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State-Space Approach to Generalized Thermoelastic Half-Space Subjected to a Ramp-Type Heating and Harmonic Mechanical Loading

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Abstract

This paper is a study of thermoelastic interactions in an elastic half-space at an elevated temperature field arising out from a ramp-type heating and harmonic loading on the bounding surface. The governing equations are taken in a unified system in which the field equations of coupled thermoelasticity as well as of generalized thermoelasticity can be easily obtained as special cases. Special attention has been paid to the finite time of rise of temperature. The problem has been solved analytically by using a state-space approach. The derived analytical expressions have been computed for a specific situation. Numerical results for the temperature distribution, thermal stress and displacement components are represented graphically. A comparison was made with the results predicted by the three theories.

Keywords: Thermoelasticity; Generalized Thermoelasticity; Ramp-Type Heating; State-Space Approach; periodical loading

NOMENCLATURE

λ , μ	Lame's constants
ρ	Density
C_E	Specific heat at constant strain
t	Time
T	Temperature
T_o	Reference temperature
σ_{ij}	Components of stress tensor
e_{ij}	Components of strain tensor
u_i	Components of displacement vector
F_i	Body force vector
K	Thermal conductivity
Q	Heat source
τ_o, υ	Relaxation times

1. INTRODUCTION

Serious attention has been paid for the last three decades to the generalized thermoelasticity theories in solving thermoelastic problems in place of the classical uncoupled /coupled theory of thermoelasticity. The heat conduction equation for uncoupled thermoelasticity without any elasticity term in appears to be unphysical, since due to the mechanical loading of an elastic body, the strain so produced causes variation in the temperature field. Moreover, the parabolic type of the heat conduction equation results in an infinite velocity of the thermal wave propagation which also contradicts the actual physical phenomena. Introducing the strain-rate term in the uncoupled heat conduction equation, Biot [1] extended the analysis to incorporate coupled thermoelasticity. In this way, although the first shortcoming was over, there remained the parabolic type partial differential equation of the heat conduction, which leads to the paradox of the infinite velocity of the thermal wave. To eliminate this paradox generalized thermoelasticity theory has been developed subsequently. The development of this theory was accelerated by the advent of the second sound effects observed experimentally by Ackerman [2, 3] in materials at a very low temperature. In heat transfer problems involving very short time intervals and/or very high heat fluxes, it has been revealed that the inclusion of the second sound effects to the original theory yields results which are realistic and very much different from those obtained with classical theory of elasticity.

Becouse of the advancement of pulsed lasers, accelerators, fast burst nuclear reactors and particle, etc. which can supply heat pulses with a very fast time-rise [4,5], generalized thermoelasticity theory is receiving serious attention of different researchers. The development of the second sound effect has been reviewed by Chandrasekharaih [6]. At present mainly two different models of generalized thermoelasticity are being extensively used-one proposed by Lord and Shulman [7] and the other proposed by Green and Lindsay [8]. The L-S theory suggests one relaxation time and according to this theory only Fourier's heat conduction equation is modified; while G-L theory suggests two relaxation times and both the energy equation and the equation of motion get modified. Contrary to the L-S theory, the G-L theory does not violate Fourier's law of heat conduction when the solid has a centre of symmetry.

A method for solving coupled thermoelastic problems by using the state-space approach in which the problem cast into the state-space variables, namely the temperature, the displacement and their gradients has been developed by Bahar and Hetnarski [9-11]. State space methods are the cornerstone of modern control theory. The essential feature of state space methods is the characterization of the processes of interest by differential equations instead of transfer functions. This may seem like a throwback to the earlier, primitive, period where differential equations also constituted the means of representing the behavior of dynamic processes. But in the earlier period the processes were simple enough to be characterized by a single differential equation of fairly low order. In the modern approach the processes are characterized by system of coupled, first order differential equations. In principle there is no limit to the order (i.e., the number of independent first order differential equations) and in practice the only limit to the order is the availability of computer software capable of performing the required calculations reliably.

The importance of state space analysis is recognized in fields where the time behavior of any physical process is of interest. The state space approach is more general than the classical Laplace and Fourier transform theory. Consequently, state space theory is applicable to all systems that can analyzed by integral transforms in time, and is applicable to many systems for which transform theory breaks down. Furthermore, state space theory gives a somewhat different insight into the time behavior of linear systems.

In particular, the state space approach is useful because: (i) linear system with time-varying parameters can be analyzed in essentially the same manner as time-invariant linear system, (ii) problems formulated by state space methods can easily be programmed on a computer, (iii) high-order linear systems can be analyzed, (iv) multiple input-multiple output systems can be treated almost as easily as single input-single output linear systems, and (v) state space theory is the foundation for further studies in such areas as nonlinear systems, stochastic systems, and optimal control. These are five of the most important advantages obtained from the generalization and rigorousness that state space brings to the classical transform theory [9-11].

Erbay and Suhubi [12] studied the longitudinal wave propagation in an infinite circular cylinder which is assumed to be made of the generalized thermoelastic material and thereby obtained the dispersion relation when the surface temperature of the cylinder was kept constant. Generalized thermoelasticity problems for an infinite body with a circular cylindrical hole and for an infinite solid cylinder were solved respectively by Furukawa et al. [13, 14]. A problem of generalized thermoelasticity was solved by Sherief [15] by adopting the state-space approach. Chandrasekharaiah and Murthy [16] studied the thermoelastic interactions in an isotropic homogeneous unbounded linear thermoelastic body with a spherical cavity, in which the field equations were taken in unified forms covering the coupled, L-S and G-L models of thermoelasticity. The effects of mechanical and thermal relaxations in a heated viscoelastic medium containing a cylindrical hole were studied by Misra et al. [17]. Investigations concerning interactions between magnetic and thermal fields in deformable bodies were carried out by Maugin [18] as well as by Eringen and Maugin [19]. Subsequently Abd-Alla and Maugin [20] conducted a generalized theoretical study by considering the mechanical, thermal and magnetic field in centro-symmetric magnetizable elastic solids.

Many problems which have been solved, were in the context of the theory of L-S; El-Maghraby and Youssef [21] used the state space approach to solve a thermomechanical shock problem using. Sherief and Youssef [22] get the short time solution for a problem in magnetothermoelasticity. Youssef [23] constructed a model of the dependence of the modulus of elasticity and the thermal conductivity on the reference temperature and solved a problem of an infinite material with a spherical cavity

It is more useful to mention here that in most of the earlier studies, mechanical or thermal loading on the bounding surface is considered to be in the form of a shock. But the sudden jump of the load is merely an idealized situation because it is impossible to realize a pulse described mathematically by a step function; even very rapid rise-time (of the order of 10-9 s) may be slow in terms of the continuum. This is particularly true in the case of second sound effects when the thermal relaxation times for typical metals are less than 10-9 s. It is thus felt that a finite time of rise of external load (mechanical or thermal) applied on the surface should be considered while studying a practical problem of this nature. Considering this aspect of rise of time, Misra et al. [24-26] solved some problems subjected to a ramp-type heating at the bounding surface.

The present investigation is devoted to a study of the induced temperature and stress fields in an elastic half space under the purview of classical coupled thermoelasticity and generalized thermoelasticity in a unified system of field equations. The semi-infinite continuum is considered to be made of an isotropic homogeneous thermoelastic material, the bounding plane surface being subjected to periodic loading and a ramp-type heating. The rationale behind the study of such a type of heating is that the temperature of the bounding surface cannot be elevated instantaneously-a finite time of rise of temperature is required for this purpose. By adopting the state-space approach [15] an exact solution of the problem is first obtained in Laplace transform space. Since the response is of more interest in the transient state, the inversions have been carried out numericaly. The derived expressions are computed numerically for copper and the results are presented in graphical form.

