

An Approximate Determination of Local Stresses in the Walls of Steel Crane Girders of an I-Section

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Abstract

In the problem solving of the stress state of the wall of a crane girder, the stress function is used for a half-plane loaded with concentrated force. Compliance with the boundary conditions along the interface between the shelves and the wall is achieved by introducing compensating stresses. The developed method for determining local stresses in the walls of I-girders under the action of concentrated loads has several advantages compared to the known solutions: in the proposed method for determining local stresses in the walls of I-girders from the action of concentrated loads, the calculation formulas are obtained in explicit form; the solution is generalized in nature, since it can be applied to calculations of a half-plane, strip, T-girder, and I-girder, including an I-girder of an asymmetric section; good agreement between theoretical calculations and experimental data is shown.

Keywords: crane girder, girder wall, concentrated pressure, compensating load, stresses.

INTRODUCTION

Crane girders operate under more severe conditions than other frame elements, since the vertical pressure of the bridge crane rollers is applied to them in the form of concentrated pressure (see Fig. 1a, b).

In crane girders, the greatest amount of damage in the form of cracks is observed during operation in the upper zone of the wall, where local stresses from the action of the bridge crane rollers have maximum values (see Fig. 1c). The problem of preventing fatigue cracks in these areas affects the quality of manufacturing girders, standardizing the sizes of permissible defects, observing operating rules, the quality of steel and welding used, and other aspects. However, one of the most important places is occupied by the task of correctly determining local stresses in the upper zone of the wall. This problem will be relevant until the problem of increasing the longevity of the crane girders to terms comparable with the longevity of the remaining frame elements is completely resolved.

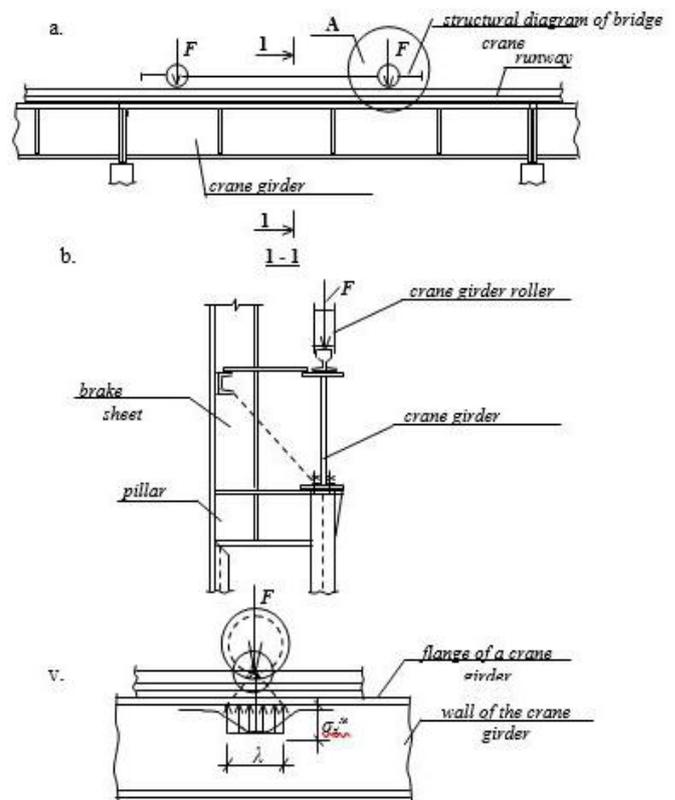


Fig. 1. Crane girder and vertical loads from crane rollers:
a - arrangement diagram of the rollers of an overhead crane on a crane run; b - crane structures in cross section; c - concentrated pressure transmission scheme

A review of studies of the stress state of the walls of steel I-girders under the action of concentrated forces showed that stresses are usually conditionally divided into two parts, namely, stresses calculated according to elementary beam theory and stresses characterizing the local effect in the zone of the point of application of concentrated force, and the latter are self-balanced inside the section. In real constructions, local stresses are commensurate and even exceed general bending stresses.

