

# Predicting Column Axial Forces in Sequential Loading Analysis Using the Difference Between One-Step and Cumulative Typical Floor Analysis Axial Forces

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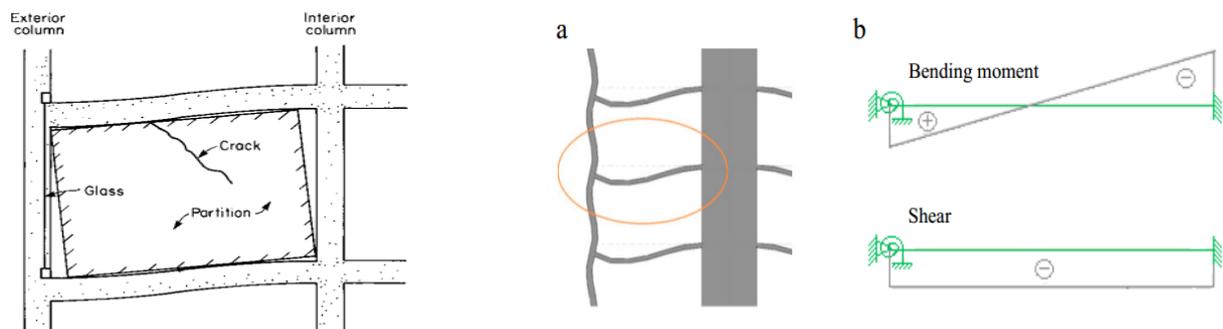
## Abstract.

The work below tried to predict the column axial forces in Sequential loading analysis (SLA) by using the differences between one-step loading analysis (OSLA) and cumulative typical floor analysis (CTFA) axial forces considering the dead loads. The CTFA, OSLA and SLA for a forty-floor concrete frame were carried out using commercial software Midas Gen 2019 according to Eurocode standard. From the numerical modelling analysis between the OSLA and SLA axial forces, differences with the OSLA and CTFA column axial forces, a linear and 2<sup>nd</sup>-degree polynomial equations were developed with 0.9877 and 0.9901 determinant of coefficient respectively. The validation results unveil a higher correlation with the predicted value ranging (between 94% and 105%) of the actual value.

**Keywords:** Construction stage analysis, Column axial shortening, Tall building, Creep, shrinkage.

## I. INTRODUCTION.

Land scarcity and improved material's engineering have been one of the drives to the increase in high rise reinforced cement concrete structures (RCC). [1]. These structure has resulted in new design challenges as a result of vertical structural elements differential axial shortening [2], which causes structural defects, beam and slab cracks, and non-structural defects such as cracks to infill walls and façade and building services [3] as shown in figure 1 if not addressed. Differential axial shortening causes self stressing (Fig.1) due to load distribution between vertical members resulting in exaggerated linear elastic analysis results which call for complex analysis [1]



Cracks on the infill walls due to Axial shortening. [2]

a) Deformation due to the axial shortening

b) Corresponding induced internal Forces. [1]

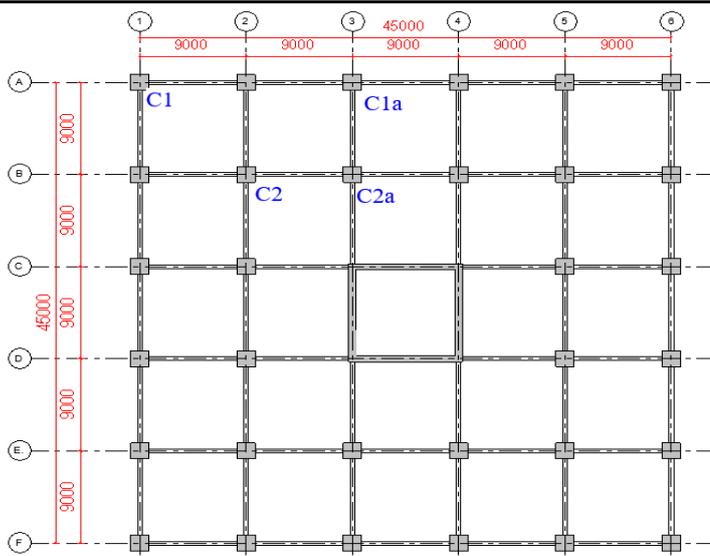
Fig. 1. Effects of Column axial shortening.

The complex analysis mentioned above entails the combination of finite element and construction stage analysis putting into consideration the RCC time-dependent properties, which is a time-consuming undertaking [4], contrary to the conventional one-step analysis approach where the material strength gain is treated as instant. Over the years, several time-dependent material properties prediction models such as CEB-FIB (1978,1990, 2010), ACI, European, GL 2000 and Portland Cement Association Standard (PCA) have been developed [5]. Sequential loading analysis results from previous research have shown that; increase in reinforcement quantity to more deformed vertical elements, offsetting vertical elements height during construction, use of outriggers beams material engineering [6] among others reduces the differential shortening. In concept designs stage it is time-saving to use a simplified method to predict the construction stage analysis results [1]. From this dimension, in this work, a straight forward numerical method was developed that used the axial forces differences between OSLA and CTFA to predict the complex SLA columns axial forces. The developed numerical model was validated by comparing the projected results with the results obtained from Midas Gen 2019 commercial software using its non-linear construction stage analysis package.