2. BASIC EQUATIONS AND FORMULATION

In the context of coupled thermoelasticity (CTE), the displacement and the thermal fields as well as the stress-strain-temperature relations for a linear homogeneous and isotropic medium [1]:

$$\mu u_{i,jj} + (\lambda + \mu) u_{j,ji} + F_i - \gamma T_{,i} = \rho \ddot{u}_i , \tag{1}$$

$$K T_{,ii} = \rho C_E \dot{T} + T_o \gamma \dot{u}_{j,j} - \rho Q , \tag{2}$$

$$\sigma_{ij} = \mu\left(u_{i,j} + u_{j,i}\right) + \left(\lambda u_{i,i} - \gamma T\right)\delta_{ij}. \tag{3}$$

In the generalized thermoelasticity (GTE) theory developed by Lord and Shulman, only the heat conduction equation given by (2.2) is modified to the form [7]:

$$K T_{,ii} = \left(1 + \tau_o \frac{\partial}{\partial t}\right)\left(\rho C_E \dot{T} + T_o \gamma \dot{u}_{i,i} - \rho Q\right), \tag{4}$$

while equations (1) and (3) remain unchanged. According to the generalized thermoelasticity (GTE) theory developed by Green and Lindsay (G-L), equations (1)-(3) are replaced by [8]:

$$\mu u_{i,jj} + (\lambda + \mu) u_{j,ji} + F_i - \gamma\left(1 + \upsilon \frac{\partial}{\partial t}\right) T_{,i} = \rho \ddot{u}_i, \tag{5}$$

$$K T_{,ii} = \rho C_E \left(1 + n \tau_o \frac{\partial}{\partial t}\right)\dot{T} + T_o \gamma \dot{u}_{j,j} - \rho Q, \tag{6}$$

$$\sigma_{ij} = \mu\left(u_{i,j} + u_{j,i}\right) + \left[\lambda u_{i,i} - \gamma\left(T + \upsilon \dot{T}\right)\right]\delta_{ij}. \tag{7}$$

All the field equations represented by (1)-(7) can be formulated in a unified system as

$$\mu u_{i,jj} + (\lambda + \mu) u_{j,ji} + F_i - \gamma\left(1 + \upsilon \frac{\partial}{\partial t}\right) T_{,i} = \rho \ddot{u}_i, \tag{8}$$

$$K T_{,ii} = \rho C_E \left(\frac{\partial}{\partial t} + \tau_o \frac{\partial^2}{\partial t^2}\right) T + \left(1 + n \tau_o \frac{\partial}{\partial t}\right)\left(T_o \gamma \dot{u}_{j,j} - \rho Q\right), \tag{9}$$

$$\sigma_{ij} = \mu\left(u_{i,j} + u_{j,i}\right) + \left[\lambda u_{i,i} - \gamma\left(T + \upsilon \dot{T}\right)\right]\delta_{ij}. \tag{10}$$

Equations (8)-(10) reduce to (1)-(3) (CI'E) when $\tau_o = \upsilon = 0$. Putting $n = 1$, $\upsilon = 0$ and $\tau_o > 0$, the equations reduce to (1), (4) and (3) for the L-S model, while when $n = 0$, $\tau_o > 0$ and $\upsilon > 0$, the equations reduce to (5)-(7) for the G-L model.

3. STATEMENT OF THE PROBLEM AND THE GOVERNING EQUATIONS

Let us consider a perfectly conducting elastic half space $x \geq 0$ of an isotropic homogeneous material medium whose state can be expressed in terms of the space variable x and the time variable t. The medium described above is considered to be exposed to ramp-type surface heating described mathematically as:

$$T(0,t) = \begin{cases} 0 & t \leq 0 \\ T_1 \dfrac{t}{t_o} & 0 < t \leq t_o \\ T_1 & t > t_o \end{cases} \tag{11}$$

T_1 being a constant. It is assumed that there are no body forces and no heat sources in the Meduim and that the plane $x = 0$ is taken to be subject to a periodic loading of frequency ω, i.e.

$$\sigma(0,t) = -\sigma_o e^{i\omega t} \tag{12}$$

where σ_o is constant.

Thus the field equations (8)-(10) in one dimensional case can be put as

$$(\lambda + 2\mu)\frac{\partial^2 u}{\partial x^2} - \gamma \frac{\partial}{\partial x}\left(1 + \upsilon \frac{\partial}{\partial t}\right) T = \rho \frac{\partial^2 u}{\partial t^2}, \tag{13}$$

$$\frac{\partial^2 T}{\partial x^2} = \frac{\rho C_E}{K}\left(\frac{\partial}{\partial t} + \tau_o \frac{\partial^2}{\partial t^2}\right) T + \frac{T_o \gamma}{K}\left(\frac{\partial}{\partial t} + n \tau_o \frac{\partial^2}{\partial t^2}\right)\frac{\partial u}{\partial x}, \tag{14}$$

$$\sigma_{xx} = (\lambda + 2\mu)\frac{\partial u}{\partial x} - \gamma\left(1 + \upsilon \frac{\partial}{\partial t}\right) T. \tag{15}$$

For convenience, we shall use the following non-dimensional variables:

$$(x', u') = c_o \eta (x, u), \quad (t', t'_o, \tau'_o, \upsilon') = c_o^2 \eta (t, t_o, \tau_o, \upsilon),$$

$$(\theta, \theta_1) = \frac{(T, T_1)}{T_o}, \ \sigma' = \frac{\sigma}{(\lambda + 2\mu)}, \ \Omega = \frac{\omega}{c_o^2 \eta}$$

where $c_o^2 = \dfrac{\lambda + 2\mu}{\rho}$ and $\eta = \dfrac{\rho C_E}{K}$.

Equations (13)-(15) assume the form (where the primes are suppressed for simplicity)

$$\frac{\partial^2 u}{\partial x^2} - a \frac{\partial}{\partial x}\left(1 + \upsilon \frac{\partial}{\partial t}\right)\theta = \frac{\partial^2 u}{\partial t^2}, \tag{16}$$

$$\frac{\partial^2 \theta}{\partial x^2} = \left(\frac{\partial}{\partial t} + \tau_o \frac{\partial^2}{\partial t^2}\right)\theta + \varepsilon\left(\frac{\partial}{\partial t} + n \tau_o \frac{\partial^2}{\partial t^2}\right)\frac{\partial u}{\partial x}, \tag{17}$$

$$\sigma_{xx} = \frac{\partial u}{\partial x} - a\left(1 + \upsilon \frac{\partial}{\partial t}\right)\theta, \tag{18}$$

where $a = \dfrac{\gamma T_o}{\lambda + 2\mu}$, $\varepsilon = \dfrac{\gamma}{\rho C_E}$ and $\gamma = (3\lambda + 2\mu)\alpha_T$

The nondimensional forms of the boundary conditions are:

$$\theta(0,t) = \begin{cases} 0 & t \leq 0 \\ \theta_1 \dfrac{t}{t_o} & 0 < t \leq t_o \\ \theta_1 & t > t_o \end{cases}, \tag{19}$$

$$\sigma(0,t) = -\sigma_o e^{i\Omega t}, \tag{20}$$

To solve the equations (16)-(18) under conditions (19) and (20) we use the method of Laplace trnasform. In the transform space the equations (16)-(18) read

$$\frac{\partial^2 \bar{u}}{\partial x^2} - \alpha_1 \bar{u} = \alpha_2 \frac{\partial \bar{\theta}}{\partial x}, \tag{21}$$

$$\frac{\partial^2 \bar{\theta}}{\partial x^2} - \alpha_3 \bar{\theta} = \alpha_4 \frac{\partial \bar{u}}{\partial x}, \tag{22}$$

$$\bar{\sigma}_{xx} = \frac{\partial \bar{u}}{\partial x} - \alpha_2 \bar{\theta}, \tag{23}$$

$$\bar{\theta}(0,s) = \alpha_5(s), \tag{24}$$

$$\bar{\sigma}(0,s) = \alpha_6(s), \tag{25}$$

where $\alpha_1(s) = s^2$, $\alpha_2(s) = a(1+\upsilon s)$, $\alpha_3(s) = s + \tau_o s^2$, $\alpha_4(s) = \varepsilon\left(s + n\,\tau_o s^2\right)$, $\alpha_5(s) = \dfrac{\theta_1\left(1-e^{-st_o}\right)}{t_o s^2}$ and

