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Optimization of a Concave Parabolic Fin for Fixed Fin Volumes

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Abstract

A concave parabolic fin for fixed fin volume is optimized using a two-dimensional analytical method. First, to ensure the reliability of this analysis, the temperature distribution and heat loss of a parabolic fin and a triangular fin are compared. When the volume of the fin is fixed, heat loss is maximized with a certain fin length and fin base height. This maximum heat loss and the corresponding fin length and fin base height are called optimum heat loss, optimum fin length, and optimum fin base height, respectively. These optimum values are presented as a function of the convection characteristic number and the fin volume. One of the results shows that optimum heat loss increases, while optimum fin length decreases as the convection characteristic number increases.

Keywords: Symmetric concave parabolic fin, Heat loss, Convection characteristic number, Fin length, Fin surface area

Nomenclature

a fin surface area, m^2

A dimensionless fin surface area, $a/(l_c l_w)$

Bi Biot number

h ambient heat transfer coefficient, W/m^2°C

k thermal conductivity, W/m°C

l_c characteristic length, m

l_e fin length, m

L_e dimensionless fin length, l_e/l_c

l_h fin base height, m

L_h dimensionless fin base height, l_h/l_c

l_w fin width, m

M convection characteristic number, $(hl_c)/k = Bi/L_h$

q heat loss from the fin, W

Q dimensionless heat loss from the fin, $q/(kl_w\phi_b)$

T temperature, °C

T_b fin base temperature, °C

T_∞ ambient temperature, °C

v parabolic fin volume, m^3

V dimensionless parabolic fin volume, $v/(l_c^2 l_w)$

x fin length coordinate, m

X dimensionless fin length coordinate, x/l_c

y fin height coordinate, m

Y dimensionless fin height coordinate, y/l_c

Greek symbols

ϕ_b adjusted fin base temperature, $(T_b - T_\infty)$

θ dimensionless temperature, $(T - T_\infty)/(T_b - T_\infty)$

λ_n eigenvalues, n=1, 2, 3, · · ·

Γ Gamma function

Subscripts

b fin base

c characteristic

e fin length

h fin base height

par parabolic fin

tri triangular fin

w fin width

∞ ambient

1. INTRODUCTION

Extended surfaces or fins play an important role in enhancing heat transfer, and various fin shapes have been used in many industrial applications such as aircrafts, electric devices, auto vehicles, and air-conditioning equipment. Various fin shapes, including pin, annular (circular), straight rectangular,

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triangular and trapezoidal fins have been investigated. For example, Yu and Chen [1] discussed the optimization of rectangular profile circular fins with variable thermal conductivity and convective heat transfer coefficients, while Abrate and Newnham [2] analyzed heat conduction in an array of triangular fins with an attached wall. Kang [3] optimized a pin fin with variable fin base thickness using a two-dimensional analytical method. Look [4] demonstrated two-dimensional effects by comparing the results of one and two-dimensional analyses of a rectangular fin on a pipe with convection from the tip as a function of the Biot number and the relative fin size. Khani and Aziz [5] developed an analytical solution for the thermal performance of a straight fin of a trapezoidal profile by using a homotopy analysis method when both the thermal conductivity and the heat transfer coefficient are temperature-dependent. Kang [6] examined the ratio of the heat loss from a symmetric trapezoidal fin to that from an asymmetric trapezoidal fin as a function of the fin length, base height and shape factor as well as the convection characteristic number.

Parabolic and hyperbolic fins have also been studied. For examples of these studies, Ullmann and Kalman [7] determined the temperature profile, the efficiency and the optimum dimensions of four different shapes of annular fins (rectangular, triangular, hyperbolic and parabolic) by solving numerically the differential equations. Moitsheki et al. [8] considered a model describing the temperature profile in a longitudinal fin with rectangular, concave, triangular and convex parabolic profiles by using optimal homotopy analysis method. Kim and Kang [9] made comparisons of heat loss between 2-D analytic method and 2-D finite difference method for two parabolic fin models. Aziz and Nguyen [10] reported two-dimensional heat transfer characteristics of individual longitudinal convecting-radiating fins of four different profile shapes: rectangular, trapezoidal, triangular and concave parabolic. Campo and Cui [11] addressed an elementary analytical procedure for solving approximately the quasi-1D heat conduction equation (a generalized Airy equation) governing the annular fin of hyperbolic profile.