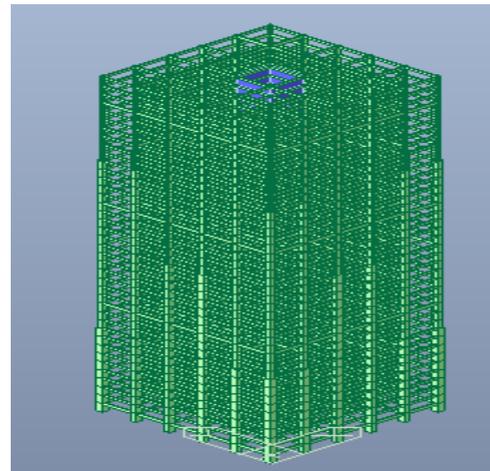
## II. METHOD AND MATERIAL.

### II.I. Analytical Model

A forty floor reinforced cement concrete frame (Fig 2) with 750x375 mm beam size, 1550x1550 mm, 1350x1350, 1100x1100, 775 x775 mm column sizes from the 1<sup>st</sup> to the 40<sup>th</sup> storey section reducing after ten floors with the core wall sizes changed together with columns from 475 mm to 400 mm with a decrement of 25 mm. Apart from the structural elements dead loads, a 7.5 kN/m<sup>2</sup> slab self-weight, 1.875 kN/m<sup>2</sup> finishes, 1.2 kN/m<sup>2</sup> partition walls and 2.5 kN/m<sup>2</sup> imposed load were used to size the structural members as per [7] standard. The lateral forces considered were, 27 m/s basic wind speed and 0.08g m/s<sup>2</sup> ground peak acceleration earthquake loads were incorporated in the member sizing. The column and core walls concrete compressive strength were 45 N/mm<sup>2</sup> (C45/55) and 35 N/mm<sup>2</sup> (C35/45) for the beams. A relative humidity of 70% was adopted, with effective section determined using CEB-FIT 2010 Equation. The SLA was carried out according to step -by step method (SSM).



Typical floor Layout Plan.



40 Storeys analytical frame 3D View.

Fig. 2. Analytical model typical floor plan and 3D view.

### II.II Analysis

Three analysis CTFA, OSLA and SLA were carried out and the columns C1, C2, C1A, and C2A axial forces recorded at every floor level. In CTFA, the columns axial force for a typical floor ( $F_t$ ) was multiplied by the number of floors ( $N$ ) above the storey level to get the cumulative axial force ( $F_T$ ) at respective storey level using Equation 1. In OSLA, material strength gain with time was instant and the typical floors loading was activated once and the observed column axial force due to the dead load was recorded at every storey level.

In SLA, typical floor loads were activated at seven days interval, after nineteen days the concrete was loaded by deactivating false work support system. This analysis was carried out with the help of a commercial software MIDAS Gen construction stage analysis package where the dead load was considered and the column axial forces recorded at 1095 days after construction.

$$F_T = N * F_t \quad (1)$$

### II.III Material Properties.

#### Strength and Elastic Modulus.

The concrete mean compressive strength with time ( $f_{cm}(t)$ ) is a function of mean compressive strength at 28 days ( $f_{cm}$ ) and age coefficient ( $\beta_{cc}(t)$ ) as shown in Equation 2&3. The variation of modulus of elasticity with time ( $E_{cm}(t)$ ) is proportional to the elastic modulus at 28 days ( $E_{cm}$ ) which was determined using Equation 4 . [7]

$$f_{cm}(t) = \beta_{cc}(t) f_{cm} \quad (2)$$

$$\beta_{cc}(t) = \exp \{0.25[1 - (28/(\frac{t}{t_1}))^{(2)}]\} \quad (3)$$

$$E_{cm}(t) = (\frac{f_{cm}(t)}{f_{cm}})^{0.3} E_{cm} \quad (4)$$

#### Creep and Shrinkage.

Creep coefficient  $\phi(t, t_0)$  is a function of creep strain ( $\epsilon_{cc}(t, t_0)$ ) at time  $t$  and elastic strain  $\epsilon_{cc}(t, t_0)$  subject to the service stress being less than  $0.4 * f_{cm}$  Equation (5) and the total shrinkage strain ( $\epsilon_{cs}$ ) is the summation  $\epsilon_{cds}$  of dry and autogenous shrinkage  $\epsilon_{cas}$  Equation (6) [7]

$$\phi(t, t_0) = \frac{\epsilon_{cc}(t, t_0)}{\epsilon_{ci}(t_0)} \quad (5)$$

$$\epsilon_{cs} = \epsilon_{CAS} + \epsilon_{CDs} \quad (6)$$

### II.IV Numerical models.

The arithmetic analysis was carried out to determine possibility of a relation between column axial forces among the prior carried out three analysis. Two sets of axial forces that is; (OSLA-SLA and OSL-CTFA ) were used to determine the existing correlation by carrying out regression analysis. Excel was used as a tool.

### II.V Validation.

#### Validation analytical model.

To validate the developed numerical model, a 40 storey RCC frame typical floor layout shown in Fig 3 [8], with 750x500 beam size, 1300x1300 mm columns size at the bottom ten floors reducing with 100 mm at every ten floors and 600 mm thick centre core wall at the bottom ten floor, reducing with 100 mm at every 10 floors and the slab dead load taken as 3.6 kN/m was used. Floor to floor height was 3000 mm and the construction sequence equal to 3 days per floor.