$\alpha_6(s) = \dfrac{-\sigma_o}{s - i\Omega}$,

and an overbar symbol denotes its Laplace transform and s denots the Laplace transform parameter, writing

$$\frac{\partial \bar{\theta}(x,s)}{\partial x} = \bar{\theta}'(x,s), \tag{26}$$

$$\frac{\partial \bar{u}(x,s)}{\partial x} = \bar{u}'(x,s), \tag{27}$$

Choosing as state variable the temperature increment, the displacement component in the x-direction and their gradient, then equations (21) and (22) can be written in matrix form as:

$$\frac{d\bar{V}(x,s)}{dx} = A(s)\bar{V}(x,s), \tag{28}$$

where

$$\bar{V}(x,s) = \begin{bmatrix} \bar{u}(x,s) \\ \bar{\vartheta}(x,s) \\ \bar{u}'(x,s) \\ \bar{\vartheta}'(x,s) \end{bmatrix}, \quad A(s) = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ \alpha_1 & 0 & 0 & \alpha_2 \\ 0 & \alpha_3 & \alpha_4 & 0 \end{bmatrix}, \tag{29}$$

the formal solution of system (28) can be written in the form

$$\bar{V}(x,s) = \exp[A(s)x]\bar{V}(0,s). \tag{30}$$

We will use the well-known Cayley-Hamilton theorem to find the form of the matrix exp (A(s) x). The characteristic equation of the matrix A(s) can be written as

$$k^4 - [\alpha_1 + \alpha_2\alpha_4 + \alpha_3]k^2 + \alpha_1\alpha_3 = 0, \tag{31}$$

the roots of this equation, namely, $\pm k_1$ and $\pm k_2$, satisfy the relations

$$k_1^2 + k_2^2 = \alpha_1 + \alpha_2\alpha_4 + \alpha_3, \tag{32}$$

$$k_1^2 k_2^2 = \alpha_1\alpha_3. \tag{33}$$

The Taylor series expansion of the matrix exponential has form

$$\exp[A(s)x] = \sum_{n=0}^{\infty} \frac{[A(s)x]^n}{n!}. \tag{34}$$

Using Cayley-Hamelton theorem again, we can express A^4 and higher orders of the matrix A in terms of I, A, A^2, and A^3, where I is the unit matrix of fourth order.

Thus, the infinite series in equation (34) can be reduced to

$$\exp[A(s)x] = L(x,s) = a_o I + a_1 A + a_2 A^2 + a_3 A^3, \tag{35}$$

where a_o-a_3 are some coefficients depending on *x* and *s*. By Cayley-Hamilton theorem, the characteristic roots $\pm k_1$ and $\pm k_2$ of the matrix *A* must satisfy equation (35), thus

$$\exp(k_1 x) = a_o + a_1 k_1 + a_2 k_1^2 + a_3 k_1^3, \tag{36a}$$

$$\exp(-k_1 x) = a_o - a_1 k_1 + a_2 k_1^2 - a_3 k_1^3, \tag{36b}$$

$$\exp(k_2 x) = a_o + a_1 k_2 + a_2 k_2^2 + a_3 k_2^3, \tag{36c}$$

$$\exp(-k_2 x) = a_o - a_1 k_2 + a_2 k_2^2 - a_3 k_2^3. \tag{36d}$$

The solution of this system is given by

$$a_o = \frac{k_1^2 \cosh(k_2 x) - k_2^2 \cosh(k_1 x)}{k_1^2 - k_2^2}, \tag{37a}$$

$$a_1 = \frac{\dfrac{k_1^2}{k_2}\sinh(k_2 x) - \dfrac{k_2^2}{k_1}\sinh(k_1 x)}{k_1^2 - k_2^2}, \tag{37b}$$

$$a_2 = \frac{\cosh(k_1 x) - \cosh(k_2 x)}{k_1^2 - k_2^2}, \tag{37c}$$

$$a_3 = \frac{k_2 \sinh(k_1 x) - k_1 \sinh(k_2 x)}{k_2 k_1 \left(k_1^2 - k_2^2\right)}. \tag{37d}$$

Now, we have

$$\exp[A(s)x] = L(x,s) = \left[\ell_{ij}(x,s)\right] \quad i,j = 1,2,3,4, \tag{38}$$

where the components $\ell_{ij}(x,s)$ are given by

$$\ell_{11} = a_0 + a_2\alpha_1,\ \ell_{12} = a_3\alpha_2\alpha_3,\ \ell_{13} = a_1 + a_3(\alpha_1 + \alpha_2\alpha_4),\ \ell_{14} = a_2\alpha_2$$

$$\ell_{21} = a_3\alpha_1\alpha_4,\ \ell_{22} = a_0 + a_2\alpha_3,\ \ell_{23} = a_2\alpha_4,\ \ell_{24} = a_1 + a_3(\alpha_3 + \alpha_2\alpha_4)$$

$$\ell_{31} = a_1\alpha_1 + a_3\alpha_1(\alpha_1 + \alpha_2\alpha_4),\ \ell_{32} = a_2\alpha_2\alpha_3,\ \ell_{33} = a_0 + a_2(\alpha_1 + \alpha_2\alpha_4),$$

$$\ell_{34} = a_1\alpha_2 + a_3\alpha_2(\alpha_1 + \alpha_2\alpha_4 + \alpha_3), \ell_{41} = a_2\alpha_1\alpha_4,\ \ell_{42} = a_1\alpha_3 + a_3\alpha_3(\alpha_2\alpha_4 + \alpha_3)$$

$$\ell_{43} = a_1\alpha_4 + a_3(\alpha_1\alpha_4 + \alpha_4(\alpha_2\alpha_4 + \alpha_3)),\ \ell_{44} = a_0 + a_2(\alpha_2\alpha_4 + \alpha_3). \tag{39}$$

Since the intent is that the solution vanishes at infinity, the positive exponentials in equations (37) should be rejected. This is done by replacing each $\cosh(k_i x)$ by $\frac{1}{2}\exp(-k_i x)$ and each $\sinh(k_i x)$ by $-\frac{1}{2}\exp(-k_i x)$, $i = 1, 2$ in equations (37).

The new values of $a_0,\ a_1,\ a_3$ and a_4 expressed as:

$$a_o = \frac{k_1^2 e^{-k_2 x} - k_2^2 e^{-k_1 x}}{2(k_1^2 - k_2^2)},\quad a_1 = \frac{\frac{k_2^2}{k_1} e^{-k_1 x} - \frac{k_1^2}{k_2} e^{-k_2 x}}{2(k_1^2 - k_2^2)},\quad a_2 = \frac{e^{-k_1 x} - e^{-k_2 x}}{2(k_1^2 - k_2^2)},\quad a_3 = \frac{k_1 e^{-k_2 x} - k_2 e^{-k_1 x}}{2 k_2 k_1 (k_1^2 - k_2^2)}. \tag{40}$$

Thus, the equation (30) become as:

$$\overline{V}(x,s) = \lfloor \ell_{ij}(x,s) \rfloor \overline{V}(0,s). \tag{41}$$

To get $\overline{u}(0,s)$ and $\overline{\theta}'(0,s)$, we use equation (41) when x = 0

$$\overline{V}(0,s) = \lfloor \ell_{ij}(0,s) \rfloor \overline{V}(0,s), \tag{42}$$

where $\overline{V}(0,s) = \begin{bmatrix} \overline{u}(0,s) \\ \overline{\vartheta}(0,s) \\ \overline{u}'(0,s) \\ \overline{\vartheta}'(0,s) \end{bmatrix} = \begin{bmatrix} \overline{u}(0,s) \\ \alpha_5 \\ \alpha_6 + \alpha_2\alpha_5 \\ \overline{\vartheta}'(0,s) \end{bmatrix}.$ (43)