In this study, a symmetric concave parabolic fin is chosen as a fin model. This fin is analyzed and then optimized for fixed fin volume using a two-dimensional analytical method. To ensure the trustworthiness of the data obtained in this analysis, the temperature and heat loss of a parabolic fin (examined in this paper) and a symmetric triangular fin are compared. The data for the triangular fin are calculated using equations for the case of ξ_{sy} = 1 in Reference [6]. When the fin volume is fixed, maximum heat loss occurs with a specific fin length and base height, although maximum heat loss may not be reached, depending on other variables. When the maximum heat loss occurs, this maximum heat loss, and the corresponding fin length and fin base height are referred as optimum heat loss, optimum fin length, and optimum fin base height, respectively. These optimum values are presented as a function of the convection characteristic number and the fin volume.

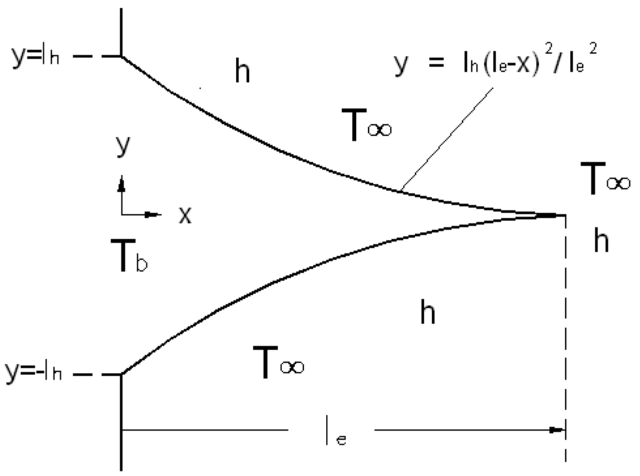

Fig. 1 Schematic diagram of a symmetric concave parabolic fin

2. TWO-DIMENSIONAL ANALYTIC METHOD

The two-dimensional governing differential equation for a symmetric concave parabolic fin as shown in Fig. 1 can be written

$$\frac{\partial^2\theta}{\partial X^2}+\frac{\partial^2\theta}{\partial Y^2}=0 \tag{1}$$

Three boundary conditions and one energy balance condition simultaneously required to solve Eq. (1) are

$$\theta\big|_{X=0}=1 \tag{2}$$

$$\left.\frac{\partial\theta}{\partial Y}\right|_{Y=0}=0 \tag{3}$$

$$\left.\frac{\partial\theta}{\partial X}\right|_{X=L_e}+M\cdot\theta\big|_{X=L_e}=0 \tag{4}$$

$$-\int_0^{L_h}\left.\frac{\partial\theta}{\partial X}\right|_{X=0}dY = M\int_0^{L_h}\theta\sqrt{\frac{L_e^2}{4L_hY}+1}\,dY \tag{5}$$