Known methods for determining local stresses from the action of concentrated pressure in the wall of a steel girder have several disadvantages that impede their use:

-many simple methods involve the determination of only individual components of the stress state at one point (in the wall under the force [10]), or along the line of conjugation of the shelf with the wall (methods of G. Stokes [20], S.P. Timoshenko [11], M. Bio [12], U. Hiroyuke[14], I. Rive [19], E.V. Parkes [18], B. M. Brode [3], A.A. Apalko [1,2], V.M. Gorpichenko [4,5] and others), therefore, they do not give a complete picture of the stress state in the wall;

- some methods are based on the use of the well-known Businessq-Flaman solution for the half-plane (methods of K. Girkman [13], I. Lukash [15.16], O.F. Ivankov, A.I. Putilov and I.E. Spengler [6] and others), however, these methods do not take into account one of the main differences between local stresses in the girder and half-plane - in the girder, the local tangential stresses change stepwise, since they complement the jump-like change in the shear stresses of the general bend, and in the half-plane the local tangential stresses change smoothly, passing at the origin through zero. Since the stress components are interconnected by differential dependences, the normal stresses in the half-plane and in the wall will also have differences;

- more informed solutions made by B.B. Lampsey [7], E. A. Ryvkin [9], X-G. Fogle [21] and others are essentially approximate, since they are associated with approximate calculations of Fourier integrals or the summation of trigonometric series, which can be calculated only with a certain accuracy. However, the more significant drawback of more informed decisions is the complexity and cumbersomeness of the calculations, the inconvenience of working with tables that make it difficult to analyze the stress state at any section or point; analysis of the stress state by numerical methods (mainly, by the finite element method) is performed only for particular cases of a constructive solution - see, for example, the work of P. Osterider and I. Oksfort [17]. Assessing the influence of individual design parameters that vary over a wide range is a laborious and therefore difficult task that has not yet been solved by numerical methods.

The proposed method for determining local stresses from concentrated pressure does not have the listed disadvantages.

METHOD.

The stress function for the wall can be represented as terms:

$$\varphi = \varphi^{\bullet} + \varphi^o + \varphi_1^k + \varphi_2^k + \varphi^o, \quad (1)$$

where $\varphi^{\bullet} = -\frac{F}{h_w t_w \pi} \cdot \bar{y} \cdot \arctg \frac{\bar{y}}{\bar{x} + \Delta \bar{x}}$ is the stress

function for the half-plane loaded with concentrated force (F is the concentrated force, x, y are the coordinates of the Cartesian system, h_w is the half-height of the wall section in

the girder, $\bar{x} = \frac{x}{h_w}$; $\bar{y} = \frac{y}{h_w}$ is the relative coordinates,

t_w is the wall thickness in the girder); φ^o - the same for efforts from general bending, from which stresses are determined under the hypothesis of flat sections by conventional methods of resistance of materials (not considered further); φ_1^k, φ_2^k - the same for compensating distributed loads; φ^o - the same for additional tangential stresses along the contours of the wall arising from the interaction and compatibility of deformations of the shelves and the wall.

The wall of the I-girder can be considered as a strip loaded along the contour with distributed loads, while the loads are adequate to the stresses acting along the interface lines of the upper and lower zones with the wall. In the proposed solution, the stresses along the contours of the strip (wall) are divided into four component parts, namely (see Fig. 2):

1. Stresses for the part of the half-plane below $\Delta \bar{x}$ the half-plane contour (see the dashed line in Scheme 1).
2. Stresses for a strip loaded on the lower wall contour with a distributed compensating load acting normally to the circuit (diagram 2). To determine local stresses, it is sufficient to consider the conditional loading diagram 2a.
3. Stresses for the strip loaded on the lower contour of the wall with tangential compensating loads (diagrams 3 and 3a).
4. Tangent stresses on the upper and lower circuits from common bending stresses (diagram 4). At the point of change in the direction of action of the tangential stresses in theoretical and experimental studies, a characteristic surge is noted, while the main stresses determined by the Zhuravsky formula (see the dashed line in diagram 4) and the additional ones presented in diagram 4a can be distinguished.

RESULTS AND DISCUSSION.

We define local stresses in the wall as the sum of local stresses according to diagrams 1-4. Let us consider alternately the diagrams presented in Fig. 2.