Thus, we get

$$\overline{u}(0,s) = \frac{\alpha_3\alpha_6 - \alpha_7 k_1 k_2}{k_1 k_2 (k_1 - k_2)}, \tag{44}$$

and

$$\overline{\theta}'(0,s) = -\frac{\alpha_5 k_1 k_2 + \alpha_3\alpha_5 + \alpha_4\alpha_7}{(k_1 + k_2)}. \tag{45}$$

By using the equations (41), (43), (44) and (45), we get

$$\bar{u}(x,s) = \frac{1}{\alpha_4\left(k_2^2 - k_1^2\right)}\left[\left(k_1^2 - \alpha_3\right)\left(\alpha_3\alpha_5 + \alpha_4\alpha_7 - \alpha_5 k_2^2\right)\frac{e^{-k_1 x}}{k_1} - \left(k_2^2 - \alpha_3\right)\left(\alpha_3\alpha_5 + \alpha_4\alpha_7 - \alpha_5 k_1^2\right)\frac{e^{-k_2 x}}{k_2}\right], \tag{46}$$

$$\bar{\theta}(x,s) = \frac{1}{\left(k_1^2 - k_2^2\right)}\left[\left(\alpha_3\alpha_5 + \alpha_4\alpha_7 - \alpha_5 k_2^2\right)e^{-k_1 x} - \left(\alpha_3\alpha_5 + \alpha_4\alpha_7 - \alpha_5 k_1^2\right)e^{-k_2 x}\right], \tag{47}$$

where $\alpha_7 = \alpha_6 + \alpha_2\alpha_5$

By using the equations (23), (46) and (47), we get the stress in the form

$$\bar{\sigma}(x,s) = \frac{1}{\alpha_4\left(k_1^2 - k_2^2\right)}\left[\left(\alpha_1 - k_2^2\right)\left(\alpha_3\alpha_5 + \alpha_4\alpha_7 - \alpha_5 k_2^2\right)e^{-k_1 x} - \left(\alpha_1 - k_1^2\right)\left(\alpha_3\alpha_5 + \alpha_4\alpha_7 - \alpha_5 k_1^2\right)e^{-k_2 x}\right]. \tag{48}$$

Those complete the solution in the Laplace transform domain.

4. NUMERICAL INVERSION OF THE LAPLACE TRANSFORM

In order to invert the Laplace transform, we adopt a numerical inversion method based on a Fourier series expansion [27], [28]

By this method the inverse $f(t)$ of the Laplace transform $\bar{f}(s)$ is approximated by

$$f(t) = \frac{e^{ct}}{t_1}\left[\frac{1}{2}\bar{f}(c) + R1\sum_{k=1}^{N}\bar{f}\left(c + \frac{ik\pi}{t_1}\right)\exp\left(\frac{ik\pi t}{t_1}\right)\right], \quad 0 < t_1 < 2t,$$

where N is a sufficiently large integer representing the number of terms in the truncated Fourier series, chosen such that

$$\exp(ct)R1\left[\bar{f}\left(c + \frac{iN\pi}{t_1}\right)\exp\left(\frac{iN\pi t}{t_1}\right)\right] \le \varepsilon_1,$$

where ε_1 is a prescribed small positive number that corresponds to the degree of accuracy required. The parameter c is a positive free parameter that must be greater than the real part of all the singularities of $\bar{f}(s)$. The optimal choice of c was obtained according to the criteria described in [27].

5. NUMERICAL RESULTS AND DISCUSSION

With a view to illustrating the analytical procedure presented earlier, we now consider a numerical example for which computational results are given. The results depict the variation of temperature, displacement and stress fields in the context of GTE (due to L-S and G-L models) and CTE. For this purpose, copper is taken as the thermoelastic material for which we take the following values of the different physical constants:

$K = 386 \text{ kg}\cdot\text{m}\cdot\text{k}^{-1}\cdot\text{s}^{-3}$,

$\alpha_T = 1.78\,(10)^{-5}\,\text{k}^{-1}$,

$\rho = 8954 \text{ kg}\cdot\text{m}^{-3}$, $T_o = 293\text{k}$,

$C_E = 383.1 \text{ m}^2\cdot\text{k}^{-1}\cdot\text{s}^{-2}$,

$\mu = 3.86\,(10)^{10}\,\text{kg}\cdot\text{m}^{-1}\cdot\text{s}^{-2}$,

$\lambda = 7.76\,(10)^{10}\,\text{kg}\cdot\text{m}^{-1}\cdot\text{s}^{-2}$,

$\omega = 10^5\,\text{s}^{-1}$.

From the above values we get the nondimensional values for our problem as:

$a = 0.01041$, $\varepsilon = 1.618$, $\Omega = 0.000007$

The field quantities, temperature, displacement and stress depend not only on the state and space variables t and x, but also depend on rise-time parameter t_o , the thermal relaxation time parameters τ_o, υ and n (for GTE) and on the loading parameter σ_o. It has been observed that in all three theories of CTE, L-S and G-L, the finite rise-time parameter t_o has significant effect on the temperature quantities even when the traction free (σ_o= 0.0) but , its effect on the dispacement and the stress quantities does not appear unless the loading on the boundary vanishes, in this case, the effect of t_o is very strong. Here all the variables/parameters are taken in nondimensional forms. In the context of the three theories, numerical analysis has been carried out by taking $\theta_1 = \sigma_o = 1,\ t = 0.3$ and the x range from 0.0 to 1.0. The numerical values for the field quantities are computed separately for each theory for a wide range of values of finite pulse rise-time t_o in the two situations $t > t_o$ and $t < t_o$ respectively. For CTE we take $\tau_o = \upsilon = 0$. For the L-S model we take n = 1, $\upsilon = 0$ and τ_o = 0.02, 0.04, 0.08 and for the G-L model we consider τ_o = 0.02, 0.04 and υ = 0.02, 0.04 in different combinations. Effects of the time of rise of temperature on the magnitude of the field quantities (in the context of the three theories) have been examined for a wide range of values of t_o. Results for the specific cases t_o = 0.1, 0.2, 0.3, 0.4 for t = 0.3 and t_o = 0.4, 0.5 , 0.6 for t = 0.5, are being shown here.

Figures 1-4, exhibit the space variation of temperature at instants, t = 0.3 for different values of t_o whereas Fig. 2 indicates the variation of temperature in CTE at different values of t_o with the observation times t = 0.3 and t = 0.5, when t_o = 0.1, 0.2, 0.3, 0.4, 0.5 and 0.6 in which we observe the following:

(i) In figure 1, temperature decreases as x increases for the three theories, whatever the value of t greater or smaller than t_o.

(ii) In figures 1, 2, 3 and 4, the temperature at a given position x at any instant t decreases with the increase of t_o for the three theories.

(iii) In figure 1, significant difference in the value of temperature is noticed for the three theories, when the bounding plane traction free $(\sigma_o = 0.0)$ or periodical loading $((\sigma_o, \omega) \neq 0.0)$.

(iv) In figure 3 and 4 significant difference in the values of the temperature when the relaxation times τ_o and υ change

Figures 5-9, exhibit the space and the time variations of displacement for different values of t, $t_o, \tau_{o,}\ \upsilon$. It observed that:

(i). In figure 5, no significant difference in the values of displacement is noticed for the three theories when the bounding plane has a periodic loading in different values of to, Thus, a single curve for each theory has been drawn at one value of $t_o = 0.2$.

(ii). In figures 6, , 7 ,8 and 9, when the bounding plane traction free, the value of to has an essential role to change the value of the displacement at the same point of x for the three theories. We can see that, the maximum point of the displacement increases when to decreases in t = 0.3.

(iii). In figure 8 and 9, significant difference in the values of the displacement when the relaxation times τ_o and υ change.

Figures 10-14, exhibit the space and the time variations of stress for different values of t, $t_o, \tau_{o,}\ \upsilon$. It observed that:

(i). In figure 10, no significant difference in the values of stress is noticed for the three theories when the bounding palne has a periodic loading in different values of to, Thus, a single curve for each theory has been drawn at one value of $t_o = 0.2$.

