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The Comparison Analysis about Reliability Features of Software Reliability Model Using Burr-XII and Type-2 Gumbel Lifetime Distribution

Hee-Cheul Kim

Associate Professor,
Department of Industrial & Management Engineering,
Namseoul University, South Korea.

Abstract:

The software permanence can be a significant and vital influence moving the dependability of computer organizations. Software distinctive form has a varied feature compared to hardware property in terms of development characteristics. Consequently, the failure of organizations to foresee software defects may lead to significant damages to software operators. Software reliability examination methods to decrease software defects during software progress course are elementary and an indispensable procedure. The software reliability information is useful material during software progress operation. In order to analyze the software failure existence, the hazard function from the nonhomogeneous Poisson process can require a constant, increasing or decreasing propensity over a certain failure period. This coursework was associated to the reliability presentation of the software reliability model using the Type- 2 Gumbel and Burr-XII lifetime distributions subsequent with the decreasing outline over period as the hazard function first increased in the software product testing. In order to investigate the features of the software reliability model, the parametric estimation method was applied to the maximum likelihood estimation technique. Consequently, software reliability features are associated and studied by spreading software failure time data. In terms of reliability, the Type-2 Gumbel lifetime distribution displays higher reliability than the Burr-XII lifetime distribution model. In addition, the Burr-XII model has more effectual points than the Type-2 Gumbel model in terms of model judgments by means of the mean square error and determination coefficient. In the procedure of software development, numerical modeling of fault occurrence in the process of using test software or actual software can be used to make relative efficiency assessment by comparing and analyzing software usability. Moreover, it is important that the software failure examination can support the software design using relating various life distributions.

Keywords: Software Reliability Model, Type-2 Gumbel Distribution, Burr-XII Distribution, NHPP, MLE

I. INTRODUCTION

Software safety terms can be an essential and required aspect moving the reliability of computer organizations. Software distinguishing points have a dissimilar aspect to hardware safety terms in points of design faces. Thus, the failure of computer system due to software defect may result in significant loss of possessions to software operators. Software reliability inspection procedures to decrease software defects during software progress development are basic and a vital preparation.

The software reliability suggestion provides useful and realistic points during software progress operation. Therefore, the failure of the computer organizations due to the imperfect conservation and characteristic influences in the software can cause enormous time and property destruction to the software users. Hence, the software reliability forecast that study the techniques to minimize software defect aspects during software development application should be well-organized tools. In this situation, software operators should satisfy reliability expectation and should contain minimum testing cost in terms of developers.

In the course of software expansion, quantitative modeling of fault occurrence property from the procedure of using testing cost of software or actual failure of software can be used to make comparative efficiency assessment by comparing and analyzing software usability.

One of the most significant problems of software organizations is to deliver high quality facility to customers secure usability and stability. However, software development is a hard and multifaceted process. Therefore, the main apprehension of software designers is to recover the stability of software organizations. This has led to the progress of a software stability engineering studies and software reliability growth model has been studied for the last several decades. In other words, to estimate the reliability features such as the number of residual failures and the failure degree, a software reliability model grounded on the non-homogeneous Poisson process (NHPP) using the defect intensity function and the mean value function in the controlled test environment has been developed [1]. The software reliability model is a secondhand to estimate and forecast the reliability of the software, the number of failures, the failure strength, and the total software progress cost.

Many software reliability forecasting study models have been projected in this field. Among many models, the software reliability model founded on the Non-Homogeneous Poisson Process (NHPP) [1] is a dependable software model that is reliable in terms of defect detection analysis. It removes immediately a failure that occurs. It also assumes that no new defects occur. Goel and Okumoto [2] planned an exponential software reliability growth model using the mean value function that follows the exponential form of the detected number of software defects. From this model, Huang [3] projected a method to analyze software reliability founded on generalized logistic testing effort function and change-point stricture. In this study, the reliability presentation of the software reliability model founded on the preliminary study was compared using the Type- 2 Gumbel and Burr-XII lifetime distributions subsequent with the decreasing shape over time as the hazard function first increased in the software produce testing..