DIAGRAM 1. Stresses in the adopted coordinate system:

$$\sigma_{x1} = -\frac{2F}{\pi h_w t_w} \cdot \frac{(\bar{x} + \Delta \bar{x})^3}{[(\bar{x} + \Delta \bar{x})^2 + \bar{y}^2]^2}; \quad (2)$$

$$\sigma_{y1} = -\frac{2F}{\pi h_w t_w} \cdot \frac{(\bar{x} + \Delta \bar{x}) \bar{y}^2}{[(\bar{x} + \Delta \bar{x})^2 + \bar{y}^2]^2}; \quad (3)$$

$$\tau_{xy1} = -\frac{2F}{\pi h_w t_w} \cdot \frac{(\bar{x} + \Delta \bar{x})^2 \bar{y}}{[(\bar{x} + \Delta \bar{x})^2 + \bar{y}^2]^2}. \quad (4)$$

The distance $\Delta\bar{x}$ from the condition of equality of the maximum normal stresses σ_{x1} in the girder along the interface line between the flange and the wall and in the half-plane at $y = 0$. The maximum normal stresses in the girder will be assigned according to the well-known solution of B. M. Brode

$$[3], \text{ then } \Delta\bar{x} = \frac{2\lambda}{\pi h_w}$$

where $\lambda = 3.25 \cdot \sqrt[3]{\frac{J_{\Sigma ft}}{t_w}}$ is the conditional length of the distribution of concentrated pressure in the wall of the steel girder; - $J_{\Sigma ft}$ the sum of the moments of inertia of the shelf and rail.

DIAGRAM 2a. Consider the equilibrium of a portion of a strip of unit thickness under the action of compensating loads located within $\bar{y} \div \infty$,

$$M = \frac{Fh_w}{\pi} \left[\bar{y} \left(\frac{\pi}{2} - \arctg \frac{\bar{y}}{2 + \Delta\bar{x}} \right) - 2 - \Delta\bar{x} \right]; \quad (5)$$

$$Q = \frac{\partial M}{\partial \bar{y}} = \frac{F}{\pi} \left[\frac{\pi}{2} - \frac{(2 + \Delta\bar{x})\bar{y}}{(2 + \Delta\bar{x})^2 + \bar{y}^2} - \arctg \frac{\bar{y}}{2 + \Delta\bar{x}} \right]; \quad (6)$$

$$q_x = \frac{\partial Q}{\partial \bar{y}} = -\frac{2F}{\pi h_w} \cdot \frac{(2 + \Delta\bar{x})^3}{[(2 + \Delta\bar{x})^2 + \bar{y}^2]^2} \quad (7)$$

We set the stress function in the form

$$\varphi_2 = h_w [F_1(\bar{y})\bar{x}^3 + F_2(\bar{y})\bar{x}^2 + F_3(\bar{y})\bar{x} + F_4(\bar{y})], \quad (8)$$

then

$$\sigma_{x2} = h_w [F_1(\bar{y})''\bar{x}^3 + F_2(\bar{y})''\bar{x}^2 + F_3(\bar{y})''\bar{x} + F_4(\bar{y})'']; \quad (9)$$

$$\sigma_{y2} = \frac{1}{h_w} [6F_1(\bar{y})\bar{x} + 2F_2(\bar{y})]; \quad (10)$$

$$\tau_{xy2} = - \left[3F_1(\bar{y})' \bar{x}^2 + 2F_1(\bar{y})'x + F_3(\bar{y})' \right]; \quad (11)$$

Internal forces (M, N, Q) in the cross section of the wall:

$$M = \int_0^2 \sigma_{y2} \bar{x} \bar{\alpha} = h_w [16F_1(\bar{y}) + 4F_2(\bar{y})]; \quad (12)$$

$$N = \int_0^2 \sigma_{y2} \bar{\alpha} = 12F_1(\bar{y}) + 4F_2(\bar{y}); \quad (13)$$

$$Q = \int_0^2 \tau_{xy2} \bar{\alpha} = h_w [-8F_1(\bar{y})' - 4F_2(\bar{y})' - 2F_3(\bar{y})']; \quad (14)$$

Equating external (5) and internal (12) moments, and taking into account that $N = 0$, we find:

$$F_1(\bar{y}) = \frac{M}{4h_w} = \frac{1}{4} \cdot \frac{F}{\pi} \left[\bar{y} \left(\frac{\pi}{2} - \arctg \frac{\bar{y}}{2 + \Delta\bar{x}} \right) - 2 - \Delta\bar{x} \right]; \quad (15)$$

$$F_2(\bar{y}) = -\frac{3M}{4h_w} = -\frac{3}{4} \cdot \frac{F}{\pi} \left[\bar{y} \left(\frac{\pi}{2} - \arctg \frac{\bar{y}}{2 + \Delta\bar{x}} \right) - 2 - \Delta\bar{x} \right]; \quad (16)$$

Condition (14) is satisfied at $F_3(\bar{y}) = 0$.