(ii). In figure 11, 12, 13 and 14, when the bounding plane traction free, the value of to has an essential role to change the value of the displacement at the same point of x for the three theories. We can see that, the magnitude of the maximum point of the stress increases when to decreases in t = 0.3

(iii). In figure 11, when $t > t_o$ and the bounding plane traction free, the stress start from zero to positive values for a small interval slowly, after this interval the curve fall to negative values rapidly till the sharp point of the curve, then the stress increases where increase of x. But when $t < t_o$, the stress is almost negative for all the values of x.

(iv). In figure 13 and 14, significant difference in the values of the stress when the relaxation times τ_o and υ change.

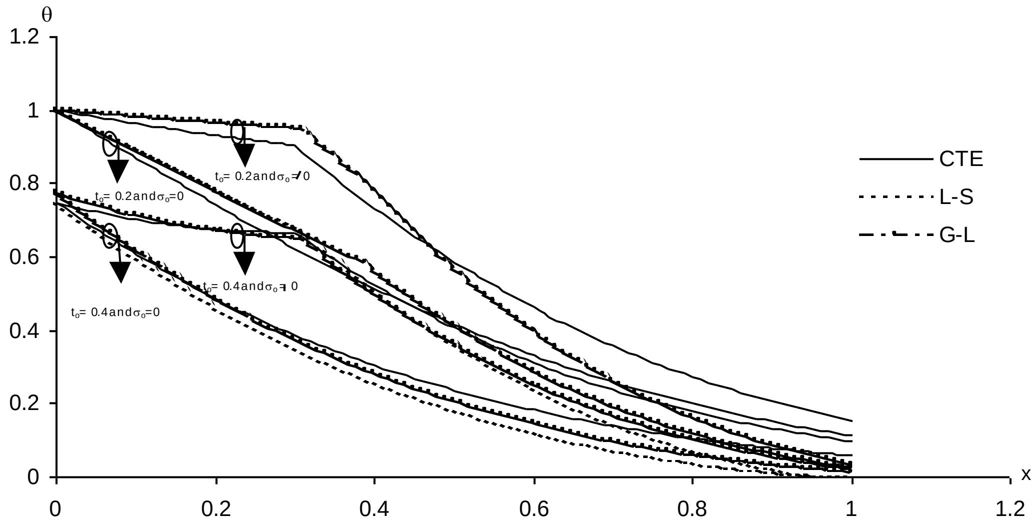

Figure 1: The temperature distribution at t = 0.3

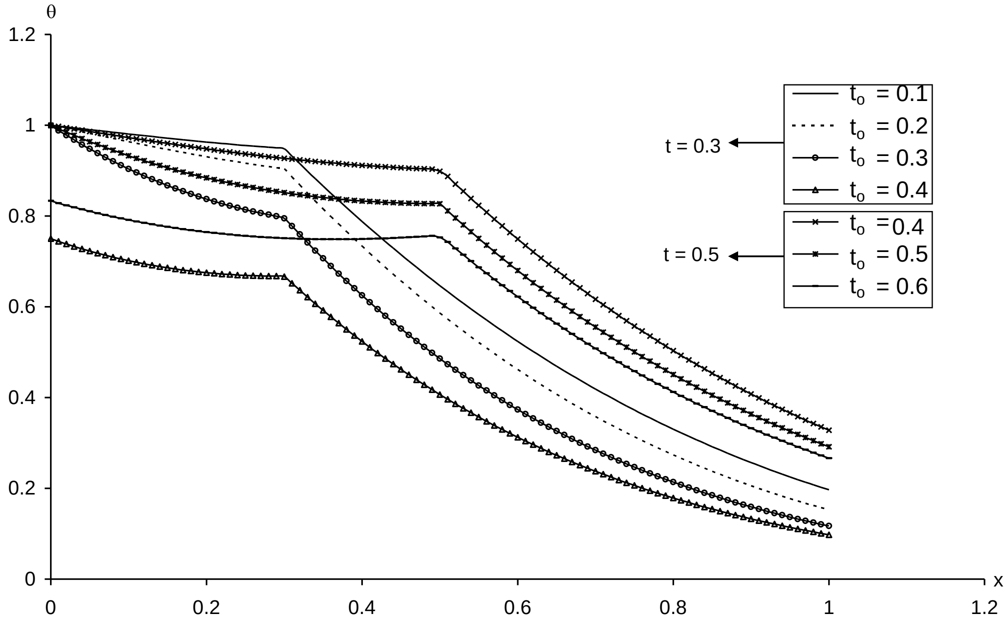

Figure 2: The temperature distributions for **CTE** at different time and different values of t_o

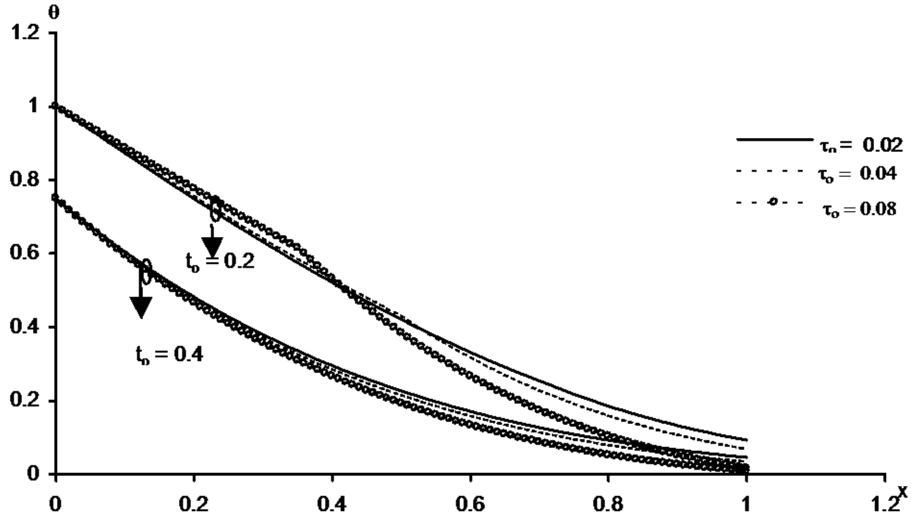

Figure 3 : The temperature distributions for **L-S** at t = 0.3 in different τ_o and t_o

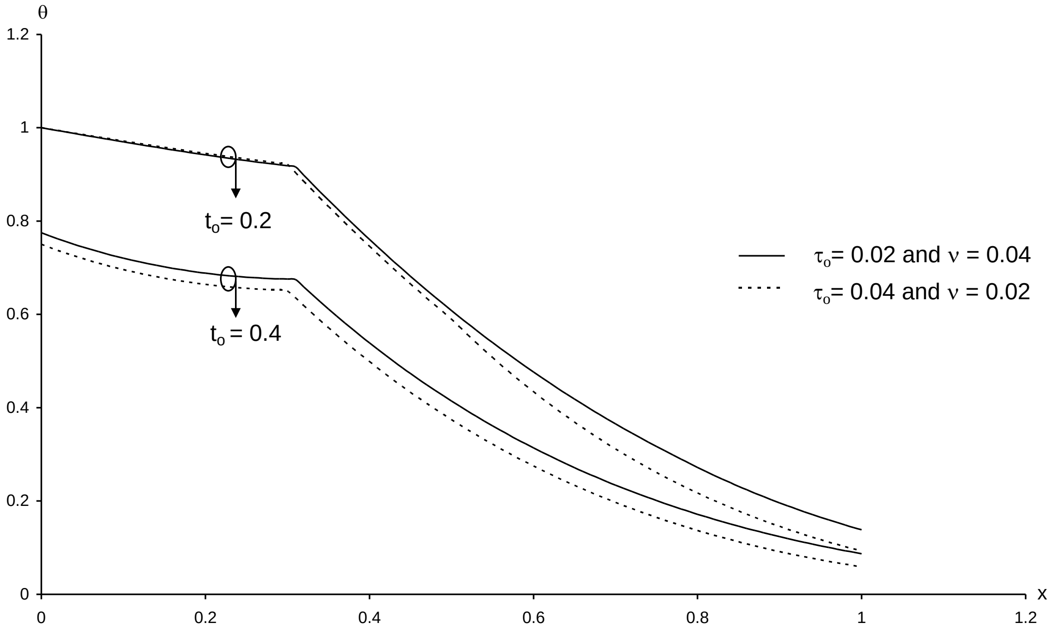

Figure 4: The temperature distributions for **G-L** at t = 0.3 and different t_o , τ_o and ν

