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Modeling of Nonlinear Control Systems for Thermoelectric Cooling and Regenerative Systems Based on Peltier Modules

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Abstract

: Increasing the efficiency of thermoelectric modules (TEM) based on Peltier elements is associated with the search for new thermoelectric materials with improved properties and the use of automatic control systems (ACS) for these modules. The analysis and synthesis of thermoelectric systems (TES) is significantly complicated due to the significant nonlinearity of the Peltier element in a wide range of temperatures and control currents. To analyze the dynamic modes of TES, it is proposed to use approximation of the system characteristics by piecewise linear functions. The method allows us to obtain generalized expressions of transients for any order of the system under study and any character of nonlinearity. An example of applying the method for modeling the transient characteristic of a TEM ACS with a single nonlinear link is considered. Based on the analysis of errors in the calculation of the transient process, the effectiveness of the proposed approach is shown.

Keywords: Peltier effect, thermoelectric module, control system, dynamic characteristic

I. INTRODUCTION

Thermoelectric modules (TEM) based on Peltier elements are actively used in cooling systems due to a number of advantages: the absence of moving parts, high accuracy of temperature control, the ability to heat and cool an object by simply switching the polarity of the voltage applied to the module [1-6]. Increasing the efficiency of TEM is associated with the search for new thermoelectric materials with improved properties [7-10] and the use of automatic control systems (ACS) [11-13].

The TEM design based on the Peltier effect is shown in Fig. 1. The device consists of two ceramic insulator plates with connected thermocouples located between them, and each side of the module, depending on the polarity, contacts either p-n or n-p junctions.

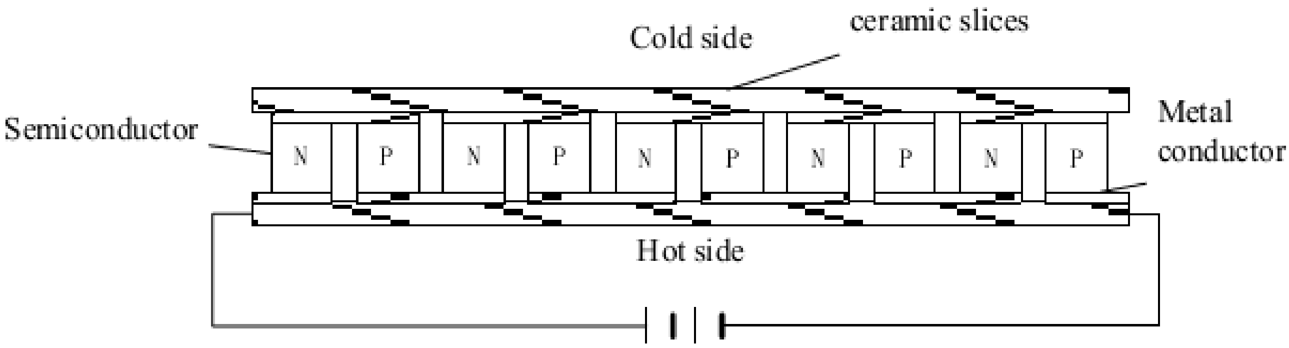

Fig. 1. The design of thermoelectric module based on the Peltier effect

The use of ACS is necessary, first of all, for the temperature control of objects in an unstable environment with a rapidly changing heat flow. The analysis and synthesis of thermoelectric systems (TES) is significantly complicated due to the significant nonlinearity of the Peltier element in a wide range of temperatures and control currents [14-19]. To analyze the dynamic modes of TES, it is proposed to use the authors' method based on the approximation of the system' characteristics by piecewise linear functions [20-23]. This will allow us to obtain generalized expressions of transients for any order of the system under study and any character of nonlinearity [27].

I.I Functional diagram and model of the control system of current flowing through the Peltier element

The functional diagram of the control system for the current flowing through the Peltier element is shown in Fig. 2 [24]. Figure contains following notation: I_d is the desired current flowing through the Peltier element; u–output signal of regulator; v–output signal of the pulse width modulator (PWM); $S_1 \ldots S_4$ are signals controlling the switching elements (SE); U–input voltage of the smoothing filter; I–current through the Peltier element.

