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# Estimation of Missing Data and Maps of the Behaviour of Total Precipitation in Northern Colombia

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**Abstract**

The climatic conditions of a territory are a key element in the development of multiple types of projects, hence it is necessary to have relevant information that helps determine and simulate the future climate conditions that may influence such projects. At this point, the important thing is to have reliable and complete weather information records. Unfortunately, in Colombia, being able to find this type of information is not so simple, and in general the climatological data are incomplete for reasons such as failures or non-calibration in the measurement instruments, errors in the recording of measurements, meteorological extremes and difficulty in the access of the measurement areas. Hence, the determination of the missing data in the records of the climatological stations becomes a necessary question to complete the incomplete historical series. A common case is the missing data in rainfall stations, which are essential in hydrological simulation studies in order to obtain optimal hydraulic solutions. Therefore, it is almost mandatory to properly fill in missing data before performing any analysis on such data. This work presents the determination of missing data of total precipitation of 40 meteorological measurement stations located in the north of Colombia. The series analysed cover a period of 40 years of observations from 1980 to 2019, obtained by the Instituto de Hidrología, Meteorología y Estudios Ambientales (IDEAM). The estimation of missing data was carried out using Multiple Linear Regression (MLR), which allows estimating missing data based on nearby records that correlate with said data. The Beale – Little algorithm is applied which is based on the RLM method, using all the available information and leads to a simultaneous estimation of the missing values. The estimated precipitation dataset is useful for future hydrological analysis of studies that require meteorological forecasts in regions with losses of hydrological information, such as the meteorological stations located in the north of Colombia.

**Keywords:** Monthly rainfall, Missing data, Extreme rainfall, Multiple Linear Regression, Beale–Little algorithm.

## I. INTRODUCTION

Hydrology is the science that deals with water, its occurrence, its circulation, distribution, its properties and its relationship with the environment and living beings [1]. Hydrology allows a rational and efficient use of the planet's water resources. Through this, it is possible to plan and carry out studies that lead to the proper management of the water resources of a region.

Hydrology provides the technical and scientific elements that will allow you to adequately understand the water cycle [2]. These scientific methods generally require input data according to the models used and the specific conditions of the project. Rainfall is considered a main variable in hydrological studies, as it is the most important source for calculating the water balance and generating early warnings for risk of drought or flooding in a geographic region [3].

Rainfall analyses require continuous, homogeneous data that cover the maximum possible time interval. Unfortunately, the database available in the collection centres, most of the time, presents missing information due to the absence of reading, failure of the recording instrument, transcription error, among others, which limits the analyses and many times, it is constituted as a source of error [3]. Multiple Linear Regression (MLR) is a statistical technique that can be used in hydrology with the intention of determining the missing data on runoff characteristics, calculated in basins with hydrometric to sites or basins where such information is necessary and there are no gauges. [4]. In a complete way, the Beale-Little algorithm is used to simultaneously estimate the missing data in the records of the rainfall stations of the geographical area under study. With the use of this technique it is possible to obtain the data lost in the series of rainfall data, in order to carry out more reliable hydrological analyses and studies.

Once an area or region of the territory has been evaluated, the results of the analyses can be presented graphically. Being one of the most common and practical forms, show them in spatial distribution schemes where the hydrological variables analysed are represented. In the case of rainfall, the use of GIS tools for this purpose has become very popular. To carry out this, it is necessary to know the spatial variation in the study area, for which precipitation fields must be constructed, using interpolation methodologies, among which there is the Kriging method and the IDW method (Inverse Distance Weighted) [6]. Once the modelling has been carried out, the diagrams will be obtained showing the distribution and spatial interpolation of the rainfall in the study areas.

The study area presents different hydrological conditions throughout the entire territory, where there are geographically mountainous areas and others with swampy areas and river

discharge points, making their environmental conditions very varied. The information analysed includes precipitation data from 40 stations that have been active in the last 40 years and are located in the department of Sucre. The historical records of total rainfall were obtained from the data of the measurements of rainfall stations that rest in the Instituto de Hidrología, Meteorología y Estudios Ambientales (IDEAM).

The objective of this work is to present the RLM method through the use of the Beatle-Litle algorithm, for the determination of missing values of total rainfall in the stations located in the department of Sucre. From this, it was possible to fill in the missing rainfall for the forty stations in the study period, which was 40 years. In this way, it is possible to obtain reliable information that hydraulic researchers and designers require to develop their projects. Similarly, distribution maps of average and maximum rainfall of the study area were created. This allows the obtained results to be viewed in a clearer way and is of great use to professionals interested in studying the precipitation conditions of the area under study.