II. BACKGROUND RESEARCH

2.1. Burr-XII distribution

The Burr-XII distribution [4] was originally planned by Burr (1942). This distribution, due to its extensive application in a variation of fields including reliability, failure period modeling and acceptance sampling planning, has been applied. Using this distribution, the probability density function and cumulative distribution function are as ensuing construction.

$$\backslash \ f(t\,|\,a,b)=\frac{abt^{a-1}}{(1+t^{a})^{b+1}}\ ,\quad F(t\,|\,a,b)=\left(1-(1+t^{a})^{-b}\right) \qquad (1)$$

Whereas $t\in(0,\infty]$ $a>0$ and $b\geq 1$ are the shape parameter.

Using the equations (1), the hazard function [5] can be specified as next progression.

$$h(t\,|\,a,b)=\frac{f(t\,|\,a,b)}{1-F(t\,|\,a,b)}=\frac{\left(\dfrac{abt^{a-1}}{(1+t^{a})^{b+1}}\right)}{(1+t^{a})^{-b}}=\frac{abt^{a-1}}{(1+t^{a})} \qquad (2)$$

In this paper, the situation where the shape parameter ($b=1$) was used for the express of the distribution function more briefly while keeping the characteristic of the hazard function of a Burr-XII distribution.

2.2. Type-2 Gumbel distribution

The Type-2 Gumbel distribution is a distribution that can be represented numerous reliability structures. The probability density function and cumulative distribution function rendering to the shape parameter (a) and shape parameter (b) was documented as the following construction [6].

$$f(t\,|\,a,b)=a\,b\,t^{-a-1}e^{-b\,t^{-a}}\ ,\ F(t\,|\,a,b)=e^{-b\,t^{-a}} \qquad (3)$$

Note. $t\in(0,\infty]$, $a>0$ and $b>0$ are the shape parameter

Using the equations (3), the hazard function [5] can be represented as ensuing structure.

$$h(t\,|\,a,b)=\frac{f(t\,|\,a,b)}{1-F(t\,|\,a,b)}=abt^{-a-1}\frac{e^{-b\,t^{-a}}}{1-e^{-b\,t^{-a}}} \qquad (4)$$

In the Type-2 Gumbel distribution, the circumstance where the shape parameter $a=1$ was used in order to more effortlessly represent the distribution functions while upholding the structure of the hazard function.

III. FINITE-FAILURE NHPP MODEL AND PARAMETER ESTIMATION

3.1. Burr-XII distribution NHPP model

In finite failure NHPP model, θ was specified the expected value of faults that would be discovered observing time $(0,t]$. The intensity function and the mean value function of the NHPP of the Burr-XII distribution model can be the result ensuing relationship set-up [5].

$$\lambda(t\,|\,a,b)=\theta\, f(t\,|\,a,b)=\theta\frac{abt^{a-1}}{(1+t^{a})^{b+1}}\ ,$$

$$m(t\,|\,a,b)=\theta\, F(t\,|\,a,b)=\theta\left(1-(1+t^{a})^{-b}\right) \qquad (5)$$

The likelihood function by means of the equation (5) can be detailed ensuing relation [7].

$$L_{NHPP}(\Theta|\underline{x})=\left[\prod_{i=1}^{n}\theta\frac{abx_i^{a-1}}{(1+x_i^{a})^{b+1}}\right]\exp\left[-\theta\left(1-(1+x_n^{a})^{-b}\right)\right] \qquad (6)$$

Note. $\underline{x}=(x_1\leq x_2\leq x_3\leq\ldots,\leq x_n)$, $\Theta=\{\theta,a,b\}$ specifies parameter space.