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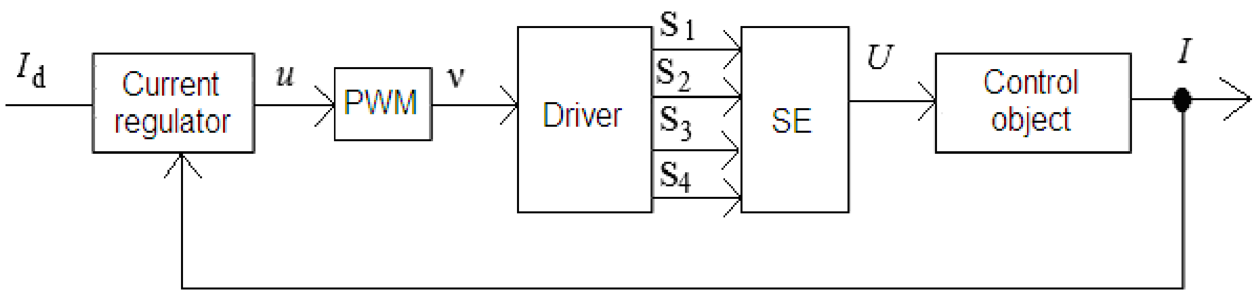

Fig. 2. Functional diagram of the control system for current flowing through the Peltier element

In order to analyze the scheme (Fig. 2), we present this scheme as a model of amplitude-phase signal transformer (APT) [22,23]. The model of the transformer with deviation control (Fig. 3) includes a similar APT*, a control unit (CU), a control path (CP), and a weight distributor (WD). The control unit controls the amplitude and (or) phase of the input signal. The CP consists of a detector for the deviation of the amplitude and (or) phase of the signal (D), as well as a filter (F). The values of the WD's coefficients determine the transmission of signals from its inputs to its outputs. The diagram shows: U_1, u_1 и U_2, u_2 -input (main, additional) and output signals, u-control signal [28].

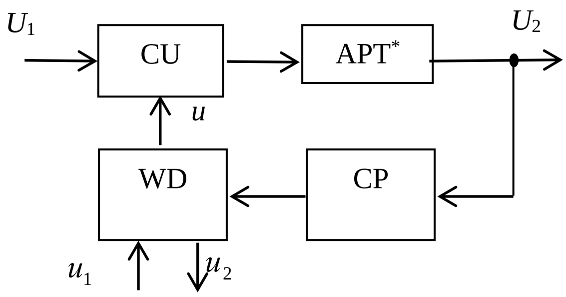

Fig. 3. Model of the amplitude-phase signal transformer with deviation control for analysis of TES

The equivalent disclosure of the APT* block similar to the presented model allows us to use this model for analysis of various thermoelectric control systems containing several control channels, local and general feedback loops, as well as power supply systems based on the principle of trigeneration which provides simultaneous generation of electricity (to power electronic devices), as well as heat and cold (to create the necessary microclimate of premises) [25].

The use of piecewise linear functions to approximate the system characteristics [22-25] allows us to obtain generalized expressions of dynamic modes of thermal power plants for any order of the system under study and any character of nonlinearity.

I.II Expressions for the analysis of dynamic modes of nonlinear TEM control systems

The heat conduction equation (Fourier differential equation) for a three-dimensional non-stationary temperature field in the Peltier element has the form [26]

$$\frac{\partial T}{\partial t} = a\nabla^2 T + \frac{q_V}{c\rho},$$

where $a = \dfrac{\lambda}{c\rho}$ is the thermal diffusivity, λ is the coefficient of conductivity, c is specific heat capacity, ρ is density, $\nabla^2 T = \dfrac{\partial^2 T}{\partial x^2} + \dfrac{\partial^2 T}{\partial y^2} + \dfrac{\partial^2 T}{\partial z^2}$ is a Laplace operator, q_V is the amount of heat generated per unit volume of the medium per unit of time.

The sensors' inertia of the control system can be described by the equation

$$T_{Snj} + \tau_{nij}\frac{dT_{Snj}}{dt} = k_{nij} T_{Pni},$$

where T_{Snj}–temperature value measured by sensor, T_{Pnj}–the true value of temperature generated by a point source, τ_{nij} - time constant (measure of inertia) of the climate sensor, k_{nij}–regulation ratio. Based on the last equation, the transfer function of the sensor has the form

$$H_{nij}(p) = \frac{T_{Snj}}{T_{Pni}} = \frac{k_{nij}}{1+\tau_{nij}p}$$

Which corresponds to an aperiodic dynamic link.