## II. EXPERIMENTAL DESIGN, MATERIALS AND METHODS

### II.I Study area description

The Department of Sucre is located in the Colombian Caribbean plain, north of the Central and Western mountain ranges, it has an area of 10,364 square kilometres. It limits to the north and east with the Department of Bolívar, to the south with the Departments of Antioquia and Córdoba, to the west with the Department of Córdoba and to the northeast with the Caribbean Sea. A little more than a third of its territory forms the flood-prone depression of the Bajo Magdalena, Cauca and San Jorge rivers, characterized by numerous swamps, especially along the San Jorge river. To the northwest, on the other hand, a strip of hills can be seen, corresponding to the San Jacinto or Montes de María mountains. Between these hills and the San Jorge depression lie more or less flat savannas. In the Department of Sucre the climate is warm, dry towards the sea and humid towards the depression. The various reliefs in the Department of Sucre have a warm thermal floor with temperatures that range between 25.5 °C and 28.7 °C average per year. The precipitation regime is determined by the geographical location, and by the influence of some factors, such as atmospheric circulation, relief, the interaction between land and sea and the influence of jungle or wooded areas. In Sucre, the annual average rainfall varies between 1,000 mm for the less humid areas in the north, up to 2,800 mm in the rainiest areas of the south [7].

### II.II Material and methods

The determination of the missing rainfall data was carried out by applying the Beale-Little algorithm, which is a Multivariate Analysis (AM) technique that allows to simultaneously estimate the missing data in records of hydrometric or pluviometric stations of a geographical area, which show a significant correlation, but do not have persistence. The technique is appropriate to estimate missing data that were lost randomly, that is, when there was an absence of the operator or a temporary suspension, maintenance or poor calibration of the equipment, or, due to improvements in the installation [8] [9].

The first step in the methodology is to collect the available information and organize it in a matrix arrangement where the rows represent the recording periods and the columns represent the meteorological stations, adding an asterisk to the missing data or leaving the spaces in blanks. Next, the initial substitution of the missing data is made, for this, the arithmetic average of each column is calculated and this value is replaced in the spaces with missing data. From these, the MLR for each record is calculated and the missing data is estimated. After applying the RLM, the differences between the old data and the new data are calculated, which would constitute the first cycle of the Beale-Little algorithm. This process continues for the number of cycles that is necessary until the difference between the new data and the previous one takes a sufficiently small value or is almost negligible; so that if this degree of precision is not reached, the new estimates are replaced by the old ones and new estimates are made using the same procedure.

The mathematical formulation of the RLM [10] [11] is shown below. It is assumed that there are n observations of Y, $X_1$, $X_2$,…, $X_p$, considering that there are p independent variables or regressors of the Regression, whose model is the following:

$$Y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \cdots + \beta_p X_p + \varepsilon \quad (1)$$

In matrix notation, we have:

$$Y = X.\beta + \varepsilon \quad (2)$$

$$Y = \begin{bmatrix} Y_1 \\ Y_2 \\ \vdots \\ Y_n \end{bmatrix} X = \begin{bmatrix} 1 & X_{11} & X_{12} & \cdots & X_{1p} \\ 1 & X_{21} & X_{22} & \cdots & X_{2p} \\ \vdots & \cdots & \cdots & \cdots & \vdots \\ 1 & X_{n1} & X_{n2} & \cdots & X_{np} \end{bmatrix}$$

$$\beta = \begin{bmatrix} \beta_0 \\ \beta_1 \\ \vdots \\ \beta_n \end{bmatrix} \varepsilon = \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \vdots \\ \varepsilon_n \end{bmatrix}$$

The matrix $X$ contains the $X_{ij}$ observations of the meteorological data, showing the i-th periods of the observations and the j-th independent variables. The method seeks that the sum of the squared errors ($\varepsilon_i$) is minimized according to the expression:

$$\sum_{i=1}^{n} \varepsilon_i^2 = \sum_{i=1}^{n} (Y_i - \beta_0 - \beta_1 X_{i1} - \beta_2 X_{i2} - \ldots - \beta_p X_{ip})^2 \quad (3)$$