From log-likelihood function using the equation (6), when the shape parameter $b=1$, $\hat{\theta}_{MLE}$ and $\hat{a}_{MLE}$ can be measured as the solutions of the ensuing relations.

$$\frac{\partial \ln L_{NHPP}(\Theta \mid \underline{x})}{\partial \theta} = \frac{n}{\theta} - \left[1-\left(1+x_n^a\right)^{-b}\right] = 0 \tag{7}$$

In equation (7), solving for θ,

$$\hat{\theta}_{MLE} = \frac{n}{\left[1-\left(1+x_n^a\right)^{-b}\right]}$$

$$\frac{\partial \ln L_{NHPP}(\Theta \mid \underline{x})}{\partial a} = \frac{n}{a} - \sum_{i=1}^{n} \ln x_i - (b+1)\sum_{i=1}^{n} \frac{x_i^a \ln x_i}{1+x_i^a} - \theta b(1+x_n^a)^{-b-1} x_n^a \ln x_n = 0 \tag{8}$$

3.2. Type-2 Gumbel distribution NHPP model

In the finite-fault NHPP model, the intensity function and the mean value function of the NHPP of the Type-2 Gumbel distribution model by means of the equation (5) and the equation (6) can be assessed in the ensuing relationship set-up [8].

$$\lambda(t \mid \theta, a, b) = \theta f(t) = \theta\, a\, b\, t^{-a-1} e^{-bt^{-a}} \tag{9}$$

$$m(t \mid \theta, a, b) = \theta F(t) = \theta e^{-bt^{-a}} \tag{10}$$

Note that θ the expected value of the defect that can be specified finite failure time. If the time truncated model [9] is used to the reflection time $(0, t]$, the likelihood function can be resulting structure by the ensuing relation by means of the equations (9) and (10).

$$L_{NHPP}(\Theta \mid \underline{x}) = \prod_{i=1}^{n} \lambda(x_i) \exp[-m(x_n)] = \prod_{i=1}^{n} \theta \left[a\, b\, x_i^{-a-1} e^{-b x_i^{-a}}\right] \times \exp\left[-\theta e^{-b x_n^{-a}}\right] \tag{11}$$

Note. $\underline{x} = (x_1 \le x_2 \le x_3 \le \ldots, \le x_n)$, $\Theta = \{\theta, a, b\}$ specifies parameter space.

When the shape parameter $a=1$ is can be fixed, the estimator $\hat{\theta}_{MLE}$ and $\hat{b}_{MLE}$ must be assessed the following structure for the maximum likelihood estimation about all parameter by means of the equation (11).

$$\frac{\partial \ln L_{NHPP}(\Theta \mid \underline{x})}{\partial \theta} = \frac{n}{\theta} - e^{-b x_n^{-1}} = 0 \tag{12}$$

From equation (12), solving for θ,

$$\hat{\theta}_{MLE} = \frac{n}{e^{-b x_n^{-1}}}$$

$$\frac{\partial \ln L_{NHPP}(\Theta \mid \underline{x})}{\partial b} = \frac{n}{b} - \sum_{i=1}^{n} x_n^{-1} + \theta x_n^{-1} e^{-b x_n^{-1}} = 0 \tag{13}$$

IV. ANALYSIS OF RELIABILITY ATTRIBUTES USING SOFTWARE FAILURE TIME

In this section, the reliability structures of the software reliability model were studied using the software failure time data [10]. The failure time data is revealed in Table 1. Furthermore, a trend test should be headed in order to assure reliability of data [8, 11]. In this study, the trend analysis used was the Laplace trend test. The consequences of the Laplace trend test in Figure 1 displays that the approximations of the Laplace factor were spread between -2 and 2, which means that extreme values scarcely occur, therefore it is sensible to recommend a reliability model using this data [8, 11].