To obtain analytical expressions of the dynamic characteristics of a non-linear TEM ACS, we approximate the nonlinear characteristic of the D f_D based on piecewise-linear functions [20-23]. After approximation, the characteristic has the form of the sum of M linear sections, where M is the maximum number of the approximation node. The equation of the approximating segment for the node m: m: $f_{Dm}=K_m y+B_m$, where y is the parameter (amplitude or phase) of the output signal of the transformer, K_m, B_m are the approximation coefficients. Let's obtain the transfer coefficient of APT* equal to 1, and u_1=u_2=0. We assume that the switching elements are ideal (switching without time delays). Equation of the transformer on the section m [Y_m; Y_{m+1}]

$$y_m = x_{\text{m}} - K_y n_p M(p)\left(K_m y_m + B_m\right),$$

where x is the parameter (amplitude or phase) of the input signal, $\tilde{M}(p)$ transmission coefficient of the F, p=d/dt is operator, t is time, K_y is transmission coefficient of the CU, n_p is transmission coefficient of the WD.

We denote $N_{\text{m}} = K_y n_p K_m$ - the regulation coefficient, and $B_m^* = K_y n_p B_m$. The Laplace representation of the output parameter on the section m will take the form

$$y_m = x_m - M(p)\left(N_m y_m + B_m^*\right). \tag{1}$$

This is a linear differential equation of the APT in the interval [m, m+1] presented in operator form. The general nonlinear equation of the transformer can be represented by the sum of equations (1) for all approximation segments.

To get its analytical solution, we present the influence in the operator form: $X_m(p) \leftarrow x_m(t)$. Piecewise linear approximation of the influence $x_{\text{k}}(t)$ allows us to obtain an operator image of an arbitrary influence based on the general analytical expression:

$$X_m(\text{p}) = \frac{1}{\Delta_t \text{p}^2}\left\{ p\text{x}_m(t_0)e^{-pt_0} - \sum_{\text{j}=0}^{J-1} \Delta_{m,j}^{(x)}\left[e^{-p(t_j+\Delta_t)} - e^{-pt_j}\right]\right\}, \tag{2}$$

where t_j are the approximation nodes of the time function, J is the number of nodes, Δ_t is time step, and $\Delta_{m,j}^{(x)} = x_m(t_{j+1}) - x_m(t_j)$ is approximation coefficient. The first addend of the image takes into account the constant component of the impact.

We represent the filter's transfer function as

$$M(p) = A(p)/B(p) = \sum_{i=0}^{I} \alpha_i p^i \Big/ \sum_{i=0}^{I} \beta_i p^i, \tag{3}$$

where α_i, β_i are the filter's coefficients.

Substitute (3) into equation (1), which after transformation will take the form

$$y_m \sum_{i=0}^{I} \beta_i p^i = x_{\text{m}} \sum_{i=0}^{I} \beta_i p^i - \sum_{i=0}^{I} \alpha_i p^i \left(\text{N}_{\text{m}} y_m + B_m^*\right).$$

In the last expression we replace the Laplace operator p by differential d/dt

$$\sum_{i=0}^{I} \beta_i \frac{d^i y_m}{dt^i} = \sum_{i=0}^{I} \beta_i \frac{d^i x_{\text{m}}}{dt^i} - \sum_{i=0}^{I} \alpha_i \frac{d^i\left(\text{N}_{\text{m}} y_m + B_m^*\right)}{dt^i}. \tag{4}$$

We derive an operator equation for (4) taking into account the initial conditions:

$$y_m(t) \leftarrow Y_m(p),\ \ y'_m(t) \leftarrow pY_m(p) - y_m(0),$$

$$y''_m(t) \leftarrow p^2 Y_m(p) - py_m(0) - y'_m(0),$$

$$y^{i'}{}_m(t) \leftarrow p^i Y_m(p) - p^{i-1} y_m(0) - p^{i-2} y'_m(0) - \ldots$$

$$- y_m{}^{i-1}(0) = p^i Y_m(p) - \sum_{k=1}^{i} y_m{}^{(k-1)}(0) p^{i-k}.$$

We assume that all derivatives except the first one are zero: $y^{(i)}{}_m(0) = 0, i$ >1. The assumption will not cause a large error in the calculation of transients. Similarly, we present the image of the influence and its derivatives. Then

$$\text{x}^{(i)}{}_m(t) \leftarrow p^i X_m(p) - p^{i-1} x_m(0) - p^{i-2} y'_m(0). \tag{5}$$