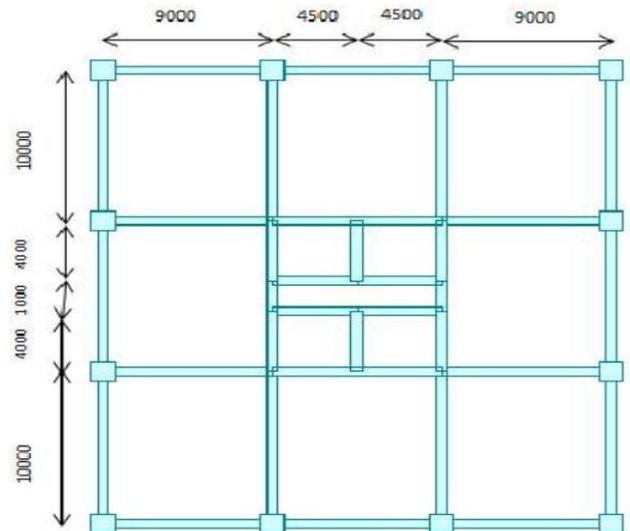


Fig. 3. Validation model Typical Floor Layout Plan [8]

The analysis was carried out using the MIDAS Gen construction stage analysis package with the material time-dependent properties determined using the PCA standard. The accuracy of the validation model was affirmed by comparing the column shortening results with the previous research on Predicting Axial Shortening of Vertical Elements in High Rise Buildings by Using PCA Method in 2016 [8]. The detailed material properties used are tabulated in Table 1.

#### Validation model material properties.

The equations below were used to determine the material properties with time were as shown below. [8] [9]

$$\text{Compressive strength } f'_c(t) = \frac{f_{28t}}{4.0+0.85t} \quad (7)$$

$$\text{Elastic Modulus } E_c(t) = 0.43W^{1.5}\sqrt{f'_c(t)} \text{ MPa} \quad (8)$$

$$\text{Shrinkage strain } \epsilon_{sh,t} = (\epsilon_{sh})_u * SH_{V:S} * SH_t * SH_H * SH_R \quad (9)$$

$$SH_t = \frac{t-t_s}{20.0e^{0.36(V:S)+(t-t_s)}} \quad (10)$$

$$\text{Creep strain } E_{cr,t} = \sigma * (\epsilon_{cr})_u * CR_{tr} * CR_{V:S} * CR_t * CR_H * CR_R \quad (11)$$

$$CR_t = \left( \frac{(t-t_r)^{0.6}}{10.6+(t-t_r)^{0.6}} \right) \quad (12)$$

Where:

- $t$  = Age of the concrete in Days
- $f_{28}$  = strength of concrete at 28 days
- $w$  = unit weight of concrete
- $(\epsilon_{sh})_u$  = Ultimate shrinkage strain.,
- $SH_{V:S}$  = Coefficient of volume to the surface area of the member

- $SH_t$  = Shrinkage with time
- $SH_H$  = Shrinkage with relative humidity
- $SH_R$  = Residual shrinkage of reinforced concrete.
- $t_s$  = time in days of initial wet curing.
- V: S Volume to surface area ratio.
- $\sigma$  = Acting stress
- $(\epsilon_{cr})_u$  = Specific creep
- $CR_{tr}$  = Age of concrete at loading
- $CR_{V,S}$  = Member size
- $CR_t$  = Creep with time
- $CR_H$  = Creep with relative humidity
- $CR_R$  = Residual creep of reinforced concrete
- $t'$  = time in days to first loading after concrete pouring

### III. RESULTS AND DISCUSSION.

From the CTFA, OSLA and SLA, the difference in axial forces due to the dead load for the four columns highlighted in Fig 2. were as tabulated in Table 2. The three analyses gave different values at every floor level as shown in Fig 4. From the observed value, the difference between OSLA with CFA and OSLA with SLA at specific level increased with an increase in the number of storeys above the level as shown in Fig 5 & 6. These findings are attributed to differential shortening resulting to axial load distribution by the horizontal elements between among the vertical elements [6]. Regression analysis was carried out between the two data which unveiled result in existence of a polynomial and linear relationship as shown by Equation (13&14) and Fig (7&8) with a coefficient of the determinant of 0.9901 and 0.9877 respectively.

$$\hat{y}_{cl} = \frac{2}{10^5} x_{cl}^2 + \frac{4199}{1000} x_{cl} - 61.228 \quad (13)$$

