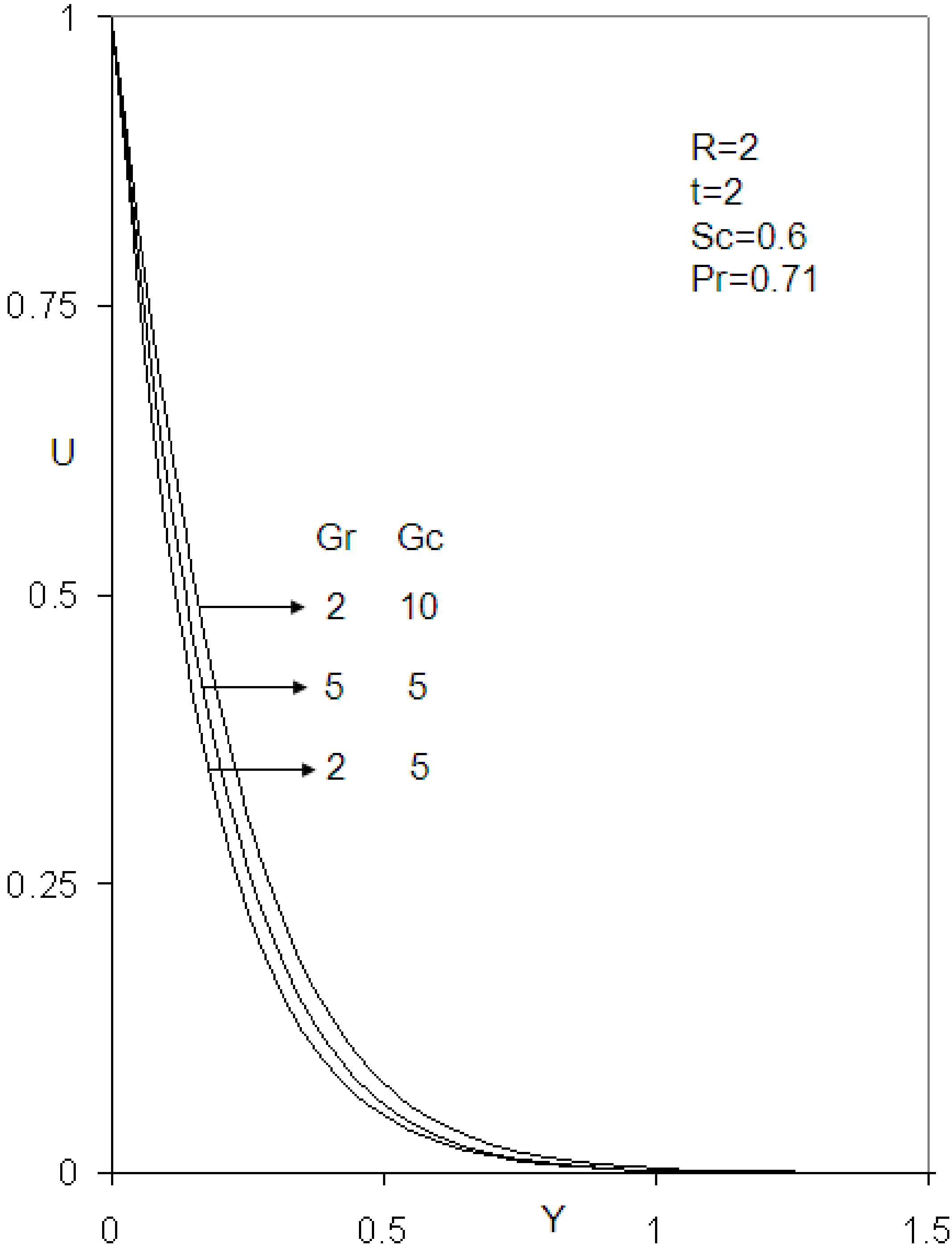

Fig.2. Velocity profiles for different Gr and Gc

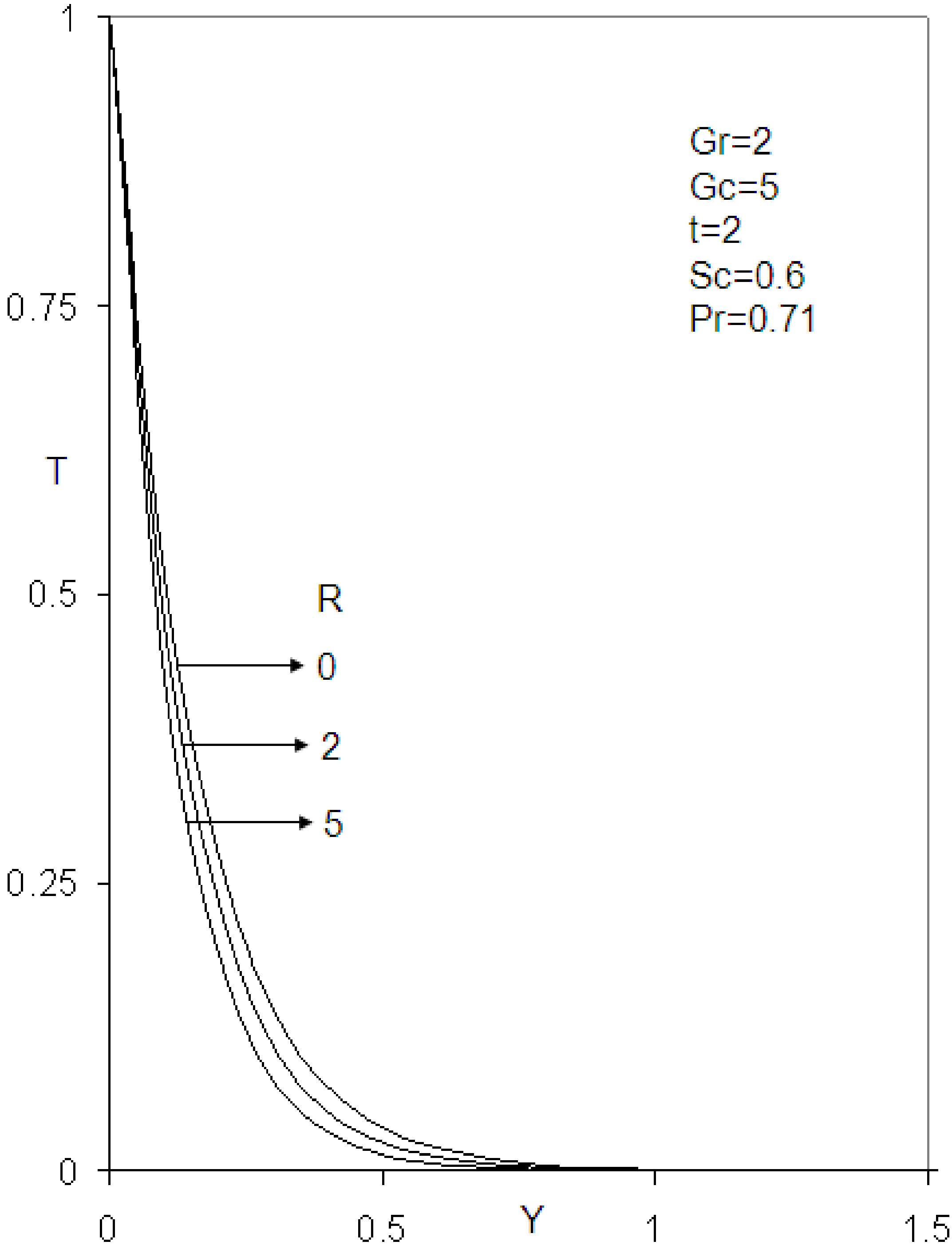

Fig.3. Temperature profiles for different R

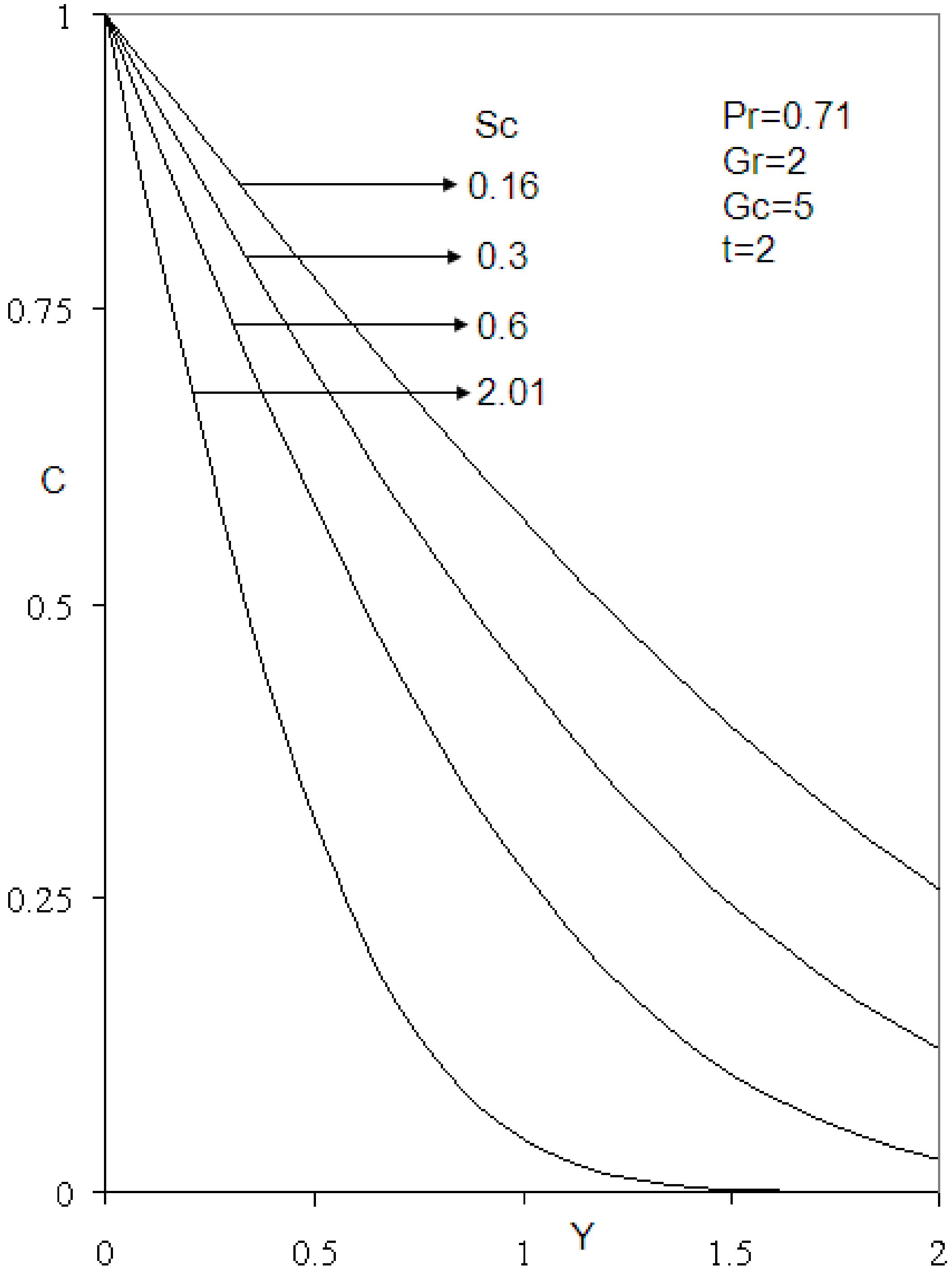

Fig.4. Concentration profiles for different Sc

5. CONCLUSIONS

Finite difference study has been carried out for unsteady flow past an impulsively started infinite vertical plate with uniform heat and mass flux in the presence of thermal radiation. The dimensionless governing equations are solved by an implicit scheme of Crank-Nicolson type. The effect of velocity, temperature and concentration for different parameters are studied. It is observed that the velocity decreases in the presence of thermal radiation.

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