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New Theory for the Low Frequencies Nonlinear Acoustic Radiation of Plate at Large Vibrations Amplitudes

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Abstract

In this paper, a new approach is presented for the estimation of the non-linear acoustic radiation of plates exhibiting a geometrical non linear behavior at low frequencies, in the neghborhood of the first mode of vibration. The explicit analytical expression for the plate non-linear forced response in the the vinicity of the fundamental mode, obtained in previous works, is substituted in the classical expressions for the acoustic indicators, such as the pressure, the velocity, the impedance, and the efficiency. This leads to new expressions for these indicators involving the effect of the geometrical non-linearity on the plate acoustic radiation indicators. Then, the indicators, computed numerically, have been compared with the corresponding classical linear ones. The comparison between the numerical and analytical results and those available from previous studies showed a good agreement. The results allowed the estimation of the effect of non-linearity on the classical acoustic parameters and showed a higher increase in the acoustic indicators, compared with those predicted by the linear theory. This confirms the necessity of taking into account the geometrical non-linearity in order to get an accurate estimate of plates sound radiation at large vibration amplitudes.

Keywords - Acoustic radiation, Forced excitation, Indicators, Nonlinear vibration, Plate.

I. INTRODUCTION (12 BOLD)

In many practical situations, the sound radiation from vibrating structures is of a great technical importance and a major environmental concern. Due to obvious security and comfort reasons, engineers in various industrial fields, such as loudspeakers manufacturing, road, rail, marine and airborne vehicles design, need to be provided with efficient tools in order to obtain reasonably accurate estimates of the noise radiation due to the vibration of the structural components involved. In many practical modern situations, geometrically non-linear structural vibrations occur, making the classical analytical and numerical tools, developed within the frames of linear theories, unable to predict properly the corresponding sound radiation parameters.The estimation of the radiation efficiency was presented by Jean-claude Pascal and Jing-Fang Li [1] using an approach which does not require a complete knowledge of the vibration field. A method for the analytical calculation of the acoustic radiation of rectangular thin plates of arbitrary boundary conditions was proposed by A. Berry, J. Nicolas and J.-L. Guyader [2]. The translational rigidity of the limits is the parameter dominating the radiation under the critical frequency. R.L.C. Lemmen, R.J. Panuszka [3] have given expressions for the numerical evaluation of the radiation efficiencies and the power radiation of baffled plates. Numerical results for simply supported plates were presented and compared with results obtained by a statical technique. In 1962, Maidanik [4] proposed approximate formulae for the modal radiation efficiency in different frequency regions of simply supported rectangular vibrating beams and panels set in an infinite rigid baffle. In 1972, Wallace [5] determined theoretically the radiation resistance corresponding to the natural modes of a finite rectangular panel from the total energy radiated to the farfield. Kadiri and Benamar [6] have developed a semi analytical method based on Hamilton's principle and spectral analysis, for determination of the geometrically non-linear free and forced response of thin straight structures. Using a thorough mathematical description of the modal radiation efficiency, Leppington [7] obtained approximate expressions of the acoustic radiation efficiency in the large wave number region, especially in the neighborhood of high critical frequencies. The results confirm that the acoustic radiation efficiency has different asymptotic forms in different regions of the plate wavenumber space. Li [8] obtained an analytical solution for the self- and mutual radiation resistances in the form of power series of the non-dimensional acoustic wave number, which appeared to be extremely efficient in comparison with the traditional numerical integration scheme. Also, the effect of the baffle on the modal radiation efficiency was presented by Laulagnet [9].The purpose of this work was the development of an easy practical tool to calculate the acoustic radiation indicators, such as the pressure, the velocity, the impedance, and the efficiency of a simply supported plate subjected to large vibration amplitudes in the neighborhood of one of its modes of vibration. To do so, the explicit analytical expression for the plate non-linear forced response in the neighborhood of the mode considered, obtained in previous works, is first substituted in the classical expressions for the acoustic indicators mentioned above. This leads to new expressions for these indicators involving the effect of the geometrical non-linearity on the plate acoustic radiation. Finally, the indicators are computed numerically and plotted in order to enable comparisons to be made with the corresponding classical linear ones.

II.GEOMETRICALLY NON-LINEAR STEADY STATE HARMONIC RESPONSE OF SIMPLY-SUPPORTED PLATES

The purpose of the present section is to make a brief review of the theory, previously developed by Benamar and his co-authors, for the geometrically non-linear steady state harmonic response of a simply-supported plate in the neighborhood of one of its mode shapes [6]. This is made in order to introduce the analytical expressions for the non-linear plate response substituted in the present work into the integrals used to calculate the non-linear acoustic indicators. For a complete presentation of the theory, the reader can be turned for example to the references mentioned above.