When $\bar{x} = 0$ $\sigma_{x2} = 0$, then $F_4(\bar{y}) = 0$.

When $\bar{x} = 0$ u $\bar{x} = 2$ $\tau_{xy2} = 0$.

When $\bar{x} = 2$ $\sigma_{x2} = q_x$.

Finally, under the action of the load according to diagram 2 for a wall of thickness tw :

$$\sigma_{x2} = -\frac{2F}{\pi h_w t_w} \cdot \frac{(2 + \Delta\bar{x})^3}{[(2 + \Delta\bar{x})^2 + \bar{y}^2]^2} \cdot \frac{\bar{x}^2}{4} (\bar{x} - 3); \quad (17)$$

$$\sigma_{y2} = \frac{3F}{2\pi h_w t_w} \cdot \left[\bar{y} \left(\frac{\pi}{2} - \arctg \frac{\bar{y}}{2 + \Delta\bar{x}} \right) - 2 - \Delta\bar{x} \right] \cdot (\bar{x} - 1); \quad (18)$$

$$\tau_{xy2} = -\frac{3F}{4\pi h_w t_w} \cdot \left[\frac{\pi}{2} - \frac{\bar{y}(2 + \Delta\bar{x})}{(2 + \Delta\bar{x})^2 + \bar{y}^2} - \arctg \frac{\bar{y}}{2 + \Delta\bar{x}} \right] \cdot \bar{x}(\bar{x} - 2). \quad (19)$$

DIAGRAM 3a. Considering the equilibrium of the wall section, we find:

$$N = -\frac{F}{\pi} \cdot \frac{(2 + \Delta\bar{x})}{(2 + \Delta\bar{x})^2 + \bar{y}^2}. \quad (20)$$

Equations (12) - (14) as applied to diagram 3a for a similar stress function will have the form:

$$16F_1(\bar{y}) + 4F_2(\bar{y}) = -2N; \quad (21)$$

$$12F_1(\bar{y}) + 4F_2(\bar{y}) = -N; \quad (22)$$

$$8F_1(\bar{y})' + 4F_2(\bar{y})' + 2F_3(\bar{y})' = 0. \quad (23)$$

Solving together (21) and (22), we find:

$$F_1(\bar{y}) = \frac{N}{4} = \frac{F}{4\pi} \cdot \frac{(2 + \Delta\bar{x})^2}{(2 + \Delta\bar{x})^2 + \bar{y}^2}; \quad (24)$$

$$F_2(\bar{y}) = -\frac{N}{2} = -\frac{F}{2\pi} \cdot \frac{(2 + \Delta\bar{x})^2}{(2 + \Delta\bar{x})^2 + \bar{y}^2}; \quad (25)$$

Condition (23) is satisfied at $F_3(\bar{y}) = 0$.

Finally, for diagram 3a:

$$\sigma_{x3} = \frac{F}{2\pi h_w t_w} \cdot \frac{(2 + \Delta\bar{x})^2 [(2 + \Delta\bar{x})^2 - 3\bar{y}^2]}{[(2 + \Delta\bar{x})^2 + \bar{y}^2]^3} \cdot \bar{x}^2 (\bar{x} - 2); \quad (26)$$

$$\sigma_{y3} = -\frac{F}{\pi h_w t_w} \cdot \frac{(2 + \Delta\bar{x})^2}{(2 + \Delta\bar{x})^2 + \bar{y}^2} \cdot \left(\frac{3}{2}\bar{x} - 1\right); \quad (27)$$

$$\tau_{xy3} = \frac{2F}{\pi h_w t_w} \cdot \frac{(2 + \Delta\bar{x})^2 \bar{y}}{[(2 + \Delta\bar{x})^2 + \bar{y}^2]^2} \cdot \bar{x} \left(\frac{3\bar{x}}{4} - 1\right). \quad (28)$$

DIAGRAM 4a. Additional tangential stresses on the wall contour depend on the magnitude of the jump in tangential stresses obtained from the total bend along the interface line between the flange and the wall.