When differentiating the right hand side of equation 3, based on $\beta_0, \beta_1, \ldots, \beta_p$ separately and equaling to zero, this produces p equations with p unknown parameters, and is called normal equations. This, written in matrix notation, looks like this:

$$X^T.X.\beta = X^T.Y \quad (4)$$

The solution of equation 4 for the β_i can be expressed from Equation 5:

$$\beta = (X^T.X)^{-1}.X^T.Y \quad (5)$$

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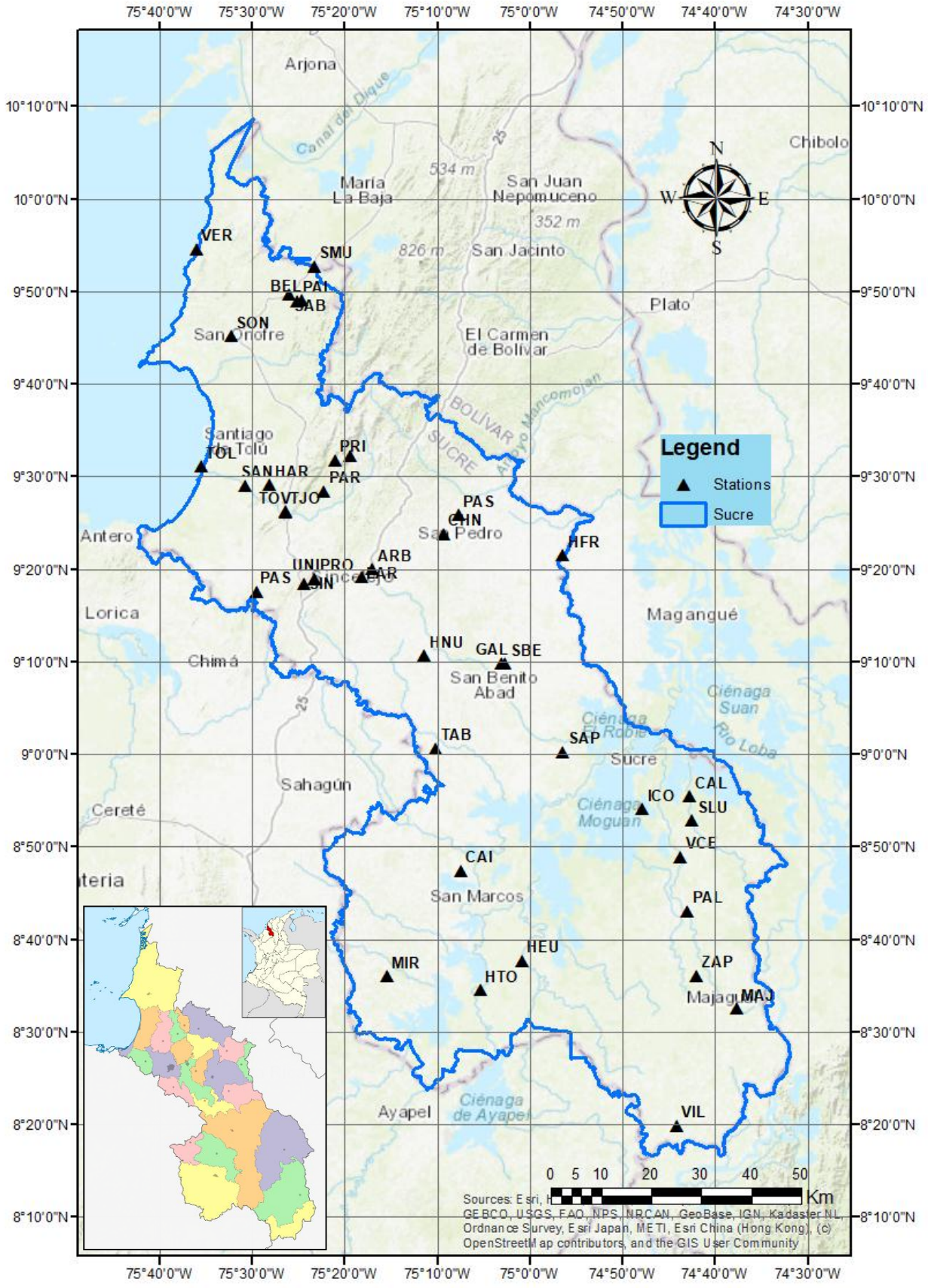


**Fig 1.** Location of meteorological stations belonging to the department of Sucre

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In equation 5, $X$ is the matrix with the observations and variables, $Y$ is the vector of dependent observations, $X^T$ is the transpose matrix of $X$, and $(X^T.X)^{-1}$ is the matrix inverse of the matrix constructed through $X^T.X$.