The transverse vibrations a simply supported plate set in an infinite rigid baffle and radiating into the fluid in a semi-infinite space are examined. The plate has the following characteristics: a, b, h: length, width and thickness of the plate, x-y: plate co-ordinates in the length and the width directions, E and ν: Young's modulus and Poisson's ratio, D and ρ: plate bending stiffness and mass per unit volume. The plate is supposed to be subjected to a harmonic force in such a manner to excite predominantly a given non-linear mode. For example, if the concentrated dimensionless excitation force of amplitude f* is harmonic, with a frequency chosen in the vicinity of the plate fundamental frequency, and is applied at the plate central point, it has been shown that the first nonlinear mode is predominant in the plate response and that the corresponding non-linear frequency response function may be presented by [6]:

$$(\omega^*/\omega_L^*)^2 = 1 + \frac{3}{2}(b_{1111}^*/k_{11}^*)a_{11}^2 - \frac{(1/k_{11}^*)f_1^*}{a_{11}} \qquad (1)$$

In which: $\omega_L^{*2} = k_{11}^*/m_{11}^*$. and m_{11}^*; k_{11}^* *and* b_{1111}^* are the dimensionless mass, rigidity and non-linear rigidity terms corresponding to the first mode of a simply supported rectangular plate defined in reference [6] as:

$$b_{1111}^* = \frac{27\pi^4}{64}\left(\alpha^8 + \frac{8\alpha^4}{9} + 1\right);\ k_{11}^* = \frac{\pi^4}{2}(\alpha^2+1)^2;\ m_{11}^* = \frac{1}{2} \qquad (2\text{-}4)$$

$$\omega^2 = \frac{EI\pi^4}{\rho Hab^4}\left(\frac{1}{2}(\alpha^2+1)^2 + \frac{81}{128}\left(\alpha^8 + \frac{8}{9}\alpha^4 + 1\right)a_{11}^2 - \frac{f_1^*}{a_{11}}\right) \qquad (5)$$

Where: $\alpha = \frac{b}{a}$

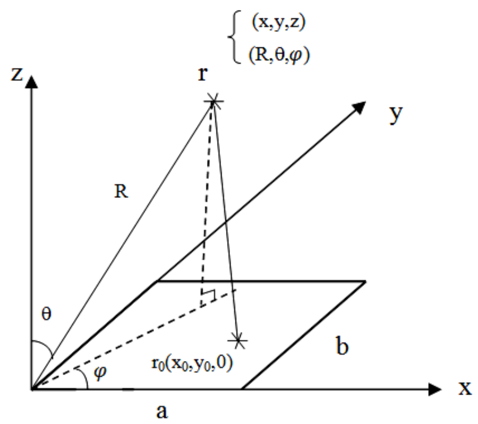

Fig.1. A rectangular thin plate in flexural vibration and its co-ordinates

III. EXPRESSIONS FOR THE BAFFLED PLATE RADIATION INDICATORS

The classical expressions for the acoustic radiation indicators, which will be used here in the non-linear case, may be found for example in reference [3]. Before presenting the modifications made, these are summarised below.

The spatially averaged mean square velocity of the plate corresponding to the (m,n) mode , is given by:

$$\langle \dot{V}^2 \rangle = \frac{1}{2S}\int_S \frac{1}{2}V_\omega^2 dxdy \qquad (6)$$

Where

$$V_\omega = V_m \sin\left(\frac{m\pi x}{a}\right)\sin\left(\frac{n\pi y}{b}\right)\quad 0 \le x \le a; 0 \le y \le b \qquad (7)$$

And

$$V_m = i\omega a_{mn} \qquad (8)$$

The sound pressure radiated in the far-field for a baffled plate using the simplified Kirchhoff–Helmholtz integral is given by:

$$P(r) = \frac{\frac{\rho_0}{2\pi}\iint_{(S_0)} \ddot{W}(r_0)G(r}{r_0} dS(r_0) \qquad (9)$$

Where ρ_0 is the density of the surrounding medium, S_0 is the area of the structural surface, r_0 the current point on the structural surface, r is the field point expressed in the spherical co-ordinates system as r = (R,θ, φ), k is the sound wave number, k =ω/c_0, with c_0 being the sound speed in the medium, and $G(r/r_0) = \frac{1}{2\pi}\frac{e^{-jk|r-r_0|}}{|r-r_0|}$ is the Green function. The above expression constitutes the basic relation between the structural response and the sound pressure radiated. In the far field, the distance between the field point r and the plate is large compared to the characteristical dimension of the plate. That allowed approximating the distance in the denominator of the Green's function by R. Then: $|r - r_0| \approx |r| = R$.