Taking into account (5), expression (4) will take the form

$$\sum_{i=0}^{I} \beta_i \left[p^i Y_m(p) - p^{i-1} y_m(0) - p^{i-2} y'_m(0)\right] =$$

$$= \sum_{i=0}^{I} \beta_i \left[p^i X_m(p) - p^{i-1} x_m(0) - p^{i-2} x'_m(0)\right] -$$

$$- N_m \sum_{i=0}^{I} \alpha_{\text{i}} \left[p^i Y_m(p) - p^{i-1} y_m(0) - p^{i-2} y'_m(0)\right] -$$

$$- \frac{B_{\text{m}}^*}{p} \sum_{i=0}^{I} \alpha_{\text{i}} p^i.$$

We express $Y_m(p)$ from the resulting equation:

$$Y_m(p) = \frac{pB(p)X(p) - B_{\text{m}}^* A(p) + pC_m(p)}{p\left[B(p) + N_m A(p)\right]}, \tag{6}$$

where $C_m(p) = C^{(\text{y})}(p) - C^{(x)}(p) - C^{(\text{G})}(p)$ - polynomial of initial conditions.

$$C^{(\text{y})}(p) = \left[\tilde{B}_1(p) + N_m A_1(p)\right] y_m(0) + \left[\tilde{B}_2(p) + N_m A_2(p)\right] y_m{}'(0)$$ - polynomial of initial response conditions,

$C^{(x)}(p) = \tilde{B}_1(p)x(0) + \tilde{B}_2(p)x'(0)$ - polynomial of initial influence conditions,

$$C^{(\text{G})}(p) = \begin{cases} B_m^* A_1(p), m \neq m_{ini} \\ 0, m = m_н, \end{cases} \quad \tilde{B}_k(p) = \sum_{i=k}^{I} \beta_i p^{i-k},$$

$x_m(0) = x(\tilde{\text{t}}_{\text{m}})$.

The initial conditions for different m take the following values:

1) $m = m_{ini}$, then $x_{m\_ini}(0) = y_{m\_ini}(0) = 0$ and $x'_{m\_ini}(0) = y'_{m\_ini}(0) = 0$.

For APT with filters of the second and higher orders ($I \geq 2$) the value $y'_{m_н}(0)=0$ corresponds to zero initial values of the influence on the reactive elements of the filter [6].

2) For each subsequent section ($m \neq m_{ini}$), the values of the APT response on the border of the sections coincide:

$$y_m(0)=Y_{m+\bar{q}_m},\ y'_m(0)=y'_{m-1+2\bar{q}_m}(t_{m-1+2\bar{q}_m}),$$

where t_m is the end time of the partial transient solution, $\bar{q}_m = q[y_m(t-\Delta)-y_m(t)]$ is the direction of the transient ($\bar{q}_m$ =1 for decreasing and $\bar{q}_m$ =0 for increasing),, $\Delta \to 0$, $q(y)$ is the piecewise linear function, equal to 1 for $y \geq 0$ and 0 for y<0. To calculate t_m, it is necessary to determine the abscissa of the intersection points of $y_m(t)$ with the straight line Y_m [3].

The initial value of the impact and its derivative on the m-th section are determined by the time shift $\tilde{t}_m$: $x_m(0)=x(\tilde{t}_m)$, $x'_m(0)=x'(\tilde{t}_m)$. For each subsequent section, the shift will consist of the sum of the duration of the transient on each of the previous sections. We denote k=0…K-1 is the number of partial transient solutions, K is the total number of partial solutions (k=0 corresponds to $m_k=m_0=m_{ini}$, $k=K$-1 corresponds to $m_{K-1}=m_{\text{end}}$. Thus, the time shift of a particular transient solution is defined as

$$\tilde{t}_{mk}=\sum_{\tilde{k}=0}^{k-1} t_{\tilde{k}},\ \tilde{t}_{m_н}=0. \quad (7)$$

This approach allows us to obtain generalized expressions of TES's transients for any order of the system under study and arbitrary nonlinearity.