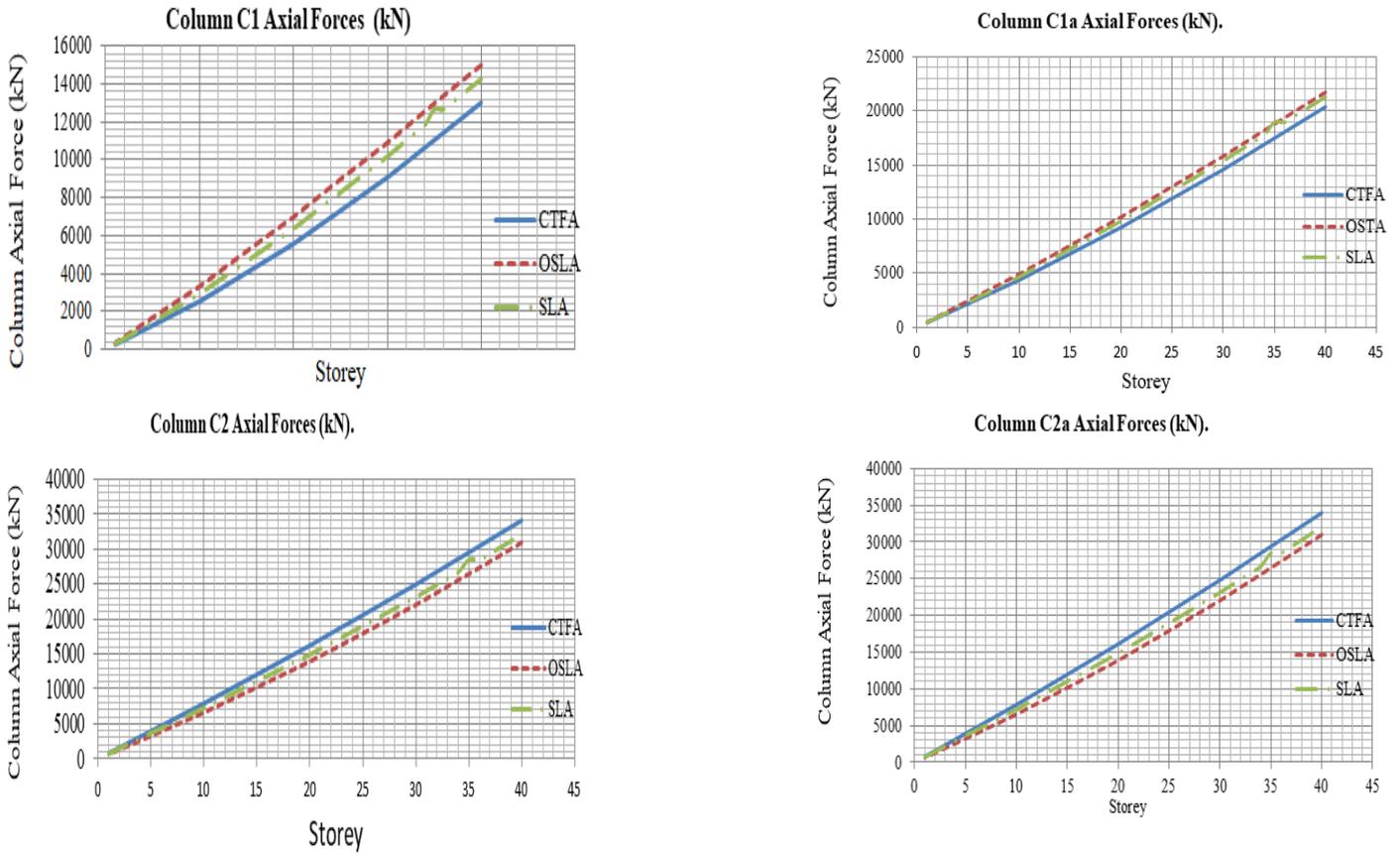
$$\hat{y}_{cl} = \frac{249}{625} x_{cl} - 17.635 \quad (14)$$

$$\hat{y}_{cl} = OSLA - SLA \quad (15)$$

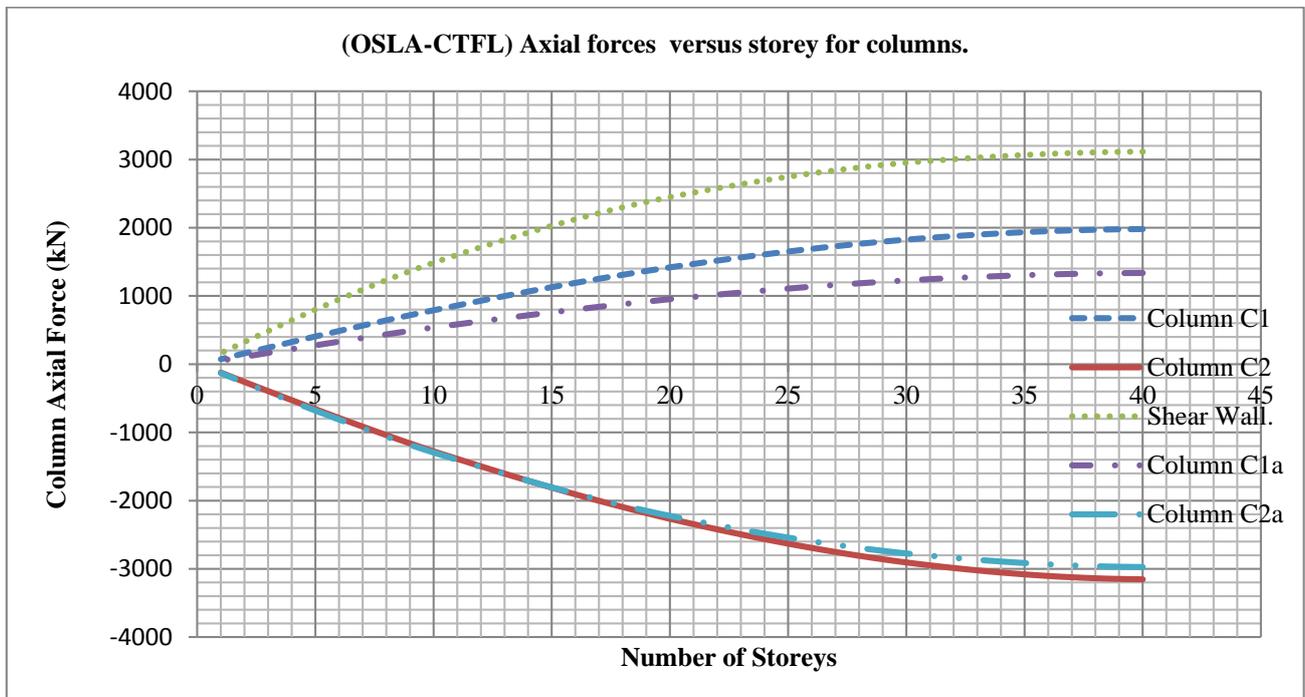
$$x_{cl} = OSLA - CTF \quad (16)$$

Table 1. Validation analytical model material data.