Using the spherical coordinates, the expression for the far-field acoustic pressure distribution can be written as:

$$P(R,\theta,\varphi) = -\rho_0\omega^2.\left(\frac{a.b}{m.n.\pi^2}\right).\frac{e^{-jkR}}{2\pi R}.\left[\frac{((-1)^m e^{-i\lambda}-1)}{\left(\frac{\lambda}{\pi m}\right)^2-1}\right].\left[\frac{((-1)^n e^{-i\beta}-1)}{\left(\frac{\beta}{\pi n}\right)^2-1}\right] \qquad (10)$$

With: $\lambda = k.\sin\theta.\cos\varphi$ and $\beta = k.\sin\theta.\sin\varphi$

The integral of the average acoustic intensity over the hemisphere in the far field yields the total acoustic power radiated by the plate, given by:

$$\Pi(\omega) = \frac{1}{2\rho_0 c_0}\int_0^{2\pi}\int_0^{\frac{\pi}{2}} R^2\,|p(R,\theta,\varphi)|^2 \sin\theta d\theta d\varphi \qquad (11)$$

The radiation efficiency can be written as:

$$\sigma = \frac{\Pi(\omega)}{\rho_0 c_0 S_0 \langle \dot{V}^2 \rangle} \qquad (12)$$

Where $\Pi(\omega)$ is the sound power radiated, ρ_0 is the density of the fluid (air in the present case), c_0 is the speed of sound, S_0 is the plate surface area and $\langle \dot{V}^2 \rangle$ is the spatially averaged mean-square normal velocity of the plate.

IV. ANALYTICAL DETAILS AND NUMERICAL RESULTS

Substituting the expression for ω^2 given in equation (5), which corresponds to the geometrically non-linear vibration of the plate considered, into equations (6) and (10) leads to the new expressions for the non-linear acoustic radiation indicators:

$$\langle \dot{V}^2 \rangle = \frac{EI\pi^4\left(\frac{1}{2}(\alpha^2+1)^2+\frac{81}{128}\left(\alpha^8+\frac{8}{9}\alpha^4+1\right)a_{11}^2-\frac{f_1^*}{a_{11}}\right)}{8\rho Hab^4}|a_{11}|^2 \qquad (13)$$

$$P(R,\theta,\varphi) = -\rho_0\frac{EI\pi^4}{\rho Hab^4}\left(\frac{1}{2}(\alpha^2+1)^2+\frac{81}{128}\left(\alpha^8+\frac{8}{9}\alpha^4+1\right)a_{11}^2 - \frac{f_1^*}{a_{11}}\right).\left(\frac{a.b}{m.n.\pi^2}\right).\frac{e^{-jkR}}{2\pi R}.\left[\frac{((-1)^m e^{-i\lambda}-1)}{\left(\frac{\lambda}{\pi m}\right)^2-1}\right].\left[\frac{((-1)^n e^{-i\beta}-1)}{\left(\frac{\beta}{\pi n}\right)^2-1}\right] \qquad (14)$$

Using the above equations, numerical results have been obtained in the case of a plate having the following parameters: a = 455 mm, b =375 mm, h =1mm. ρ=7800Kg/m^3, E=210 10^9 N/m^2.The external fluid: Air: ρ_0 = 1.29 kg/m^3, c_0 = 343 m/s.

In fig 2, the nonlinear frequency response function of the plate, i.e. equation (1) is plotted, showing a non-linear behavior of the hardening type.

The qualitative behavior obtained in fig 1 is characteristic for non-linear frequency response functions of systems with a cubic non-linearity. It includes multivalued regions in which the jump phenomena may occur in non-linear frequency response testing.