Additionally, Figures were created showing the average and maximum rainfall of the stations, in order to generate an extrapolation of the data and observe the behavior of rainfall based on the historical series with missing data filled in, being able to observe the behavior of average rainfall over the 40-year Study period.

To carry out this process, the GIS computing tool, ArcMap, was used, for which the geographic location data of the stations were imported into the program, as well as the values related to the average and maximum precipitation. Additionally, layers were imported to limit the study area (the department of Sucre) and other layers to give relief to neighboring places and general information.

The Kriging method was used to perform the spatial interpolation of the precipitation variable. This is a geostatistical method, which relates mathematics to earth sciences, therefore being ideal for estimating the rainfall data obtained from the measurement of the rain gauge. The Kriging method works with statistical models that involve autocorrelation [6].

Autocorrelation allows spatial interpolation to occur, which generally occurs when estimating a regionalized value at unsampled points, based on the weight of the regionalized values that were observed. Equation 6 shows the mathematical model that governs spatial interpolation [6].

$$Zg = \sum_{i=1}^{ns} \lambda_i Zsi \quad (6)$$

In equation 6, $Zg$ is the interpolated value in the requested areas or points, $Zsi$ is the value observed at point $i$, $ns$ is the total number of points observed and $\lambda i$ is the weight that contributes to the interpolation.

Within ArcMap this method (Kriging) was selected, because it has some advantages over the traditional interpolation methods built into GIS. If the assumptions of the theory hold, the method is less arbitrary compared to others. A very important difference to emphasize is that the weights used in the method are determined by the variogram and the configuration of the observed data [12].