A fin base boundary condition is represented by Eq. (2), which reveals that the constant fin base temperature is T_b. Equation (3) is a fin center line condition, and it reveals that there is no heat transfer through the fin center line due to its symmetric geometry. Equation (4) is the boundary condition at the fin tip, showing that the heat conduction to the tip is equal to the heat convection from the tip. The energy balance condition is given by Eq. (5), which indicates that the heat conduction through the upper half fin base is equal to the heat convection from the upper half fin surface. Solving Eq. (1) with boundary conditions (2)-(4), the dimensionless temperature distribution within the fin is

$$\theta(X,Y)=\sum_{n=1}^{\infty}A_n\ f(X)\cos(\lambda_n Y) \tag{6}$$

where

$$f(X)=cosh\ (\lambda_n X)-g_n\cdot sinh\ (\lambda_n X) \tag{7}$$

$$g_n=\frac{\lambda_n\sinh(\lambda_n L_e)+M\cosh(\lambda_n L_e)}{\lambda_n\cosh(\lambda_n L_e)+M\sinh(\lambda_n L_e)} \tag{8}$$

$$A_n=\frac{4\sin(\lambda_n L_h)}{2\lambda_n L_h+\sin(2\lambda_n L_h)} \tag{9}$$

The eigenvalues λ_n can be calculated using Eq. (10), which is arranged from Eq. (5).

$$\begin{aligned}&fn1(\lambda_n)\cdot\sin(\lambda_n L_h)\\&-\sum_{i=1}^{\infty}\sum_{j=1}^{\infty}(C1_{ij}-M\cdot C2_{ij}\cdot C3\cdot\lambda_n^{2j})\ fn2_{ij}(\lambda_n)\\&+\sum_{i=1}^{\infty}\sum_{j=1}^{\infty}\sum_{k=1}^{i+2j-2}M\cdot C2_{ij}\cdot C4_k\cdot C5\cdot fn2_{ij}(\lambda_n)\\&+\sum_{i=1}^{\infty}\sum_{j=1}^{\infty}\sum_{k=1}^{i+2j-2}C6_{ijk}(C7_k-C8_k)fn3_{ij}(\lambda_n)=0\end{aligned} \tag{10}$$

where

$$fn1(\lambda_n)=\lambda_n\sinh(\lambda_n L_e)+M\cosh(\lambda_n L_e) \tag{11}$$

$$fn2_{ij}(\lambda_n)=\frac{(-1)^{j-1}\lambda_n^{2i+2j-3}}{(2i+4j-5)\,\Gamma(2i-1)\,\Gamma(2j-1)} \tag{12}$$

$$C1_{ij}=2\,M\,L_e^{2(i-1)}\,L_h^{2j-1}\sqrt{\frac{L_e^2}{4L_h^2}+1} \tag{13}$$

$$C2_{ij}=\frac{L_e^{2(2i+2j-3)}\Gamma(2i+4j-3)}{(-16)^{i+2j-2}L_h^{2i+2j-3}\{\Gamma(i+2j-1)\}^2} \tag{14}$$

$$C3=\cdot\ln\left(\frac{\sqrt{\frac{L_e^2}{4L_h^2}+1}+1}{\sqrt{\frac{L_e^2}{4L_h^2}+1}-1}\right) \tag{15}$$

$$C4_k=\frac{(-4)^k\,\Gamma(k+1)\cdot\Gamma(k)}{\left(\frac{L_e^2}{4L_h^2}\right)^k\Gamma(2k+1)} \tag{16}$$

$$C5=\sqrt{\frac{L_e^2}{4L_h^2}+1} \tag{17}$$

$$C6_{ijk}=\frac{\left(-\frac{L_e^2}{4L_h}\right)^{i+2j-k-2}\Gamma(i+2j-3)}{(2k+1)\Gamma(k)\Gamma(i+2j-k-1)} \tag{18}$$

$$C7_k=M^2\cdot\left(\frac{L_e^2}{4L_h}\right)^{k+\frac{1}{2}} \tag{19}$$

$$C8_k=2M^2\cdot\left(\frac{L_e^2}{4L_h}+L_h\right)^{k+\frac{1}{2}} \tag{20}$$

$$fn3_{ij}(\lambda_n)=\frac{(-1)^{j-1}\,L_e^{2i-1}\,\lambda_n^{2i+2j-3}}{L_h^{i-\frac{1}{2}}\,\Gamma(2i)\,\Gamma(2j-1)} \tag{21}$$