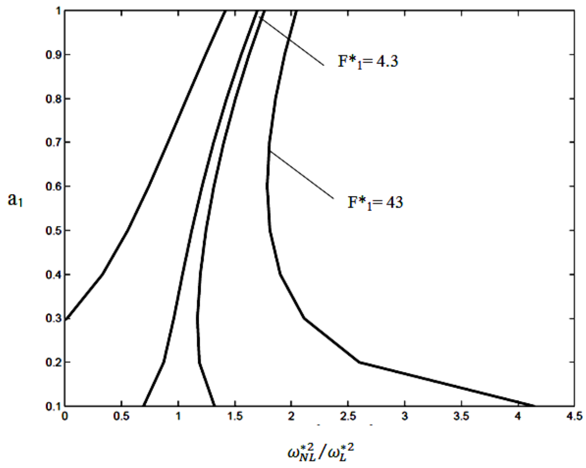

Fig.2. Forced vibration of a SSSS plate

Fig 3 presents the radiation efficiency in the low frequency region of a simply supported plate vibrating in the linear regime. It is observed that the results obtained here closely matches with those from reference [2].

It can be seen from the average radiation efficiency that the overall result below 70 Hz is determined by the first mode. The results are shown in fig 3. For a plate of a thickness h= 1mm, the first mode has radiation efficiencies from 0.05 up to 0.25. The computational results agree very well with the frequency averaged result of Maidanik.

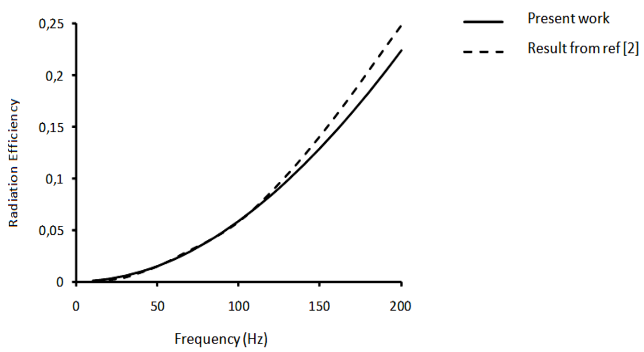

Fig.3. Radiation Efficiency of a simply supported plate

Fig 4 presents the Sound power level radiated from a simply supported plate subjected to a harmonic force applied at the center of the plate in the vinicity of its fundamental mode. It is observed that the results obtained here closely match with those from reference [2].

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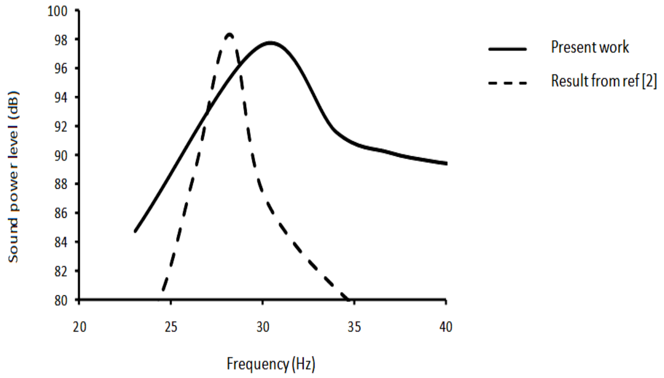

Fig.4. Linear sound power level radiated from a simply supported plate

In fig 5 and fig 6, the acoustic radiation indicators, i.e. the pressure and the radiated acoustic power, are plotted in each figure for the same value of the dimensionless excitation force f*= 1434 using both the classical linear expressions and the corresponding non-linear ones established here. It can be seen that the geometrical non-linearity induces a visible difference in the curves, indicating that the non-linearity has to be taken into account, especially as this effect is expected to increase if the amplitude of vibration increases. The pressure and the sound power of the plate, shown in fig 5 and fig 6, indicate that the first mode dominates the response in the low frequency region, and moreover it has the highest modal radiation efficiency.

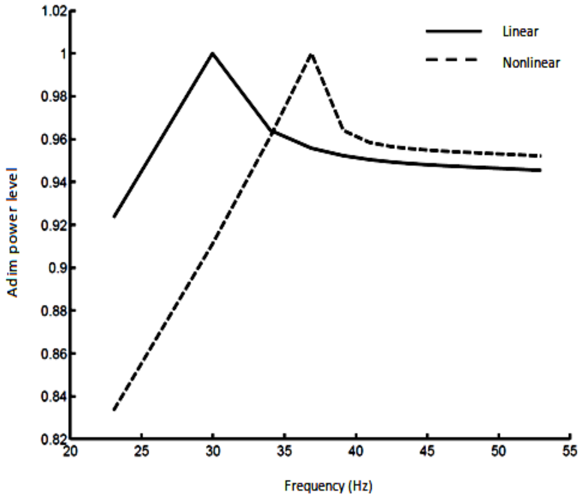

Fig.5. Normalized linear and nonlinear power level in the neighborhood of the first mode for f*=1434