Storey	Section. (mm)	Compressive Strength (N/mm <sup>2</sup> )	Relative Humidity (%)	Ultimate Creep Strain. (1/F <sub>c</sub> *10 <sup>-3</sup> )	Ultimate Shrinkage Strain. (10 <sup>-6</sup> )	Volume Strain Ration (mm)	% Reinforcement used.	E <sub>Steel</sub> (Gpa)
<b>Columns.</b>								
1-10	1300x1300	50	70	4	780	325	0.8	200
11-20	1200x1200	45	70	4	780	300	0.98	200
20-30	1100x1100	40	70	4	780	275	1.18	200
31-40	1000x100	35	70	4	780	250	1.45	200
<b>Core Wall.</b>								
1-10	600	50	70	4	780	282(9 m wall), 261(4 mm wall)	0.4	200
11-20	500	45	70	4	780	237(9 m wall), 222(4 mm wall)	0.4	200
20-30	400	40	70	4	780	192(9 m wall), 182(4 mm wall)	0.4	200
31-40	400	35	70	4	780	145(9 m wall), 140(4 mm wall)	0.4	200



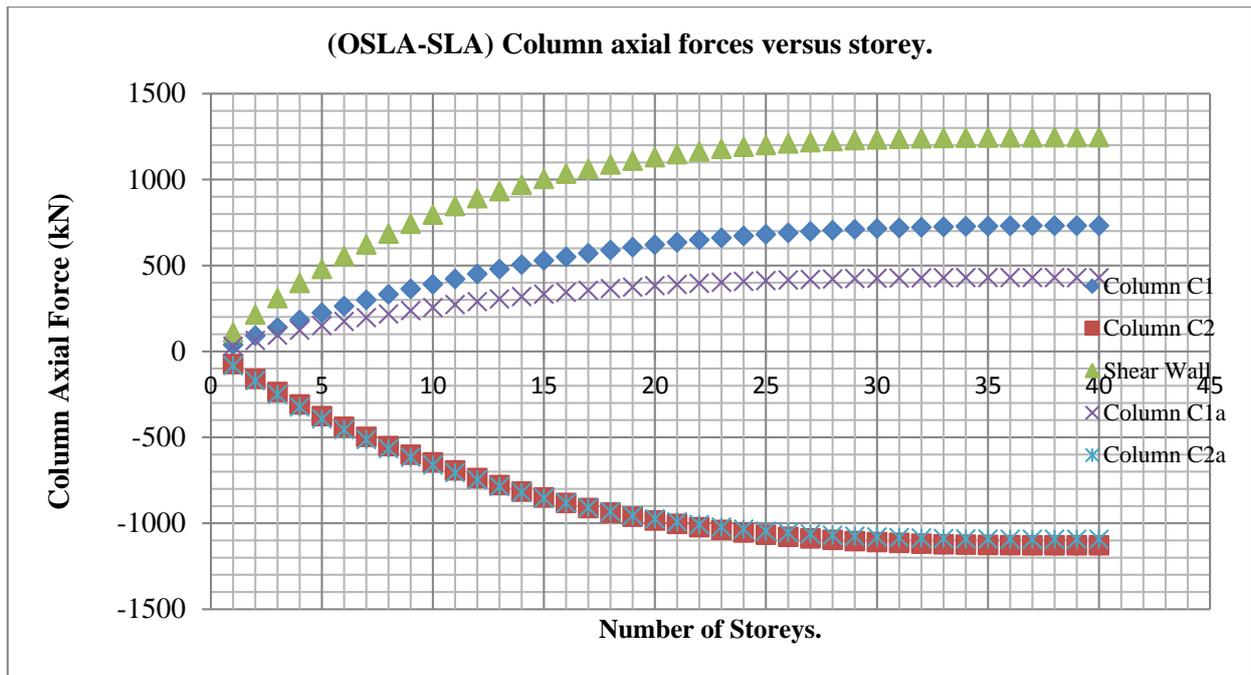
**Fig. 4.** Curve of CTFA, OSLA & SLA columns axial forces against the number of storeys.



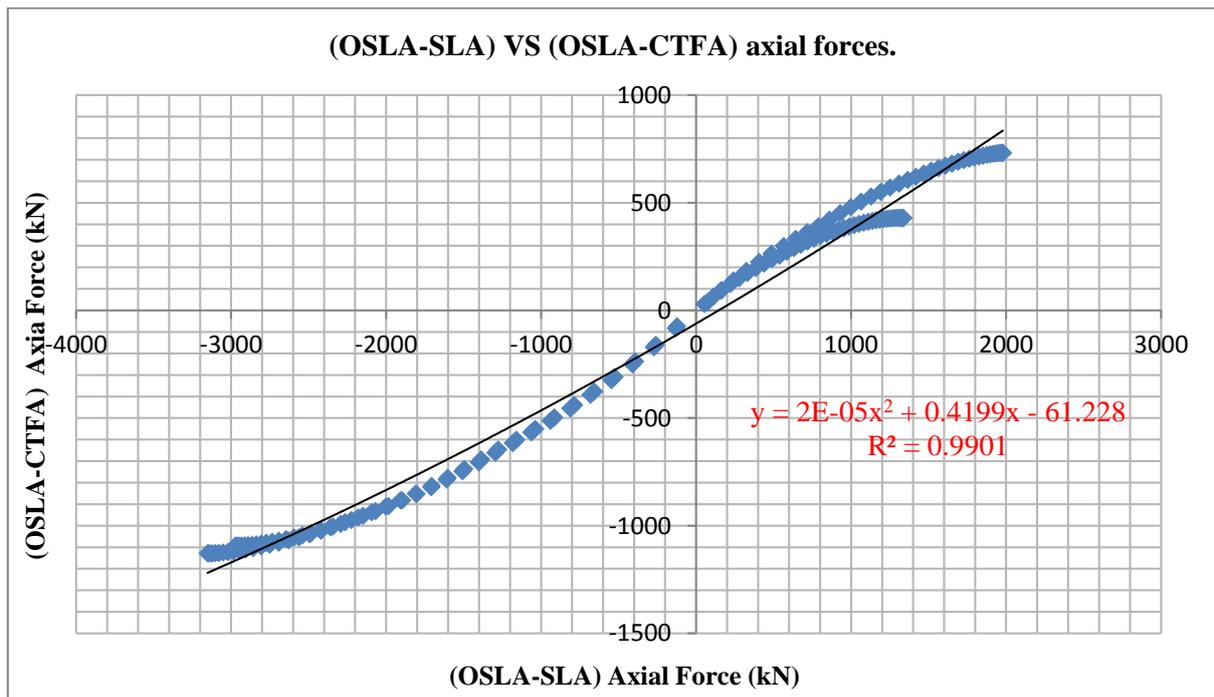
**Fig. 5.** (OSLA - CTFL) columns Axial forces versus number of storeys.