I.III TEM ACS modeling based on the APT model and piecewise linear approximation

Based on the developed method, the simulation of the transient characteristics of the TEM ACS is performed. We accept K_y=1, n_p=2, K^*=1. The characteristic of the filter F, which describes the inertia of the feedback loop of the system, is given by the expression $M(p)=(1+100p)^{-1}$. Nonlinear characteristic of the detector D is approximated by two and three straight lines (Fig. 3A and 3B). The calculated transient characteristic of the TEM ACS when approximating D with two and three linear components is shown in Fig. 4. A solid line marks the component formed by the initial section of characteristic D and described by the expression $y_1(t)=e^{-0,02t}$. The dotted line marks the subsequent section of the transition characteristic described by the expression $y_0(t-t_1)=0{,}125+0{,}375e^{-0,04t}$, where t_1 = 34 s is the time shift of the second partial solution. The result of calculating the transition process for three approximation sections of D is shown by a dashed line (time shifts of the second and third partial solutions (t_2 = 8.7 s; t_1 = 43 s). The figure shows that the error caused by representation of D by three sections is significantly less, especially at the setting stage.

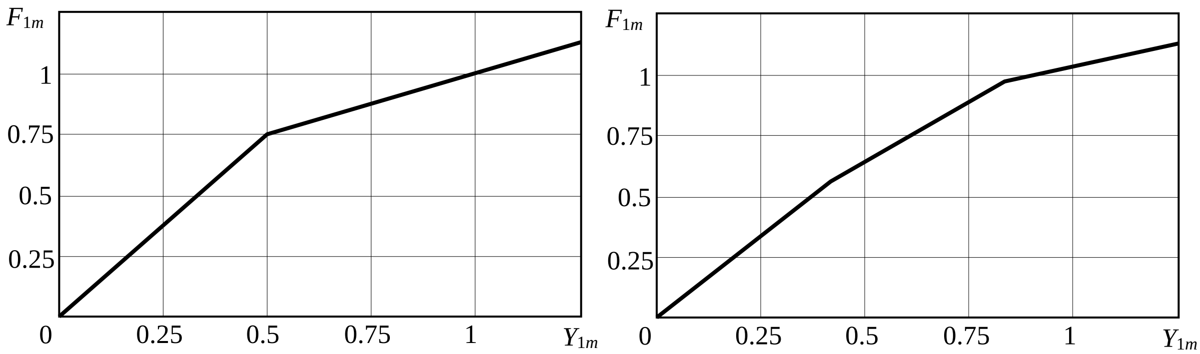

Fig. 3. Approximation of the characteristics of the nonlinear link of the TEM ACS:

a) two linear sections: b) three linear sections

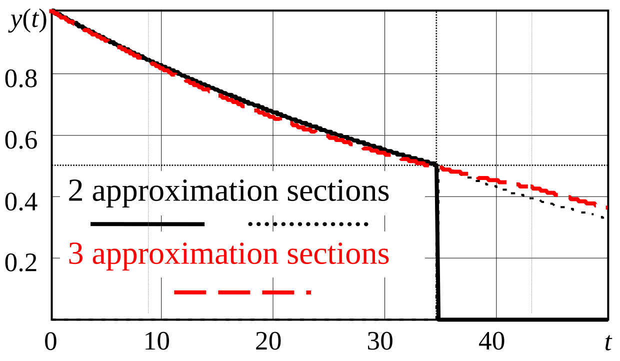

Fig. 4. Transients of the TEM ACS when approximating a nonlinear link with two or three linear sections

II. CONCLUSION

A method for modeling dynamic processes in nonlinear automatic control systems of thermoelectric modules based on the model of amplitude-phase signal transformation and piecewise linear approximation is developed. The approach makes it possible to obtain analytical solutions of dynamic processes in control systems that are valid for individual linear sections of the characteristics of nonlinear links in the system. Based on the developed method, a simulation of a TEM ACS with a nonlinear link represented by two and three linear segments was performed. It is found that the transient modeling error when representing the characteristics of a link by three approximation sections is significantly less, especially at the establishment stage. Thus, the effectiveness of the proposed approach is shown. The approach makes it possible to analyze various nonlinear thermoelectric control systems containing several control channels, local and general feedback loops, as well as power supply systems based on the principle of trigeneration which implies simultaneous generation of electricity (to power electronic devices), as well as heat and cold (to create the necessary microclimate of premises).

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