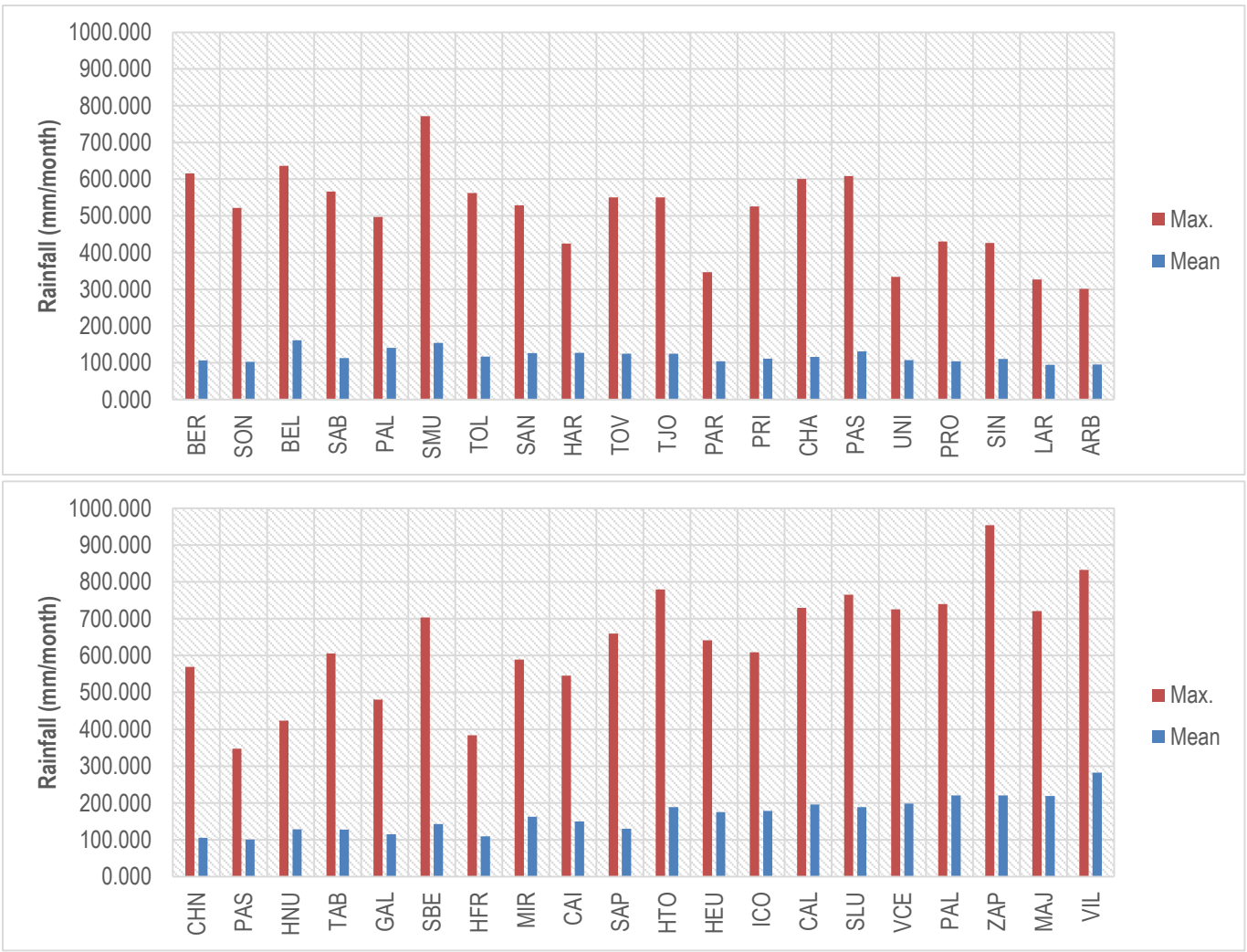


**Fig. 2.** Mean and maximum values of monthly rainfall.

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**Table 1.** General data of the Meteorological Stations

| ID | NUMBER | STATION NAME | CODE | ALTITUDE | LATITUDE | LONGITUDE | MUNICIPALITY |
|---|---|---|---|---|---|---|---|
| 1 | 13090030 | BERRUGAS | BER | 1 | 9.91027778 | -75.6005556 | San Onofre |
| 2 | 13090020 | SAN ONOFRE | SON | 55 | 9.75416667 | -75.5380556 | San Onofre |
| 3 | 13090080 | BELEN HACIENDA | BEL | 60 | 9.81944444 | -75.4102778 | San Onofre |
| 4 | 13090010 | SABANETICA | SAB | 1 | 9.82888889 | -75.4327778 | San Onofre |
| 5 | 13090110 | PALO ALTO | PAL | 50 | 9.81750000 | -75.4183333 | San Onofre |
| 6 | 29030380 | SABANAS DE MUCACAL | SMU | 10 | 9.87980556 | -75.3886944 | San Onofre |
| 7 | 13090070 | TOLU | TOL | 2 | 9.51916667 | -75.5911111 | Tolú |
| 8 | 13090090 | SANTA ANGELA HACIENDA | SAN | 20 | 9.48527778 | -75.5125000 | Tolú |
| 9 | 13090100 | ARGENTINA LA HACIENDA | HAR | 20 | 9.48583333 | -75.4686111 | Tolú Viejo |
| 10 | 13090050 | TOLUVIEJO | TOV | 60 | 9.43800000 | -75.4400000 | Tolú Viejo |
| 11 | 13097010 | TOLUVIEJO | TJO | 59 | 9.43700000 | -75.4390000 | Tolú Viejo |
| 12 | 13090060 | PARAISO EL | PAR | 100 | 9.47388889 | -75.3708333 | Colosó |
| 13 | 13095020 | PRIMATES | PRI | 200 | 9.53013889 | -75.3513611 | Colosó |
| 14 | 13090040 | CHALAN | CHA | 100 | 9.53861111 | -75.3225000 | Chalán |
| 15 | 13090230 | PASTORA LA | PAS | 58 | 9.29333333 | -75.4919444 | Sincelejo |
| 16 | 25025270 | UNISUCRE | UNI | 160 | 9.31638889 | -75.3875000 | Sampués |
| 17 | 25025350 | PUERTA ROJA | PRO | 160 | 9.31638889 | -75.3875000 | Sincelejo |
| 18 | 25020130 | SINCELEJO | SIN | 200 | 9.30816667 | -75.4068333 | Sincelejo |