The eigenfunction, Eq. (10), is too long and complicated to obtain all the eigenvalues simultaneously. Therefore, the first eigenvalue, λ_1, is directly obtained from Eq. (10) by using an incremental search method. The remaining eigenvalues (i.e., λ_2, λ_3, λ_4, . . .) are calculated from Eq. (22) by using the Newton-Raphson method. Equation (22) is derived from the orthogonality principle used in the separation of variables method and is presented in Reference [6].

$$\lambda_n=(2\lambda_1+\lambda_n)-2(\lambda_1+\lambda_n)\frac{\tan(\lambda_n L_h)}{\tan(\lambda_1 L_h)+\tan(\lambda_n L_h)} \tag{22}$$

The heat loss from the symmetric concave parabolic fin is calculated from

$$q=\int_{-l_h}^{l_h}-k\frac{\partial T}{\partial x}\bigg|_{x=0}l_w\ dy \tag{23}$$

Then, the dimensionless heat loss {i.e., $Q=q/(k\phi_b l_w)$} from the fin can be expressed by

$$Q=\sum_{n=1}^{\infty}2\,sin(\lambda_n L_h)\cdot A_n\cdot g_n \tag{24}$$

The symmetric concave parabolic fin volume, as shown in Fig. 1, is given by

$$v = 2\int_0^{l_e} \frac{l_h}{l_e^2}\left(l_e - x\right)^2 l_w \, dx \tag{25}$$

The dimensionless fin volume {i.e., $V = v/\left(l_c^2 l_w\right)$} can be expressed as

$$V = \frac{v}{l_c^2 l_w} = \frac{2}{3} L_e L_h \tag{26}$$

Fin surface area for a symmetric concave parabolic fin with neglecting z-direction area is calculated by

$$a_{par} = 2\int_0^{l_h} \sqrt{\frac{l_e^2}{4 l_h y} + 1} \; l_w \, dy \tag{27}$$

The dimensionless fin surface area is presented by

$$A_{par} = \frac{a_{par}}{l_w l_c} = \frac{L_e^2}{4L_h} C3 + 2L_h \, \mathrm{C5} \tag{28}$$

3. RESULTS

The ratios of parabolic fin temperature to triangular fin temperature along the fin center line with the variation of X for three different convection characteristic numbers are listed in Table 1. As expected, the temperature of a parabolic fin is lower than that of a triangular fin at the same location. The ratios decrease as both X and M increase, but the range of variation for this ratio is not large. The ratios for M = 0.01 are almost 100 %, while the ratios for M = 0.2 decrease from 99.92 % to 98.75 % as X increases from 0.1 to 2.

Table 1. The ratio of parabolic fin temperature to triangular fin temperature (L_h =0.1, L_e =2)

	$\theta_{par} / \theta_{tri}$		
X	M =0.01	M =0.1	M =0.2
0.1	1.0000	0.9996	0.9992
0.5	0.9998	0.9980	0.9985
1	0.9997	0.9962	0.9917
1.5	0.9996	0.9950	0.9887
2	0.9996	0.9945	0.9875

Figure 2 presents the variations of temperatures along the fin height for parabolic and triangular fins. It should be noted that the height of a parabolic fin is lower than that of a triangular fin for the same X and L_e, and the heights of both parabolic and triangular fins for given X decrease as fin length decreases. The temperatures of parabolic and triangular fins decrease, slowly at first and then somewhat rapidly, as Y increases. The temperature of a parabolic fin is lower than that of a triangular fin at the same location. The temperature difference between a parabolic fin and a triangular fin for the same X and Y increases as fin length decreases.