The author performed an analysis of various distributions of additional tangential stresses, which provide relatively good agreement with experimental data. The best match was provided by the following formulas:

- for the upper contour at $\bar{y} \geq 0$

$$q_{xy\bar{a}} = -\frac{F}{4\alpha\beta} \cdot \frac{S_f^{\bar{a}}}{J} \cdot e^{-\alpha\bar{y}} (\alpha^2 \sin \beta\bar{y} - 2\alpha\beta \cos \beta\bar{y} - \beta^2 \sin \beta\bar{y}), \quad (29)$$

then

$$\bar{q}_{xy\bar{e}} = -\frac{F}{4\alpha\beta} \cdot \frac{S_f^{\bar{e}} h_w}{J} \cdot e^{-\alpha\bar{y}} (-\alpha \sin \beta\bar{y} + \beta \cos \beta\bar{y}); \quad (30)$$

$$q'_{xy\bar{a}} = -\frac{F}{4\alpha\beta} \cdot \frac{S_f^{\bar{a}}}{J h_w} \cdot e^{-\alpha\bar{y}} [\alpha(3\beta^2 - \alpha^2) \sin \beta\bar{y} + \beta(3\alpha^2 - \beta^2) \cos \beta\bar{y}] \quad (31)$$

-for the bottom contour

$$q_{xy\bar{H}} = q_{xy\bar{e}} \cdot \frac{S_f^{\bar{H}}}{S_f^{\bar{e}}}; \quad \bar{q}_{xy\bar{H}} = \bar{q}_{xy\bar{e}} \cdot \frac{S_f^{\bar{H}}}{S_f^{\bar{e}}}; \quad q'_{xy\bar{H}} = q'_{xy\bar{e}} \cdot \frac{S_f^{\bar{H}}}{S_f^{\bar{e}}}. \quad (32)$$

Rates α and β may vary. The analysis showed that the best results are achieved when $\alpha = 0.35h_w/\lambda$; $\beta = h_w/\lambda$.

Considering the equation of equilibrium of the plot within the limits $\bar{y} \div \infty$ under the action of additional tangential stresses, we obtain:

$$N = -\int_{\bar{y}}^{\infty} q_{xy\bar{e}} \bar{\partial}\bar{y} + \int_{\bar{y}}^{\infty} q_{xy\bar{e}} \bar{\partial}\bar{y} + \int_0^2 \sigma_y \bar{\partial}\bar{x} = 0, \quad (33)$$