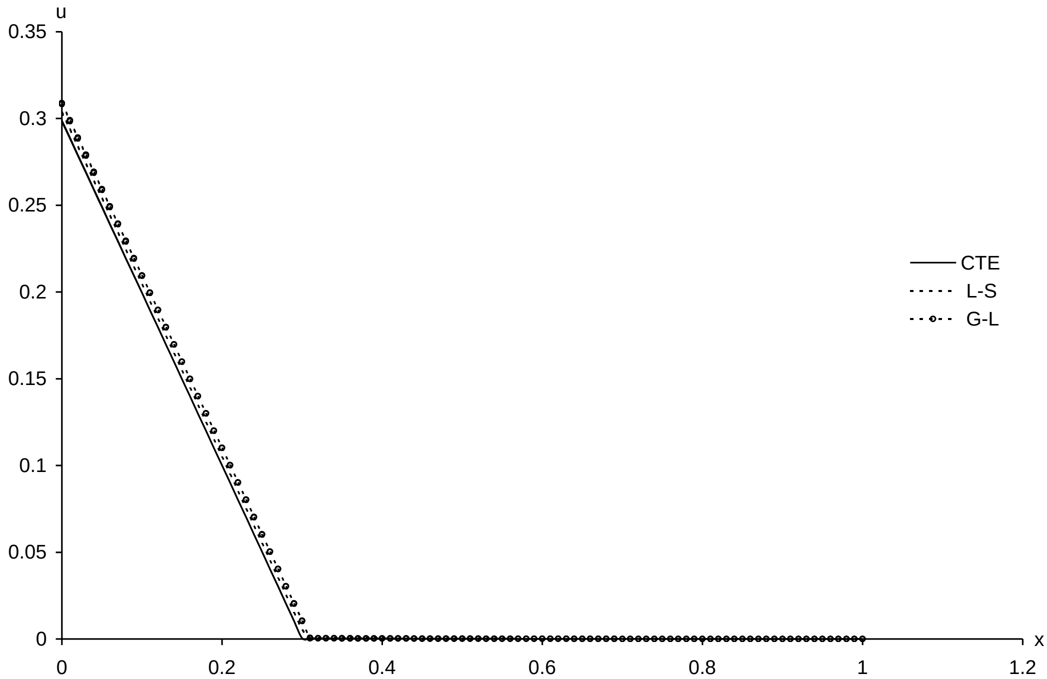

Figure 5: The displacement distribution at t = 0.3 and t_o = 0.2

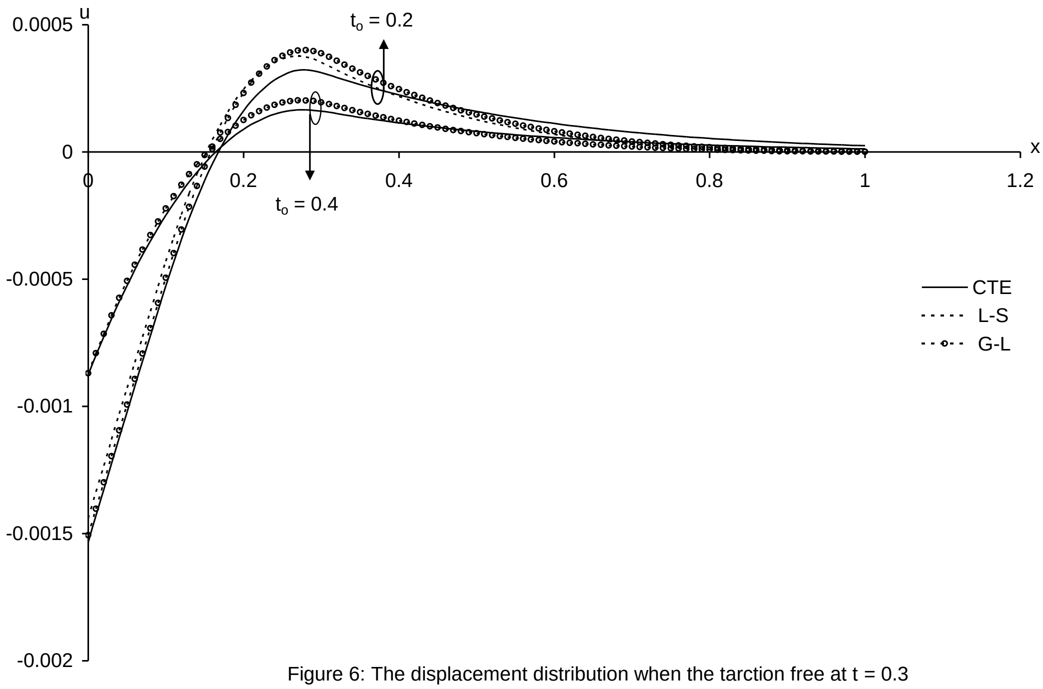

Figure 6: The displacement distribution when the tarction free at t = 0.3

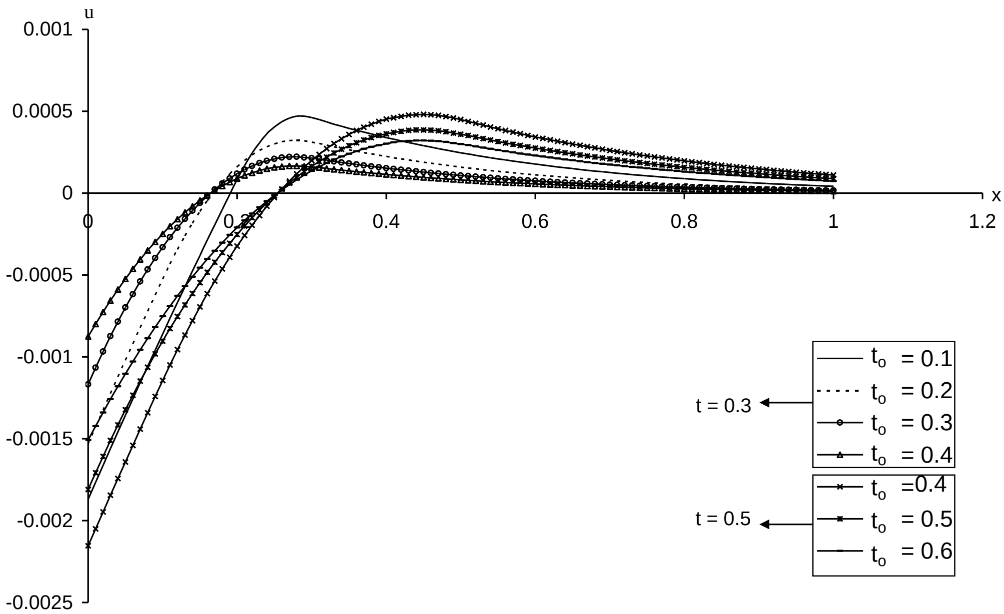

Figure 7: The displacement distribution for **CTE** at different time and different values of t_o