**Table 2.** Column axial forces differences between OSLA with CTFA and OSLA with SLA. Used in mathematical modelling.

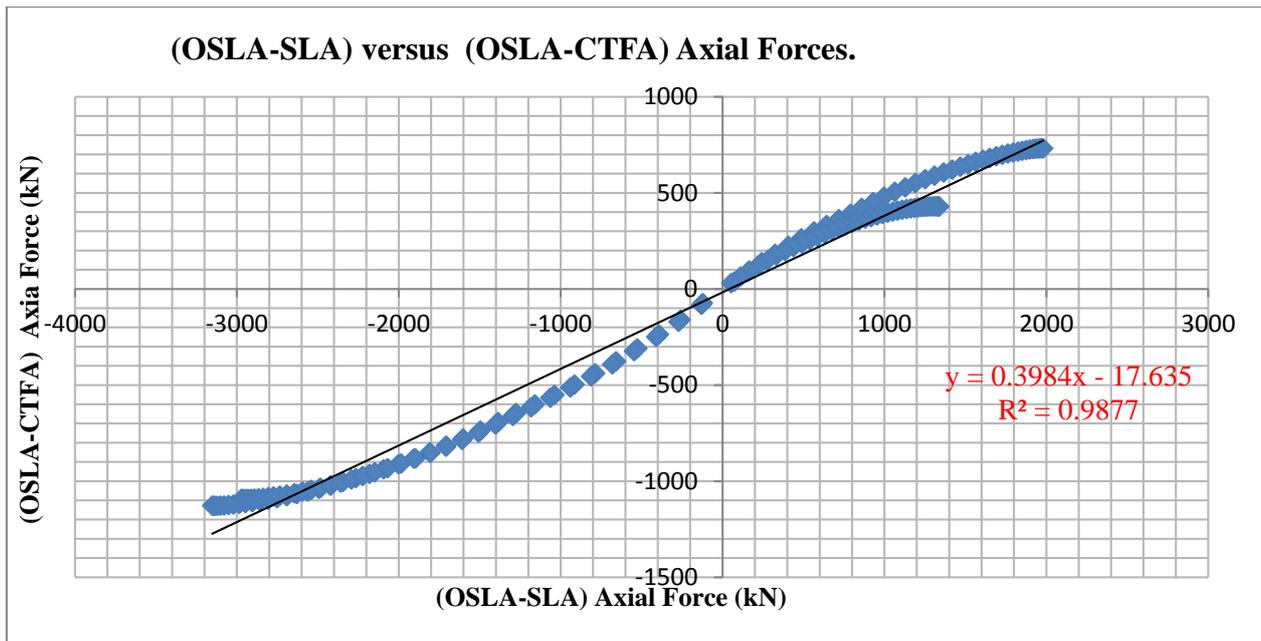
Structural Element.	Column C1 Axial Force (KN)		Column C2 Axial Force (KN)		Column C1A Axial Force (KN)		Column C2A Axial Force (KN)	
	(OTLA – CTFA)	(OSLA – SLA)	(OTLA - CTFA )	(OSLA - SLA)	(OTLA-CTFA)	(OSLA-SLA)	(OTLA - CTFA)	(OSLA-SLA)
1	1979.11	731.66	-3152.13	-1127.78	1337.4	428.91	-2972.64	-1092.54
2	1975.8	732.02	-3146.74	-1128.25	1334.99	429.25	-2968.34	-1092.82
3	1969.51	731.94	-3136.57	-1128.31	1330.45	429.48	-2960.11	-1092.92
4	1960.39	731.54	-3121.88	-1128.07	1323.93	429.69	-2948.15	-1092.83
5	1948.52	730.74	-3102.82	-1127.42	1315.48	429.78	-2932.62	-1092.46
6	1933.98	729.49	-3079.54	-1126.25	1305.19	429.72	-2913.68	-1091.73
7	1916.87	727.75	-3052.19	-1124.46	1293.13	429.47	-2891.49	-1090.54
8	1897.26	725.44	-3020.91	-1121.97	1279.37	428.97	-2866.19	-1088.79
9	1875.24	722.53	-2985.87	-1118.69	1263.99	428.18	-2837.95	-1086.41
10	1850.89	718.94	-2947.17	-1114.47	1247.06	427.06	-2806.9	-1083.29
11	1824.41	714.83	-2905.14	-1109.51	1228.71	425.67	-2773.29	-1079.48
12	1794.81	710.12	-2858.18	-1103.62	1208.22	423.95	-2735.07	-1074.85
13	1762.45	704.49	-2806.94	-1096.48	1185.93	421.77	-2692.56	-1069.2
14	1727.48	697.95	-2751.65	-1088.07	1161.95	419.11	-2646	-1062.44
15	1689.99	690.42	-2692.48	-1078.26	1136.37	415.91	-2595.58	-1054.42
16	1650.09	681.83	-2629.59	-1066.92	1109.26	412.11	-2541.52	-1045.03
17	1607.88	672.11	-2563.14	-1053.93	1080.7	407.64	-2484.01	-1034.13
18	1563.46	661.21	-2493.31	-1039.18	1050.77	402.47	-2423.24	-1021.59
19	1516.95	649.07	-2420.26	-1022.57	1019.56	396.55	-2359.43	-1007.33
20	1468.38	635.52	-2344.06	-1003.85	987.1	389.77	-2292.69	-991.13
21	1418.28	621.14	-2265.41	-983.66	953.67	382.4	-2223.59	-973.33
22	1365.18	606.07	-2181.58	-961.93	917.92	374.46	-2149.03	-953.89
23	1309.37	589.24	-2093.66	-937.73	880.57	365.53	-2069.49	-932.23
24	1251.19	570.85	-2002.07	-911.17	841.81	355.69	-1985.42	-908.31
25	1190.74	550.75	-1907.01	-882.06	801.73	344.84	-1897.1	-881.89
26	1128.17	528.87	-1808.7	-850.23	760.42	332.9	-1804.85	-852.77
27	1063.63	505.12	-1707.34	-815.51	717.99	319.82	-1708.96	-820.76
28	997.23	479.39	-1603.14	-777.72	674.5	305.48	-1609.7	-785.64
29	929.17	451.68	-1496.37	-736.81	630.08	289.88	-1507.42	-747.3
30	859.25	421.43	-1386.85	-692.1	584.7	272.73	-1402.13	-705.25
31	789.5	391.09	-1276.88	-646.15	539.04	255.05	-1295.56	-660.93
32	717.44	363.8	-1160.31	-601.08	489.28	237.56	-1182.82	-615.9
33	642.08	332.7	-1038.99	-550.82	437.81	217.99	-1063.86	-566.35
34	564.96	299.47	-914.75	-496.99	385.26	197.03	-940.28	-512.93
35	486.08	263.59	-787.71	-438.92	331.68	174.36	-812.51	-454.96
36	405.73	225	-658.32	-376.36	277.26	149.88	-681.2	-392.06
37	324.17	183.57	-526.98	-309.05	222.13	123.44	-546.98	-323.87
38	241.53	139.03	-393.96	-236.59	166.41	94.86	-410.34	-249.94
39	158.66	92.17	-260.32	-159.59	110.45	64.37	-272.33	-170.48
40	72.66	38.59	-122.96	-73.79	53.37	30.35	-131.26	-82.79