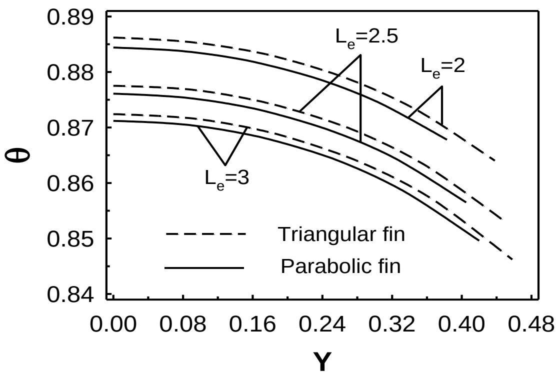

Fig. 2 Dimensionless temperature variation along fin height (M = 0.2, L_h = 0.5, X = 0.25)

Table 2 lists the ratios of fin surface area and heat loss with the variation of fin length for parabolic and triangular fins. The surface area of a triangular fin is calculated using an equation for the case of ξ_{sy} =1 in Reference [6]. The surface area of a parabolic fin is larger than that of a triangular fin, even though the volume of a parabolic fin is smaller than that of a triangular fin for a given fin length and fin base height. As expected, more heat is lost from a parabolic fin than a triangular fin since heat loss is proportional to the fin's surface area. The ratio of heat loss from a parabolic fin to heat loss from a triangular fin is slightly larger than that of the surface areas of the two fins. The ratio of fin surface area and the ratio of heat loss both decrease as fin length increases. Physically, this means that the effect of fin shape on the fin's surface area and heat loss decreases as fin length increases.

Table 2 Ratio of fin surface areas and ratio of heat loss with parabolic and triangular fins (L_h =0.1)

L_e	A_{par} / A_{tri}	M	Q_{par} / Q_{tri}
0.2	1.0266	0.001	1.0290
		0.01	1.0280
0.3	1.0149	0.001	1.0158
		0.01	1.0162
0.4	1.0092	0.001	1.0109
		0.01	1.0105
0.5	1.0061	0.001	1.0069
		0.01	1.0075
0.8	1.0025	0.001	1.0037
		0.01	1.0039
1.2	1.0011	0.001	1.0025
		0.01	1.0023

The modified heat loss, Q/M, as a function of fin length, L_e, is illustrated in Fig. 3. It is observed that modified heat loss increases rapidly as fin length decreases from about 1.35 to about 0.9. The reason for this phenomenon can be explained physically as follows. Because the dimensionless fin volume, V, is constant in this figure, L_h, which represents the half height at the fin base, will increase while L_e decreases. Obviously, the fin designs in this case are impractical, although the heat loss is very large. Another important phenomenon shown in Fig. 3 is that maximum heat loss may not be always obtainable. Optimum heat loss exists in the case of M = 0.1 and 0.15. There is no optimum heat loss in the practical range of a fin length in the case of M = 0.2. This trend can also be applied to fin volume, V. From now on, maximum heat loss will be referred to as optimum heat loss, Q^*, and the fin length and fin base height for the maximum heat will be referred as optimum fin length, L_e^* and optimum fin base height, L_h^*, respectively.

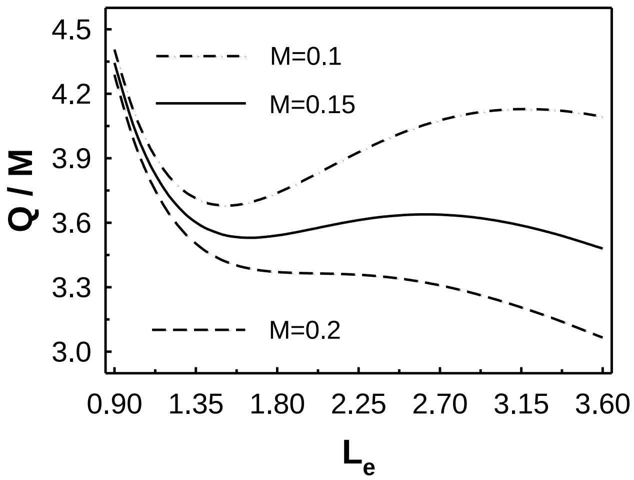

Fig. 3 Modified heat loss versus fin length (V =1.2)