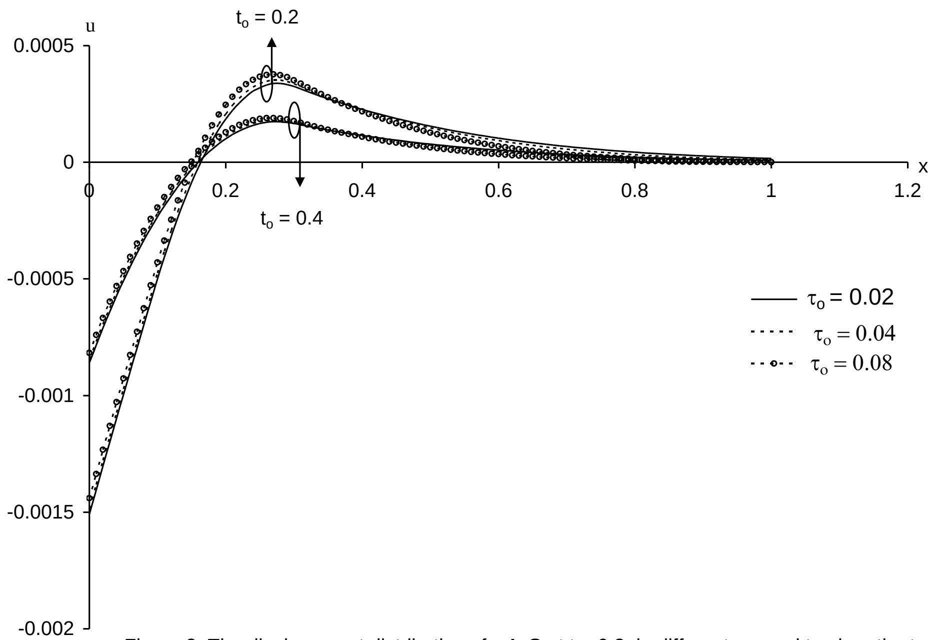

Figure 8: The displacement distributions for **L-S** at t = 0.3 in different τ_o and t_o when the traction free

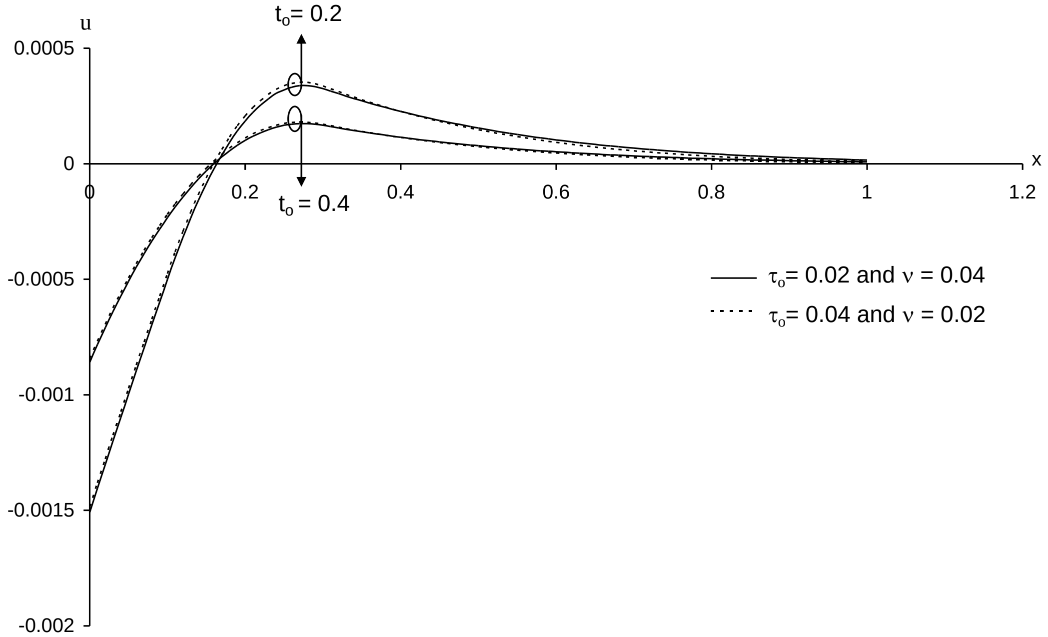

Figure 9: The displacement distributions for **G-L** at t = 0.3 and different t_o , τ_o and ν when the traction free

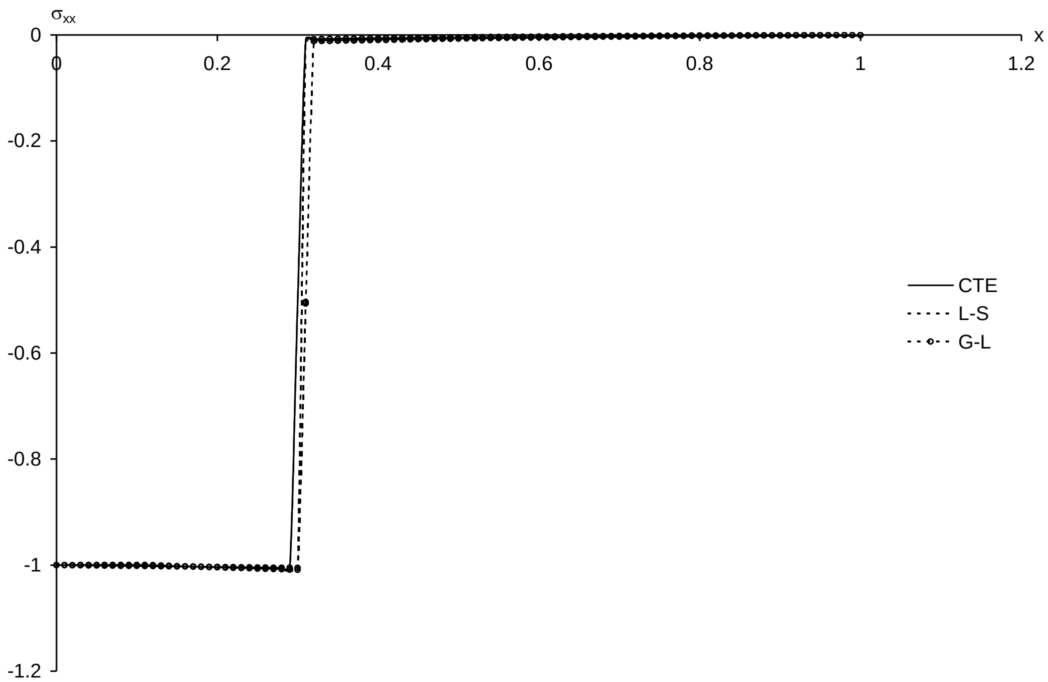

Figure 10: The stress distributions at t = 0.3 and t_o = 0.2

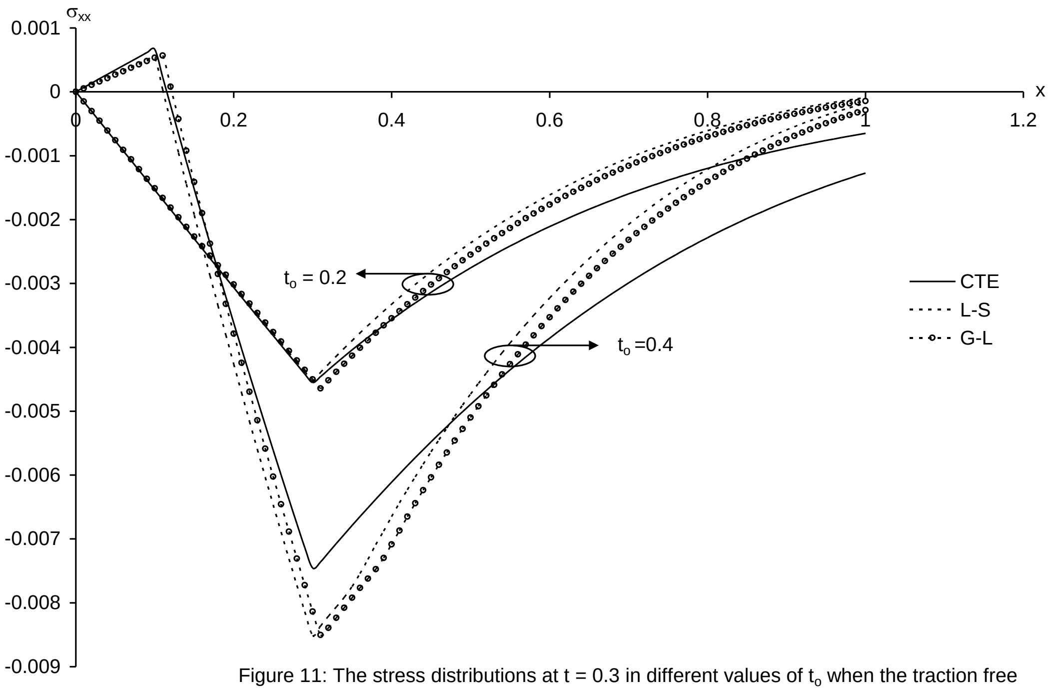

Figure 11: The stress distributions at t = 0.3 in different values of t_o when the traction free