| 19 | 25020400 | LIBRA ARRIBA | LAR | 180 | 9.32055556 | -75.3036111 | Betulia |
| 20 | 25025080 | APTO RAFAEL BARVO | ARB | 166 | 9.33388889 | -75.2830556 | Corozal |
| 21 | 25020390 | CHARCON | CHN | 150 | 9.39722222 | -75.1558056 | Los Palmitos |
| 22 | 25020190 | PUERTO ASIS | PAS | 200 | 9.43194444 | -75.1291667 | Los Palmitos |
| 23 | 25020990 | HATO NUEVO | HNU | 80 | 9.17833333 | -75.1902778 | Corozal |
| 24 | 25020750 | TABLITAS LAS | TAB | 60 | 9.01138889 | -75.1688889 | San Benito Abad |
| 25 | 25021000 | GALERAS | GAL | 70 | 9.16500000 | -75.0508333 | Galeras |
| 26 | 25025380 | SAN BENITO | SBE | 20 | 9.16388889 | -75.0447222 | San Benito Abad |
| 27 | 25020860 | FRONTERA LA HACIENDA | HFR | 100 | 9.35944444 | -74.9408333 | Buenavista |
| 28 | 25021660 | MIRASOL | MIR | 30 | 8.60083333 | -75.2572222 | San Marcos |
| 29 | 25020980 | CAIMITO | CAI | 20 | 8.79083333 | -75.1244444 | Caimito |
| 30 | 25020760 | SANTIAGO APOSTOL | SAP | 25 | 9.00472222 | -74.9405556 | San Benito Abad |
| 31 | 25021470 | TORNO EL HACIENDA | HTO | 60 | 8.57638889 | -75.0897222 | San Marcos |
| 32 | 25020740 | EUREKA HACIENDA | HEU | 20 | 8.62916667 | -75.0147222 | San Marcos |
| 33 | 25021560 | ISLA DEL COCO | ICO | 20 | 8.90277778 | -74.7986111 | Sucre (Sucre) |
| 34 | 25021360 | CAMPO ALEGRE | CAL | 20 | 8.92472222 | -74.7119444 | Sucre (Sucre) |
| 35 | 25021370 | SAN LUIS | SLU | 20 | 8.88166667 | -74.7080556 | Sucre (Sucre) |
| 36 | 25020500 | VILLA CECILIA HACIENDA | VCE | 50 | 8.81611111 | -74.7294444 | Sucre (Sucre) |
| 37 | 25020790 | PALMARITO | PAL | 50 | 8.71888889 | -74.7177778 | Majagual |
| 38 | 25020820 | ZAPATA | ZAP | 50 | 8.60277778 | -74.6997222 | Majagual |
| 39 | 25025240 | MAJAGUAL | MAJ | 26 | 8.54269444 | -74.6273333 | Majagual |
| 40 | 25020940 | VILLANUEVA | VIL | 45 | 8.33277778 | -74.7355556 | Guaranda |