or, taking into account the stress function (8),

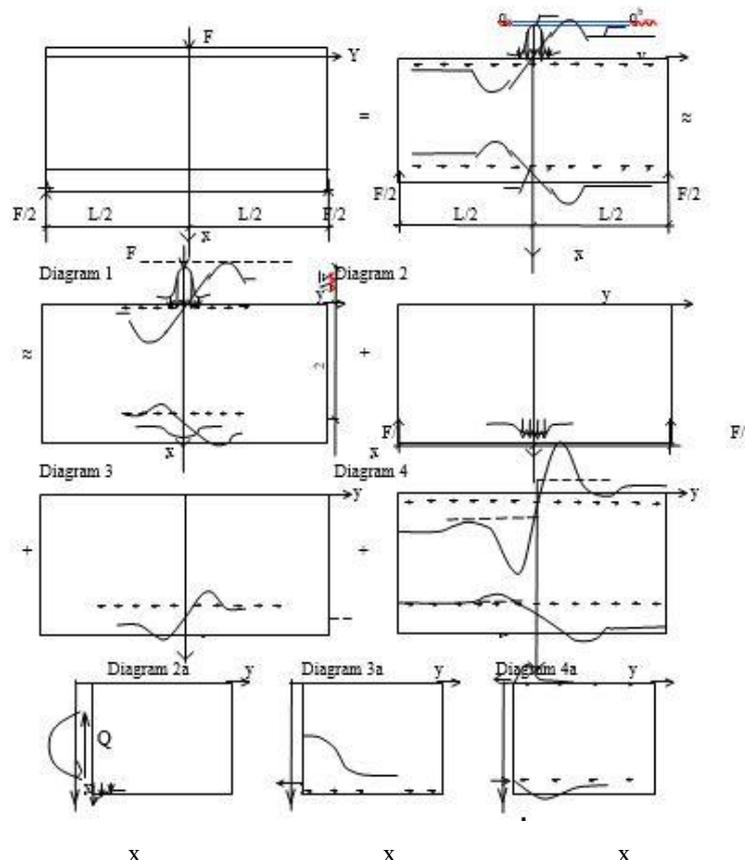


Fig. 2. Wall loading patterns under the action of concentrated pressure on the girder (the separate action of stresses along the contour is determined using the solution for the half-plane)

$$\frac{1}{h_w} [12F_1(\bar{y}) + 4F_2(\bar{y})] = \bar{q}_{xy\phi} - \bar{q}_{xyH}; \quad (34)$$

$$M = -\int_0^2 \sigma_y \bar{x} \bar{\alpha} \bar{x} - 2h_w \int_{\bar{y}}^{\infty} q_{xyH} \bar{\alpha} \bar{y} = 0, \quad (35)$$

$$\text{or } \frac{1}{h_w} [16F_1(\bar{y}) + F_2(\bar{y})] = -2\bar{q}_{xyH}; \quad (36)$$

$$Q = \int_0^2 \tau_{xy} \bar{\alpha} \bar{x} = -8F_1(\bar{y})' - 4F_2(\bar{y})' - 2F_3(\bar{y})' = 0. \quad (37)$$

Boundary conditions.

When $\bar{x} = 0$ $\sigma_x = 0$, then $F_4(\bar{y})'' = 0$;

$$\tau_{xy} = -\frac{1}{h_w} \cdot F_3(\bar{y})' = q_{xy\phi}. \quad (38)$$

When $\bar{x} = 2$ $\sigma_x = 0$, then

$$4F_1(\bar{y}) + 2F_2(\bar{y}) + F_3(\bar{y}) = 0 \text{ (see 37);}$$

$$\tau_{xy} = -\frac{1}{h_w} [12F_1(\bar{y})' + 4F_2(\bar{y})' + F_3(\bar{y})'] = q_{xyH} \text{ (see 34 and 38).}$$

Considering conditions (34), (36 ... 38), we find:

$$F_1(\bar{y}) = -0.25(\bar{q}_{xy\phi} + \bar{q}_{xyH}); \quad (39)$$

$$F_2(\bar{y}) = \bar{q}_{xy\phi} + 0.5q_{xyH}; \quad (40)$$

$$F_3(\bar{y}) = -q_{xy\phi}. \quad (41)$$

After simple transformations we get:

$$\sigma_{x4} = h_w [q'_{xy\phi} (-0.25\bar{x}^3 + \bar{x}^2 - \bar{x}) + q'_{xyH} (-0.25\bar{x}^3 + 0.5\bar{x}^2)]; \quad (42)$$

$$\sigma_{y4} = \frac{1}{h_w} [\bar{q}_{xy\phi} (-1.5\bar{x} + 2) + \bar{q}_{xyH} (-1.5\bar{x} + 1)]; \quad (43)$$

$$\tau_{xy4} = -q_{xy\phi} (-0.75\bar{x}^2 + 2\bar{x} - 1) - q_{xyH} (-0.75\bar{x}^2 + \bar{x}). \quad (44)$$

To determine local stresses, all components should be summarized, that is:

$$\sigma_x^M = \sigma_{x1} + \sigma_{x2} + \sigma_{x3} + \sigma_{x4}; \quad (45)$$

$$\sigma_y^M = \sigma_{y1} + \sigma_{y2} + \sigma_{y3} + \sigma_{y4}; \quad (46)$$

$$\tau_{xy}^M = \tau_{xy1} + \tau_{xy2} + \tau_{xy3} + \tau_{xy4}. \quad (47)$$

Equations (45) - (47) are universal:

$$\text{-if } J_{\Sigma f_i} = 0 \quad (\Delta \bar{x} = 0), \quad S_f^e = 0; \quad S_f^H = 0,$$

then we get a solution for a strip loaded with concentrated force, and if, moreover $h_w \rightarrow \infty$, we get a well-known solution for a half-plane;