Table 1. Failure time data

Failure Number	*Failure Time(hours)*	**Failure Number**	*Failure Time(hours)*
1	9	16	92
2	21	17	95
3	32	18	98
4	36	19	104
5	43	20	105
6	45	21	116
7	50	22	149
8	58	23	156
9	63	24	247
10	70	25	249
11	71	26	250
12	77	27	337
13	78	28	384
14	87	29	396
15	91	30	405
		31	540

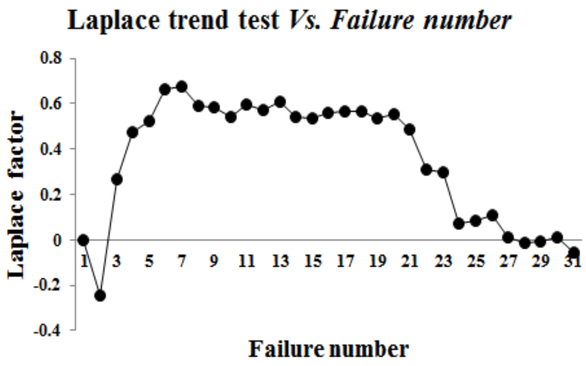

Fig 1. Test outcome of Laplace trend

The hazard function using (2) and (4) is abridged in Figure 2. In this figure, both the Type-2 Gumbel and Burr-XII distribution models are increasing at first and then decreasing. However, the Burr-XII model has the higher estimated value than the Type-2 Gumbel during the applied failure time.

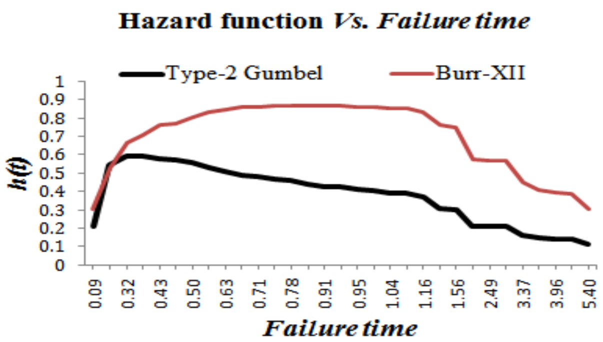

Fig 2. Trend of hazard function

Table 2. Parameter estimation of each model

Model	*MLE*		Model Comparison	
			MSE	R^2
Burr-XII	$\hat{\theta}_{MLE} = 32.726$	$\hat{a}_{MLE} = 1.713$	2.144	0.978
Type-2 Gumbel	$\hat{\theta}_{MLE} = 34.093$	$\hat{b}_{MLE} = 0.691$	2.217	0.976

Note. MLE Maximum likelihood estimation.
MSE : Mean square error. R^2 : Coefficient of determination.

The parameter approximation was used to the traditional maximum likelihood method and numerically changes the original failure time data $(Failure\ time \times 10^{-2})$ to safeguard the convergence of the parameter approximation. In the calculating method of nonlinear equations the bisection technique was used which is a numerical method. A result of the parameter approximation was reached from the Table 2. In this section, results of parameter approximation were itemized in Table 2. These controls solve the root exactly, since the initial values were specified 0.001 and 5.000, and the tolerance value for the measurement of interval (10^{-5}) were specified, with an accomplished replication of 100 times using C-language checking satisfactory convergent. The mean squared error (*MSE*), which was used in the model contrast in Table 2, is the measure of the difference of the between the actual observation and the forecast value [7, 12].

$$MSE = \frac{\sum_{i=1}^{n} [m(x_i) - \hat{m}(x_i)]^2}{n-k} \tag{14}$$

Similarly, R^2 (coefficient of determination) [9, 12] stipulates the predictive degree of the difference among the predicting values

$$R^2 = 1 - \frac{\sum_{i=1}^{n} [m(x_i) - \hat{m}(x_i)]^2}{\sum_{i=1}^{n} \left(m(x_i) - \sum_{j=1}^{n} m(x_j)/n \right)^2} \tag{15}$$

Note that $m(x_i)$ is the occupied cumulated number of the faults noticed in $(0, x_i]$ and $\hat{m}(x_i)$ approximating full cumulated number of the faults noticed in $(0, x_i]$, n specifies the number of realizing values and k is the number of the parameter. In other words, if the mean square error value is small in terms of the comparative degree, it is regarded as a relatively effectual model. As a result, since the Burr-XII distribution model has a smaller value than the Type-2 Gumbel model for the mean square error estimate on the scale used in Table 2, the Burr-XII distribution model is an effectual model. In order to settle this situation, a summary picture of the comparison of estimated values of square error ($SE = [m(x_i) - \hat{m}(x_i)]^2, i = 1, 2 \ldots, 30$) for each time points are abridged in Figure 3.