**Fig. 6.** (OSLA-SLA) Column axial forces versus the number of storey curves



**Fig. 7.** (OSLA-SLA) versus (OSLA-CTFA) axial forces



**Fig. 8.** (OSLA-SLA) versus (OSLA-CTFA) axial forces

The model validation results were as tabulated in Table 3 with the predicted results from both equation 13 & 14 showing a good correlation with the validation model column axial

forces as shown in figure 9 with values ranging between 94% and 105%) as shown in figure 10.

**Table 3.** Column C3, True and the predicted axial forces using the developed, polynomial and linear equations

Storey Level.	SLA Actual Value. (KN)	Linear Equation (KN)	Polynomial Equation. (KN)	Storey Level.	Summation (KN)	Linear Equation (KN)	Polynomial Equation.
40	400.4	379.736	423.337	20	8417.46	7835.53	7877.66
39	806.92	743.222	786.779	19	8829.26	8231.44	8273.4
38	1210.55	1106.4	1149.92	18	9239.56	8627.46	8669.24
37	1612.58	1469.62	1513.11	17	9648.63	9023.58	9065.19
36	2012.86	1832.86	1876.31	16	10056.42	9419.81	9461.23
35	2411.44	2196.12	2239.55	15	10462.95	9816.14	9857.37
34	2808.33	2559.44	2602.82	14	10868.2	10212.5	10253.6
33	3203.55	2922.79	2966.13	13	11272.14	10609	10649.8
32	3597.15	3286.2	3329.51	12	11674.78	11005.5	11046.1
31	3988.74	3649.62	3692.88	11	12075.93	11402	11442.4
30	4395.58	4027.94	4071.13	10	12494.14	11815.8	11856
29	4802.61	4406.57	4449.69	09	12911.77	12229.5	12269.4
28	5207.86	4785.28	4828.3	08	13327.82	12643.2	12682.9
27	5611.7	5164.11	5207.05	07	13742.45	13056.8	13096.2
26	6014.12	5543.06	5585.91	06	14567.11	13883.6	13922.6
25	6415.15	5922.15	5964.9	05	14567.11	13883.6	13922.6
24	6814.81	6301.37	6344.01	04	14977.03	14296.8	14335.5
23	7213.12	6680.73	6723.26	03	15385.1	14709.8	14748.3
22	7610.13	7060.22	7102.63	02	15791.98	15122.4	15160.8
21	8005.62	7439.82	7482.1	01	16193.69	15535	15573.1