- if the cross-section of the I-girder is symmetrical ($\bar{q}_{xy\phi} = \bar{q}_{xyH}$), then the stresses are σ_{x4} , σ_{y4} , τ_{xy4} determined by the formulas

$$\sigma_{x4} = h_w \cdot q'_{xy\phi} \bar{x} (-0.5\bar{x}^2 + 1.5\bar{x} - 1); \quad (48)$$

$$\sigma_{y4} = \frac{1}{h_w} \cdot \bar{q}_{xy\phi} 3(-\bar{x} + 1); \quad (49)$$

$$\tau_{xy4} = -q_{xy\phi} (-1.5\bar{x}^2 + 3\bar{x} - 1). \quad (50)$$

-If the section is T-shaped, $S_f^H = 0$, $\bar{q}_{xyH} = 0$,

then:

$$\sigma_{x4} = h_w \cdot q'_{xy\phi} \bar{x} (-0.25\bar{x}^2 + \bar{x} - 1); \quad (51)$$

$$\sigma_{y4} = \frac{1}{h_w} \cdot \bar{q}_{xy\phi} (-1.5\bar{x} + 2); \quad (52)$$

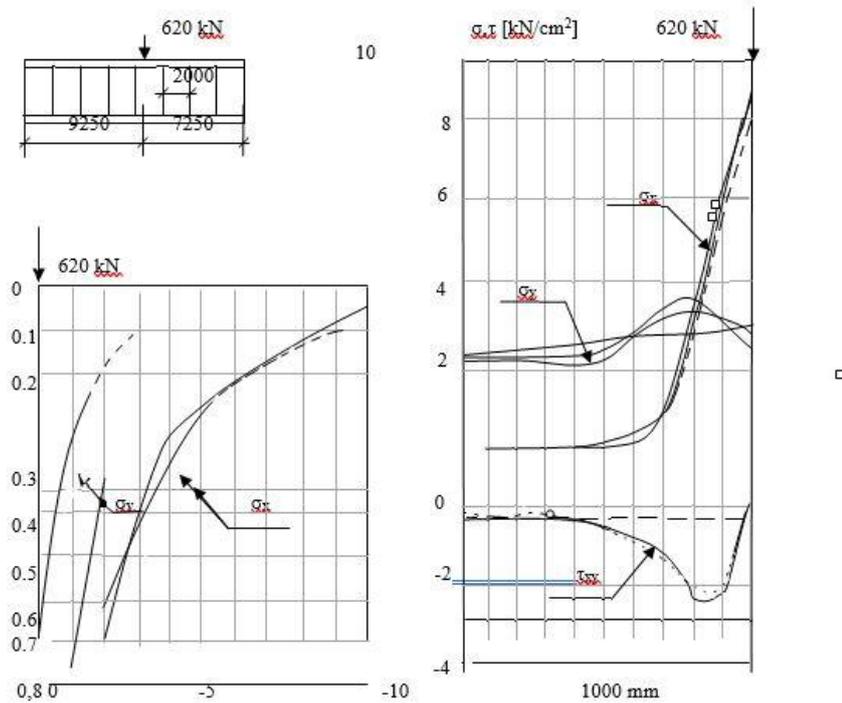
$$\tau_{xy4} = -q_{xy\phi} (-0.75\bar{x}^2 + 2\bar{x} - 1). \quad (53)$$

A comparison of theoretical calculations by formulas (45), (46) and (47) with experimental data [21] was performed in graphical form in Fig. 3. It can be noted that the results are acceptable from a practical point of view: at a horizontal level, located at a distance of 40 mm from the upper zone in the wall, the differences in stress σ_x^M τ_{xy}^M do not exceed 5%, and stresses σ_y^M 10%.

CONCLUSION

The developed method for determining local stresses in the walls of I-girders under the action of concentrated loads has a number of advantages compared with the known solutions:

1. Computational efficiency. All calculation formulas are obtained explicitly and do not require the use of numerical methods.
2. A larger number of parameters are taken into account in relation to known solutions. The solution is generalized in nature, so it can be applied to calculations of a half-plane, strip, T-girder and I-girder, including asymmetric section.
3. Good agreement between theoretical and experimental data was confirmed.



Designations:

..... experimental data

..... theoretical data

----- General bending stresses (elementary)

Fig. 3. Comparison of experimental data [21] with theoretical

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