In the comparison of the estimated values of the square error in Figure 3, the Type-2 Gumbel model illustrates a relatively higher estimated value in the first half than the Burr-XII distribution model in the first half, and the Burr-XII distribution model is higher in the latter half. That is, if the decision coefficient estimation value would be large in comparison, it becomes a relatively effective model.

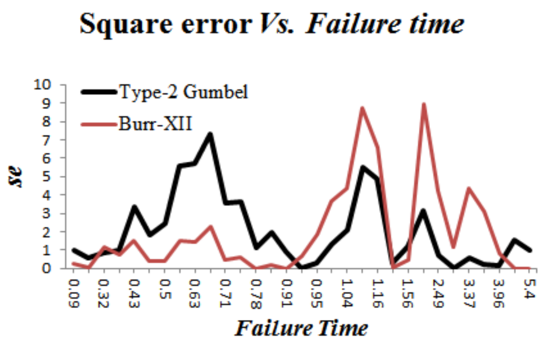

Fig.3: Estimation of square error for each time

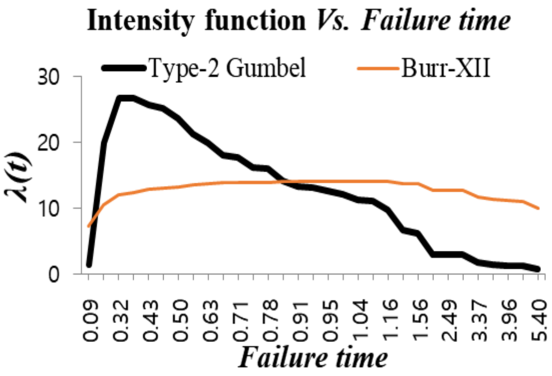

Fig 4. Pattern of intensity function for each model

And, the resulting estimation value of intensity functions are abridged in Figure 4. In this figure, intensity functions of Type-2 Gumbel have the propensity of increasing at first and then decreasing. However, the Burr-XII has the tendency of a closely constant form. Figure 5 displays tendencies in the pattern for the mean value function following the Burr-XII distribution model and the Type-2 Gumbel model. The second half of the Burr-XII distribution model was predicted to have a smaller difference from the observed value than the Type-2 Gumbel model in the first half. The Type-2 Gumbel distribution model was different from the true value in the second half. Overall, the difference from observed value Burr-XII distribution model was smaller than Type-2 Gumbel model.

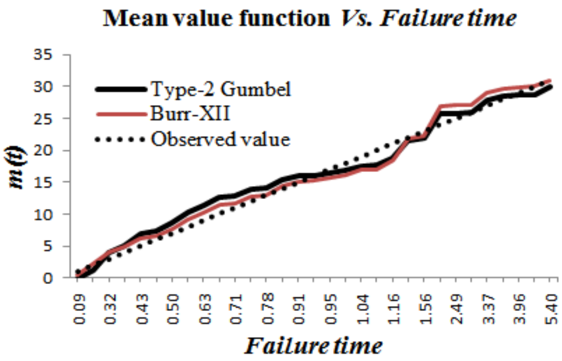

Fig 5. Pattern of mean value function for each model

In the NHPP model, a software failure occurs at the time of testing x_{31} and reliability, which is the probability that a software failure does not occur between 540×10^{-2} and $540\times10^{-2}+t$ (where t is the mission time, $540\times10^{-2}=5.4$) can be stated using the ensuing construction [7].