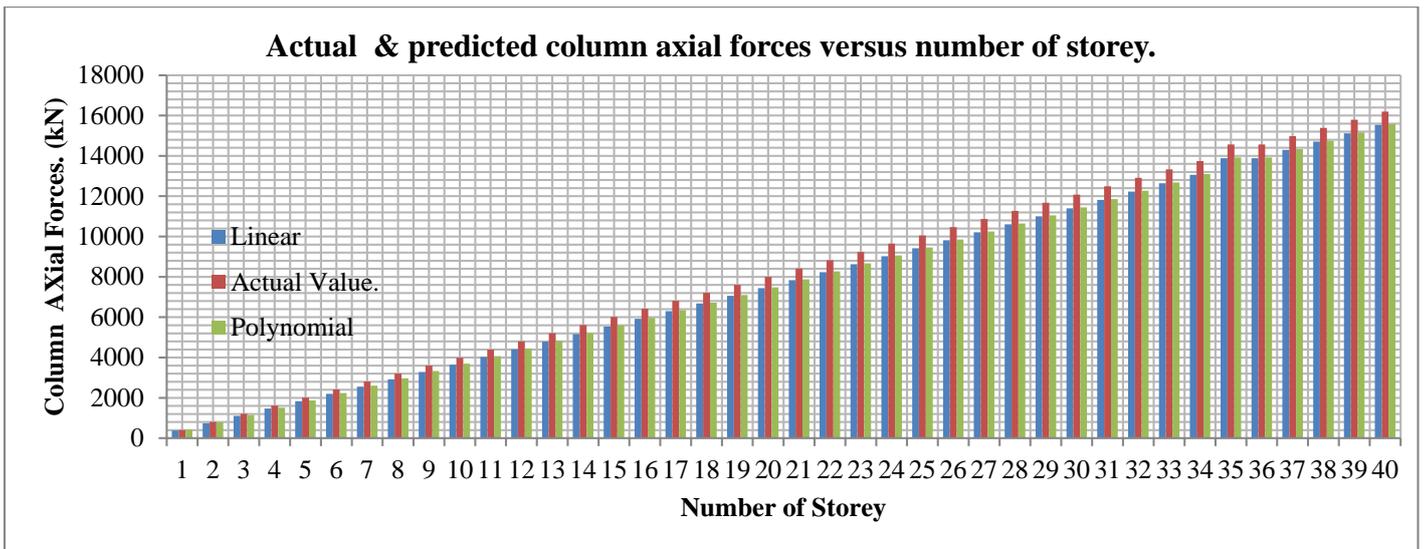


Fig. 9. Correlation between Column C3 true value and predicted axial force from the developed linear and polynomial equations

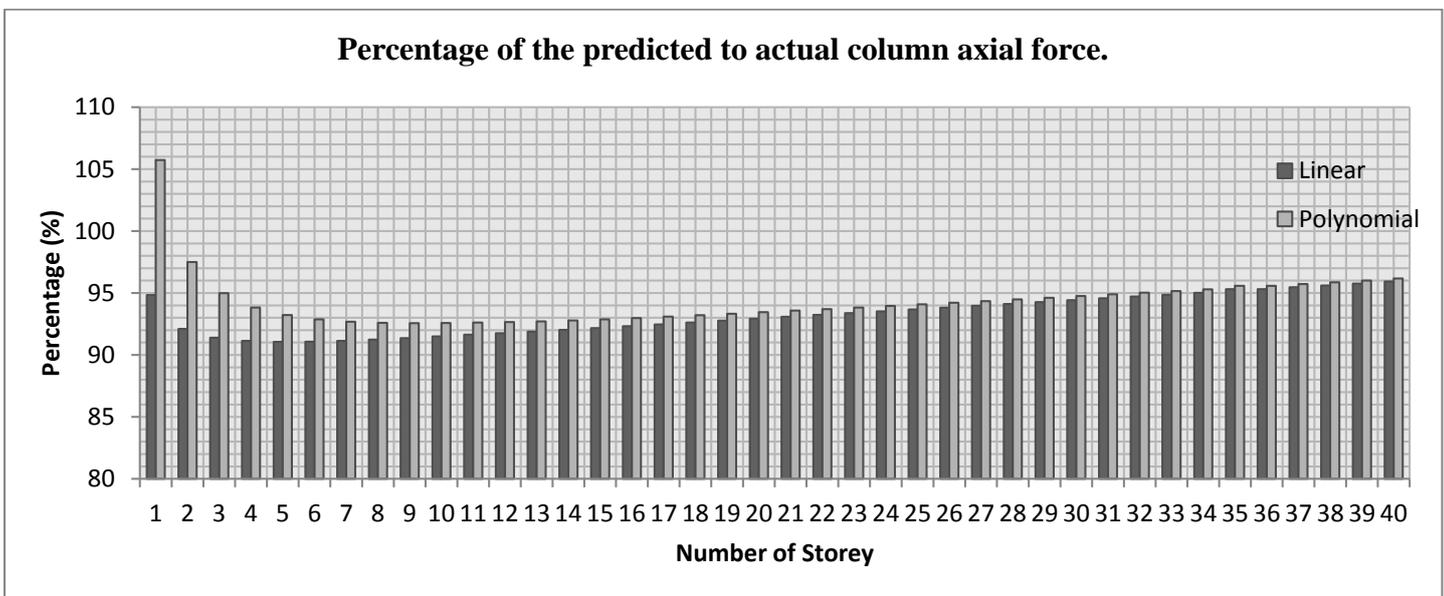


Fig. 10. Percentage (%) of the predicted Column C3 axial force to the true value.

#### IV. CONCLUSION.

From the result cumulative typical floor, one step loading and sequential loading analysis for the forty story building, the column axial forces due to dead loads were analysed and the results showed that:

1. In relation to the sequential loading analysis results, one-step loading analysis results are exaggerated on the higher side for external columns that would result in overdesigning whereas, in cumulative typical floor loading, the effect is reversed. In regard to the inner columns, the effect is the vice versa thus the need for sequential analysis for high rise buildings.
2. There exist a 2<sup>nd</sup>-degree polynomial and a linear equation between the column axial forces difference between the

one-step loading with sequential loading and one step loading with cumulative typical floors analysis.

3. Further work is to be carried out to determine the effect in variations of the number of floors, concrete loading age and humidity).

#### V. ACKNOWLEDGEMENT

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