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**Table 2.** Statistical Description of the Data of the Meteorological Stations

| CODE | Observations | Minimum (mm/month) | Maximum (mm/month) | Mean (mm/month) | Standard deviation (mm/month) |
|---|---|---|---|---|---|
| BER | 480 | 0.161 | 615.500 | 106.155 | 103.801 |
| SON | 480 | 0.554 | 521.600 | 102.349 | 83.100 |
| BEL | 480 | 0.100 | 636.208 | 161.092 | 113.493 |
| SAB | 480 | 0.080 | 566.000 | 113.084 | 98.949 |
| PAL | 480 | 0.233 | 497.167 | 140.471 | 97.700 |
| SMU | 480 | 0.304 | 771.125 | 154.512 | 113.475 |
| TOL | 480 | 0.277 | 562.500 | 116.658 | 101.510 |
| SAN | 480 | 0.708 | 528.667 | 126.011 | 104.181 |
| HAR | 480 | 0.312 | 425.017 | 127.547 | 95.134 |
| TOV | 480 | 0.100 | 550.000 | 124.710 | 98.335 |
| TJO | 480 | 0.055 | 550.000 | 125.031 | 98.848 |
| PAR | 480 | 0.400 | 346.742 | 103.775 | 75.258 |
| PRI | 480 | 0.736 | 525.858 | 111.484 | 89.300 |
| CHA | 480 | 0.200 | 600.375 | 116.309 | 94.523 |
| PAS | 480 | 0.040 | 608.417 | 131.407 | 103.449 |
| UNI | 480 | 0.273 | 334.194 | 107.471 | 75.653 |
| PRO | 480 | 1.000 | 430.000 | 104.139 | 72.846 |
| SIN | 480 | 1.220 | 426.000 | 110.153 | 77.261 |
| LAR | 480 | 0.896 | 326.690 | 94.822 | 71.803 |
| ARB | 480 | 0.088 | 301.700 | 95.164 | 68.821 |
| CHN | 480 | 0.468 | 569.500 | 105.440 | 80.147 |
| PAS | 480 | 0.175 | 347.375 | 100.986 | 70.993 |
| HNU | 480 | 1.000 | 423.208 | 128.674 | 96.774 |
| TAB | 480 | 0.500 | 605.500 | 127.729 | 98.497 |
| GAL | 480 | 0.331 | 480.667 | 115.122 | 92.116 |
| SBE | 480 | 0.146 | 703.158 | 143.126 | 113.750 |
| HFR | 480 | 0.583 | 383.583 | 109.328 | 79.578 |
| MIR | 480 | 0.723 | 589.000 | 162.386 | 126.330 |
| CAI | 480 | 0.430 | 545.375 | 150.230 | 117.609 |
| SAP | 480 | 0.200 | 660.000 | 130.086 | 109.792 |
| HTO | 480 | 0.391 | 779.792 | 188.760 | 154.414 |
| HEU | 480 | 1.000 | 641.792 | 175.472 | 134.026 |
| ICO | 480 | 1.458 | 608.875 | 178.659 | 137.756 |
| CAL | 480 | 0.886 | 729.375 | 195.968 | 155.168 |
| SLU | 480 | 0.898 | 765.417 | 188.797 | 150.821 |
| VCE | 480 | 0.106 | 725.292 | 198.295 | 155.765 |
| PAL | 480 | 0.036 | 740.000 | 220.580 | 175.848 |
| ZAP | 480 | 0.997 | 954.042 | 220.417 | 169.662 |
| MAJ | 480 | 0.700 | 720.917 | 218.938 | 163.761 |
| VIL | 480 | 1.417 | 832.167 | 282.592 | 199.068 |

## III. RESULTS

This section contains Figures and Tables of the precipitation data that were analyzed from the information obtained from 40 meteorological stations located within the territory of the Department of Sucre (Colombia).

Fig. 1 shows the map of the location of the department of Sucre and the location of the meteorological stations that were selected for this work. The names, codes, the geographic coordinates of the location, the elevations and the municipalities where the IDEAM meteorological stations are located were tabulated in Table 1. The information on the

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statistical description of the data of the selected meteorological stations is shown in Table 2, where you can see the nomenclature given to each station, the number of observations considered during the study period (40 years), as well as the maximum, average values and the corresponding standard deviation, related to the Total monthly rainfall for each station.

Fig. 2 shows the historical average of the mean and maximum annual rainfall in the 40-year period for the department of Sucre. The stations are organized in a North-South direction, thus allowing us to look at their behavior.

In Fig. 3 and 4, a graphical representation has been made through the ArcMap program of the average and maximum precipitations of the area under study, in this way it can be easily appreciated which are the parts of the department where it has rained the most in the last 40 years and which have been the places with less precipitation, which gives an idea of the spatial and temporal distribution of the rains during the study period.