$$\hat{R}(t\,|\,5.4) = e^{-\int_{5.4}^{5.4+t}\lambda(\eta)\,d\eta} = \exp\left[-\{m(t+5.4)-m(5.4)\}\right] \qquad (16)$$

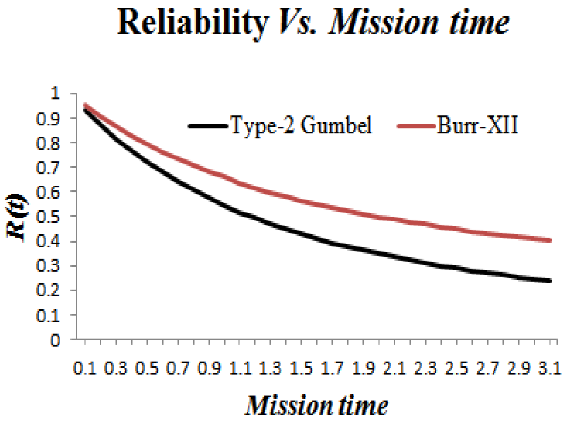

Fig 6. Transition of reliability Pattern

As shown in Figure 6, the Type-2 Gumbel model is further reliable than the Burr XII distribution over mission time by means of the equation (16).

V. CONCLUSION

Software stability can be a vital and essential factor which affects the reliability of computer systems. Software consumes a dissimilar aspect to hardware stability in terms of design characteristics. Thus, a failure of a computer organization due to a software defect may result in marvelous damage of property to software operators. Software reliability analysis schemes to decrease software defects during software development procedure are elementary and important.

Software organizations have become a fundamental part of our life style. One of the most significant problems of software organizations is to offer high quality service to customers with secure usability and stability. However, software development is a difficult and multifaceted procedure. Therefore, the main apprehension of software designers is to progress the stability of software organizations. This has led to the progress of a software stability engineering studies and software reliability growth model has been studied for the last several decades. In other words, to estimate the reliability characteristics such as the number of residual failures and the failure rate, a software reliability model founded on the non-homogeneous Poisson process (NHPP) using the defect intensity function and the mean value function in the controlled test situation has been developed. In the procedure of software development, numerical modeling of fault occurrence in the process of using test software or actual software can be used to make relative efficiency assessment by comparing and analyzing software usability.

In this study, was compared the Type-2 Gumbel distribution model and the Burr-XII distribution model that follow the lifetime distribution that software designers can use to grasp software fault characteristics.

The findings of this study were as follows: First, in terms of the mean squared error estimation value which is a measure of the difference for between the actual observation value and the prediction value, since the Burr-XII distribution model has a smaller value than the Type-2 Gumbel model, the Burr-XII distribution can be judged as an efficient model. In addition, Burr-XII distribution model is further efficient than Type-2 Gumbel model because Burr-XII distribution model displays higher estimation value than Type-2 Gumbel in terms of estimated coefficient of determination. Second, in hazard function, both the Type-2 Gumbel and Burr-XII distribution models are increasing at first and then decreasing. However, the Burr-XII model has a higher estimated value than the Type-2 Gumbel for the particular failure time. In addition, in terms of intensity function, intensity function pattern of Type-2 Gumbel has the propensity of increasing at first and then decreasing. However, the Burr-XII has the tendency of a closely constant form. Third, in the mean value function following the Burr-XII distribution model and the Type-2 Gumbel model, the second half of the Burr-XII distribution model was predicted to have a smaller difference points from the observed data than the Type-2 Gumbel model in the first half. The Type-2 Gumbel and Burr-XII distribution model were different from the observed data in the second half. Overall, in terms of mean value function about difference from true value, Burr-XII distribution model has smaller estimation value than Type-2 Gumbel model. Fourth, reliability of Type-2 Gumbel model is higher than Burr-XII distribution over mission time. Using content of this study, it can be concluded that the software design segment can assist the software design by using the software failure analysis and applying several life distributions.

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