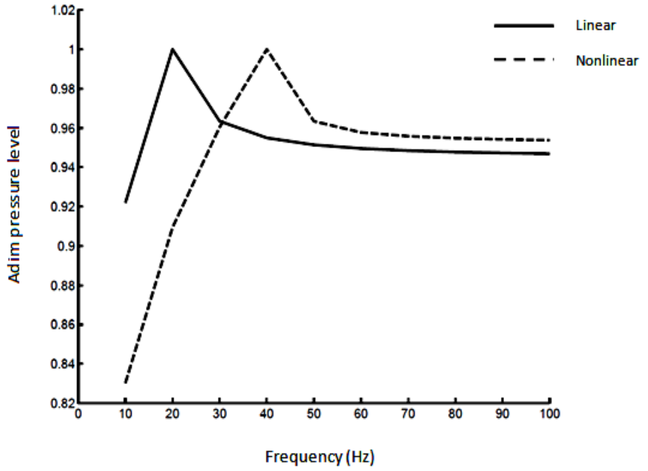

Fig.6. Normalized linear and nonlinear pressure level in the neighborhood of the first mode for f*=1434.

In fig 7 and fig 8 the nondimensional power level and the pressure level are plotted for different values of the dimensionless excitation force. It can be seen that the effect of the non linearity increases with the amplitude of the excitation as may be expected.

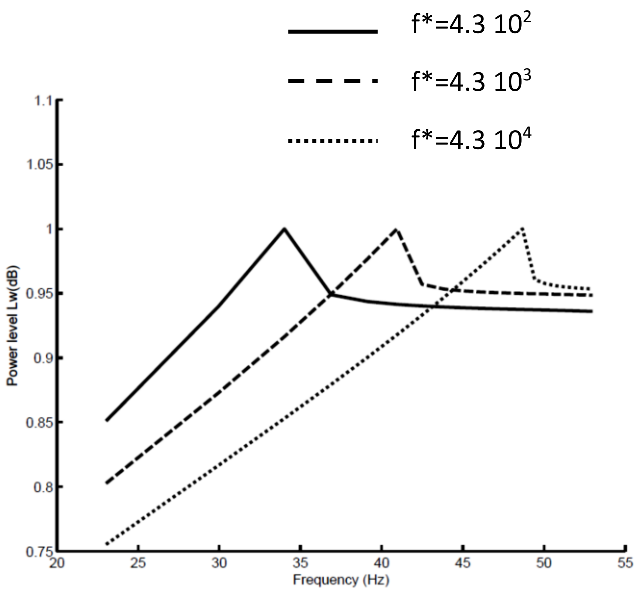

Fig.7. Normalized nonlinear power level in the neighborhood of the first mode for different values of the dimensionless excitation force

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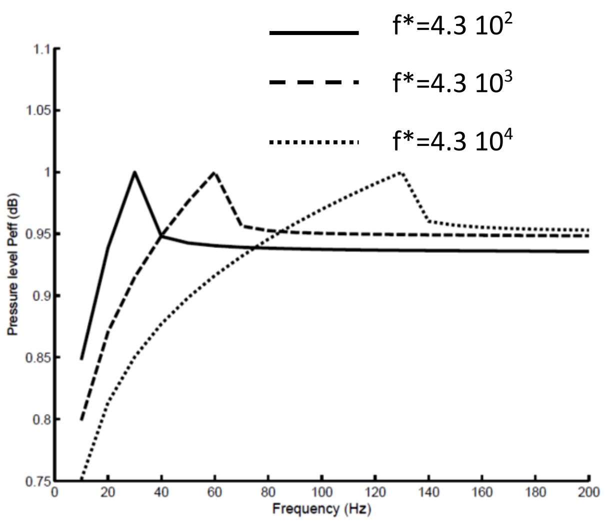

Fig .8. Normalized nonlinear pressure level in the neighborhood of the first mode for different values of the dimensionless excitation force

V. CONCLUSION

The above results are typical of what occurs when the non linear frequency response function is used in the expressions for the velocity, the pressure, and the radiated power level of a simply supported rectangular plate in the neighborhood of the non-linear first mode. It can be noticed that both the values of the sound radiation indicators and their distributions predicted by the present non-linear theory can be different from those usually obtained by the classical linear approaches. Consequently, it appears that the extension of the present works to the higher modes and to other structures, such as plates with other geometries, end conditions and materials, may be useful to the engineers working in the field of sound control. Also, the use of the multimode approach is the expression for the plates nonlinear frequency response functions is expected to yield more accurate results.

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