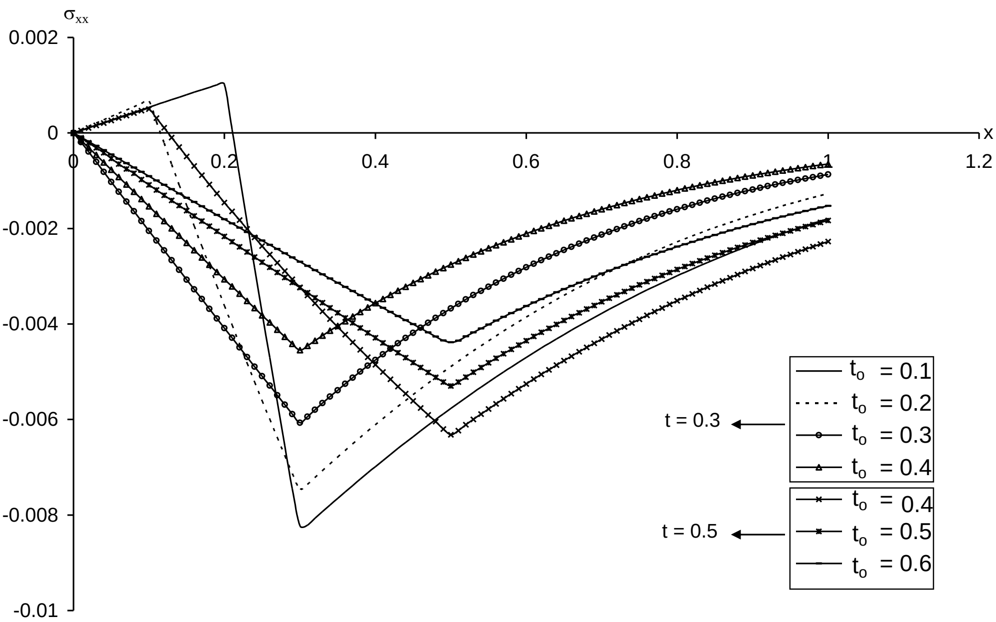

Figure 12: The stress distribution for **CTE** at different time and different values of t_o

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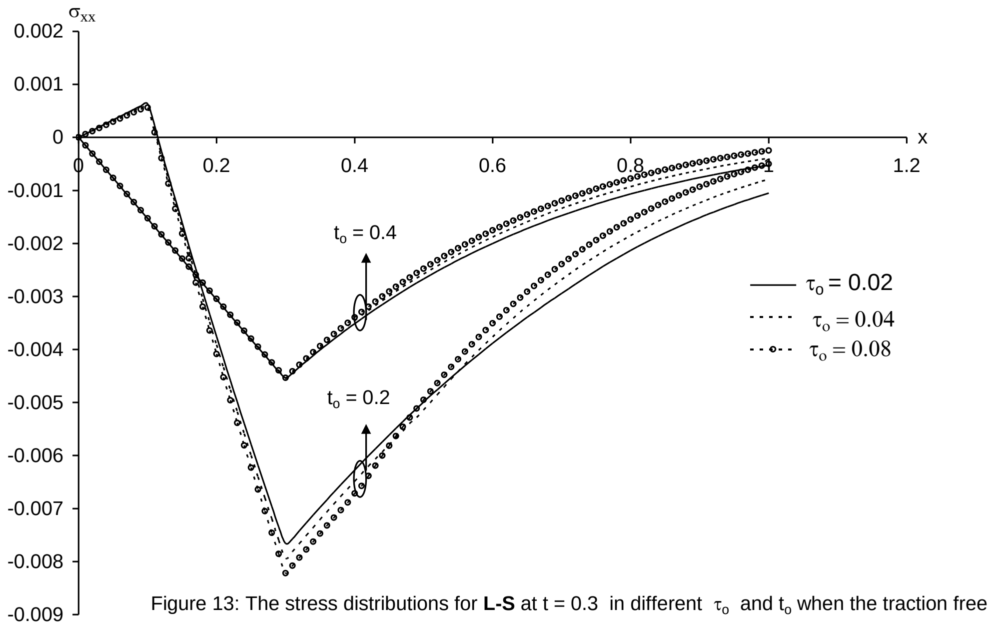

Figure 13: The stress distributions for **L-S** at t = 0.3 in different τ_o and t_o when the traction free

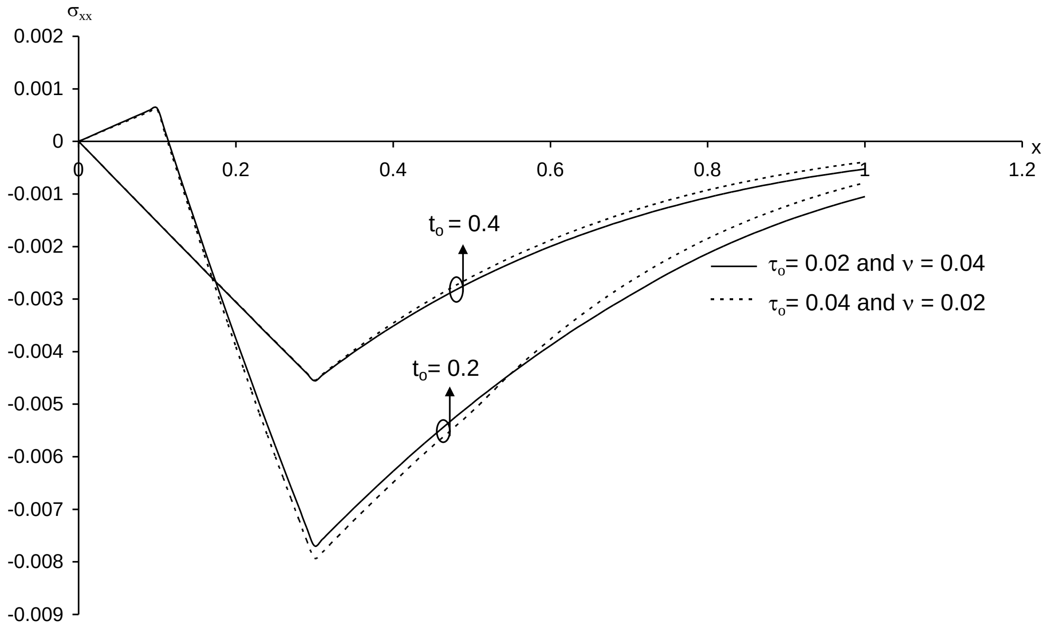

Figure 14: The stress distributions for **G-L** at t = 0.3 and different t_o , τ_o and ν when the traction free

6. CONCLOUSION

Temperature, stress, and displacements fields in homogeneous elastic half-space due to linear temperature ramping have been examined within the framework of the generalized thermoelasticity theories of Lord and Shulman and Green and Lindsay. Comparisons with predictions of the classical coupled thermoelasticity theory, in which only a coupling term in the parabolic heat conduction equation, were also made. For the range of rise time parameter t_0 considered herein, we find essentially no differences between the predictions of either theory. For small values t_0, which imply a slower temperature rise on the boundary, the predictions from the CTE differ from the generalized theories. We can say that, the speed of wave propagation has not finite value at large distance x in the context of CTE which make GTE is more agreeable with the physical properties of the solid materials in small value of temperature rise time. For large values of t_0, which are associated with much more rapid temperature rise on the boundary, the CTE almost are very closed to the GTE so any model of them may be used.

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