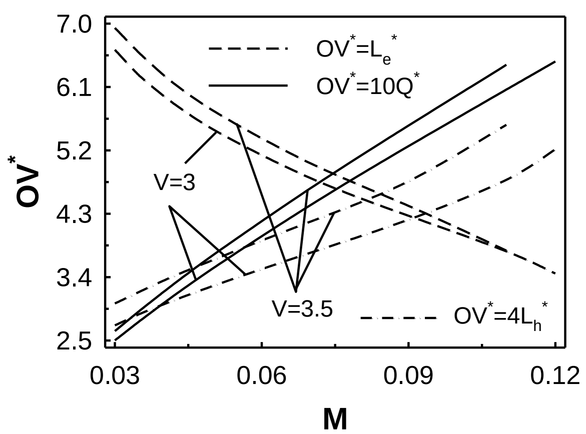

Fig.4 Optimum values as a function of the convection characteristic number

Figure 4 represents the variations of the optimum heat loss, the corresponding optimum fin length, and fin base height as a function of the convection characteristic number in the case of V = 3 and 3.5. It implies that the optimum heat loss exists for about $M \leq 0.1$ in the case of V = 3.5. Both the optimum heat loss and fin length increase as the convection characteristic number increases but the increasing ratio of the optimum heat loss is larger than that of the optimum fin length. Optimum fin base height decreases as the convection characteristic number increases because fin volume is fixed.

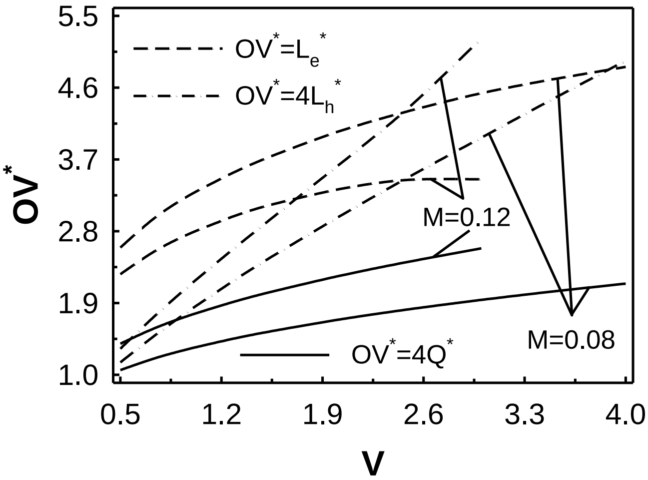

Fig. 5 Optimum values as a function of dimensionless fin volume

The variations of optimum heat loss, the corresponding optimum fin length, and optimum fin base height as a function of V are shown in Fig. 5. As expected, an increase in V enhances the optimum heat loss. It shows that optimum fin length increases rapidly at first and then levels off, while optimum fin base height increases steadily as fin volume increases. It can be noticed that the increasing ratio of optimum fin base height is larger than that of optimum fin length with the increasing of fin volume. Physically, this means that the optimum shape of a parabolic fin becomes "fatter" as fin volume increases.

4. CONCLUSIONS

From the optimization of a symmetric concave parabolic fin for a fixed fin volume, and the comparison of heat loss between a parabolic fin and a triangular fin, the following conclusions can be drawn.

1) As expected, the heat loss from a parabolic fin is a little but not much more than the heat loss from a triangular fin since the surface area of a parabolic fin is larger than that of a triangular fin for the same fin length and base height.
2) The ratio of heat loss from a parabolic fin to that from a triangular fin is slightly larger than the ratio of fin surface area of a parabolic fin to that of a triangular fin.

3) Optimum heat loss increases, while optimum fin length decreases as the convection characteristic number increases.

4) Optimum heat loss increases linearly as fin volume increases; optimum fin length increases rapidly at first and then levels off as fin volume increases.

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