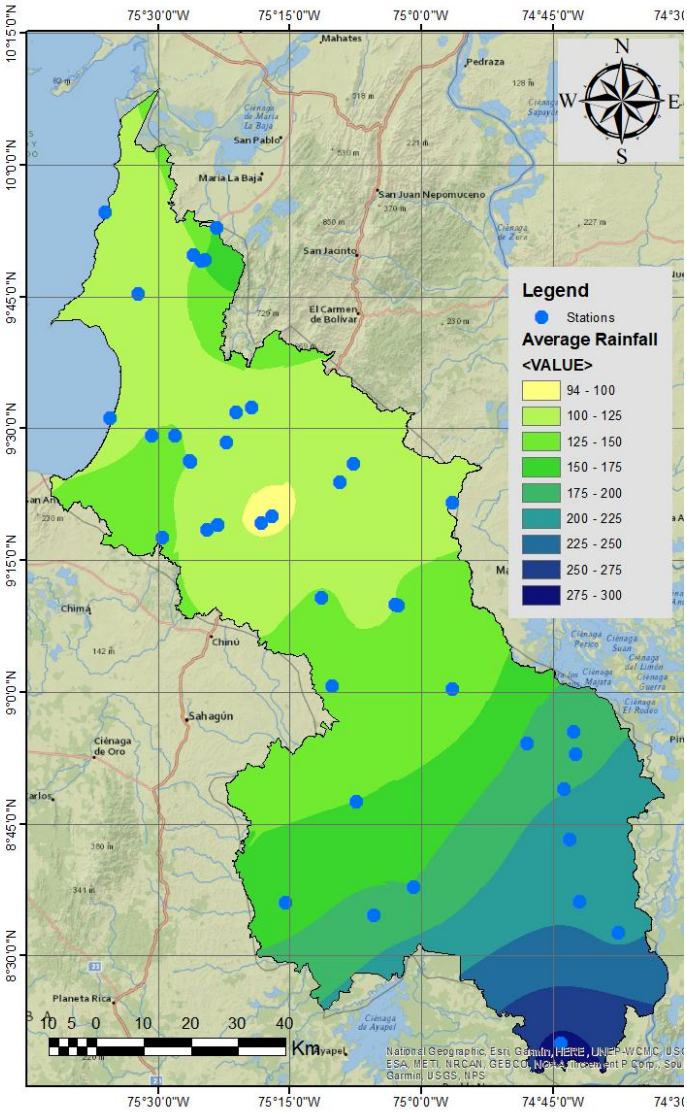


**Fig. 3.** Comportamiento de la precipitación promedio mensual en el departamento de Sucre

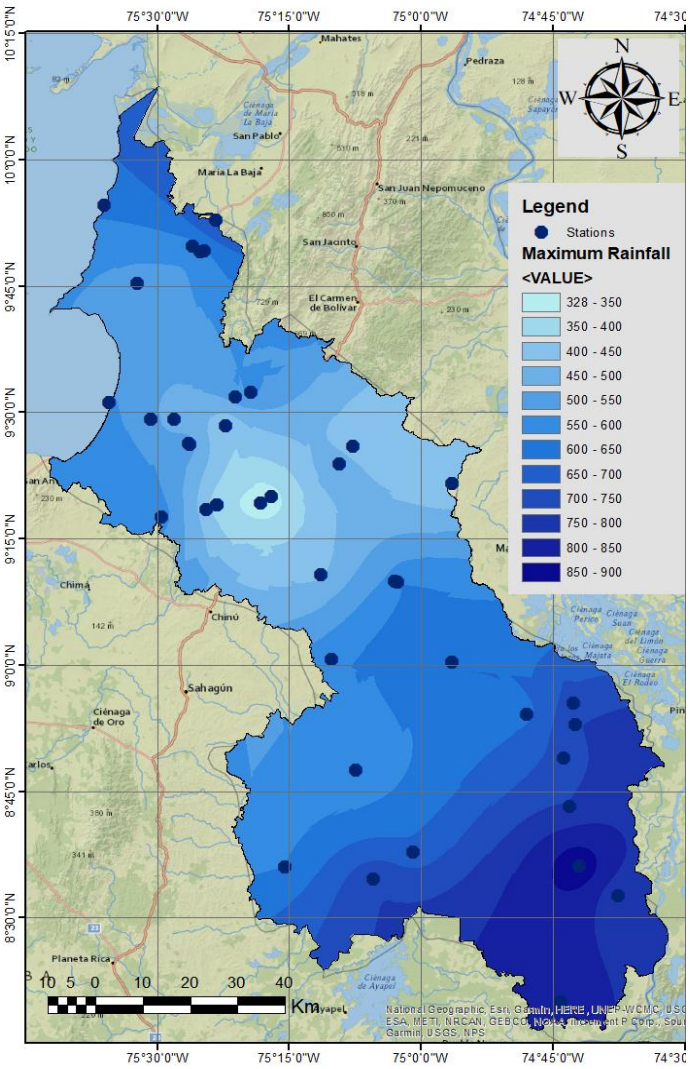


**Fig. 4.** Comportamiento de la precipitación máxima mensual en el departamento de Sucre

## IV. CONCLUSIONS

From the work carried out, the missing monthly precipitation data could be obtained for the 40 selected meteorological stations. This information is of particular interest to be taken as an essential input for the hydrological characterization of the engineering projects that are intended to be developed within the department of Sucre.

Additionally, maps were obtained that show the behavior of monthly average and maximum monthly rainfall in the department of Sucre, information that allows establishing the rainfall characteristics of a project, according to its geographic location.

### Acknowledgments

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(Colombia). Authors also thank the engineer Edilberto Elias Contreras Sierra for the contributions made in the development of the manuscript.

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