Estimation of Clear-Sky Solar Radiation Using ASHRAE Model for Aligarh, India

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Abstract

Solar radiation is a function of various parameters viz., solar altitude, location (latitude and longitude), site altitude, albedo, atmospheric transmittance and cloudiness. However, clear sky solar radiation is independent of cloudiness index. Analyzing the solar radiation variation on a clear day provides a basis for comparison of cloudiness index for days falling under different sky conditions. Solar radiation (global, diffuse and direct) variation on horizontal surfaces is presently calculated using ASHRAE clear-sky model. The calculated values are validated against the measurements for Aligarh, India (27.88˚N, 78.08˚E, 192 m.s.l.). Mean bias error, root mean square error and t-statistic methods are used to analyze model performance. The results show that the ASHRAE clear-sky model is suitable to estimate hourly solar radiation in Aligarh, India.

Index Terms - ASHRAE, clear-sky, estimation, India, statistics.

I. INTRODUCTION

Knowledge of global solar radiation and its components (direct and diffuse solar radiation) is required for analysis of solar energy conversion systems. In many solar energy applications, the solar energy incident on the surface of the earth is an essential requirement for assessment of the performance and evaluation of efficiency of solar energy systems. Also, detailed analysis of solar radiation data is necessary to estimate solar energy potential of the site [1]. For this, radiation measurements of high quality at several stations covering all major climate zones of a region are necessary. But
because of high economical cost and maintenance of the equipments only a limited stations have been setup around the world for monitoring of solar radiation.

Estimation of solar radiation received at ground level is thus an important task. This is often complicated by varying sky conditions. Solar irradiance and sky luminance distributions are highly influenced by many factors, including solar altitude, sky turbidity and pollution level of the atmosphere and cloud type and amount [2]. These factors all together cause interactions of sunlight and skylight leading to intermittent nature of solar radiation.

In most practices, only bright sunshine hours and global solar radiation are measured and used for development of models [3]. Consequently, solar radiation components such as direct and diffuse were estimated and derived using an empirical formula. Estimations of the clear sky solar radiation for limited locations are present in various works [4-6].

Although, it is interesting to note that estimation of clear sky radiation forms the basis of comparison of cloudiness index and various other associated indices [7].

In the present work, the ASHRAE clear-sky model is used to estimate the monthly average hourly global solar radiation on horizontal surfaces in Aligarh, India (27.88˚N, 78.08˚E, 192 m.s.l). The task here is to present the theoretical values of the estimation of hourly clear sky solar radiation for Aligarh based on the ASHRAE method. The results of the present study will enable us to predict how much solar radiation can be expected on average at Aligarh, on hourly basis, during any given month of the year.

The measurements of the global radiation on a horizontal surface have been monitored using Kipp and Zonen CM-11 pyranometer and recorded continuously in its datalogger (Kipp and Zonen LogBox Datalogger) connected with a computer in the Solar Energy Laboratory, Department of Mechanical Engineering, Aligarh Muslim University, Aligarh (India).

II. METHOD FOR ESTIMATION OF SOLAR RADIATION
ASHRAE [8] recommended the estimation of hourly global radiation \( I \), hourly beam radiation in the direction of rays \( I_{b} \), and hourly diffuse radiation \( I_{d} \) on the horizontal surface for clear day using following equations, respectively:

\[
I = I_{b} \cos \theta_{z} + I_{d} \\
I_{b} = A \exp(-B/\cos \theta_{z}) \\
I_{d} = CI_{b}
\]

where, \( A \) is the apparent solar-radiation constant, \( B \) is the atmospheric extinction coefficient, and \( C \) is the diffuse sky factor.

Cosine of the zenith angle \( \cos \theta_{z} \) is given by the equation:

\[
\cos \theta_{z} = \cos \delta \cos \phi \cos \omega + \sin \delta \sin \phi
\]

where, \( \delta \) is the solar declination, \( \phi \) is the latitude of the station, and \( \omega \) is the hour angle. The solar declination \( \delta \) is given as [9]:

\[
\delta = 23.43 \sin \left[ \frac{360\left(284 + 0.669 \cos \omega - 0.626 \sin \omega \cos \omegaight)}{365} \right]
\]
where, ‘n’ is the n\textsuperscript{th} day of the year starting from 1\textsuperscript{st} January. The hour angle (ω) is an angular measure of time and is equivalent to 15° per hour with morning (-) and afternoon (+). It is measured from noon-based local solar time (ST) from the equation given by:

\[
ω = 15(12.0 − ST)
\] (6)

The local solar time (ST) is calculated from the local standard time (LT) and the equation of time (ET) is given later by:

\[
ST = LT + ET/60 - 4/60(LS - LL)
\] (7)

where, L\textsubscript{S} is the standard meridian (= 45°) for the local time zone (longitude of the time zone) and L\textsubscript{L} is the longitude of the location in degrees west (0° < L\textsubscript{L} < 360°). The equation of time (ET) is obtained from formulae given by Tasdemiroglu [10] as:

\[
ET = 0.87 \sin B − 7.53 \cos B - 1.50 \sin B
\] (8)

where B is given as:

\[
B = \frac{360 (n − 81)}{365}
\] (9)

ASHRAE clear-sky model is, thus, completely specified by substituting the values of the following nine parameters, namely: A, B, C, ET, LS, LL, and local standard time (LT). The model solar data for parameters A, B and C given by ASHRAE [9] for each month are given in Table 1. Other parameters are specific to the location of interest; their values for Aligarh are given in within this article. The final parameter; namely the local standard time (LT) is the only varying parameter which is input for calculations at any required time in the day.

**Table 1. Constants for ASHRAE equations for the 21st day of each month**

<table>
<thead>
<tr>
<th>Months</th>
<th>A, Wm\textsuperscript{-2}</th>
<th>B, dimensionless</th>
<th>C, dimensionless</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jan 21</td>
<td>1229.475</td>
<td>0.142</td>
<td>0.058</td>
</tr>
<tr>
<td>Feb 21</td>
<td>1213.713</td>
<td>0.144</td>
<td>0.060</td>
</tr>
<tr>
<td>Mar 21</td>
<td>1185.340</td>
<td>0.156</td>
<td>0.071</td>
</tr>
<tr>
<td>Apr 21</td>
<td>1134.900</td>
<td>0.180</td>
<td>0.097</td>
</tr>
<tr>
<td>May 21</td>
<td>1103.375</td>
<td>0.196</td>
<td>0.121</td>
</tr>
<tr>
<td>Jun 21</td>
<td>1087.613</td>
<td>0.205</td>
<td>0.134</td>
</tr>
<tr>
<td>Jul 21</td>
<td>1084.460</td>
<td>0.207</td>
<td>0.136</td>
</tr>
<tr>
<td>Aug 21</td>
<td>1106.528</td>
<td>0.201</td>
<td>0.122</td>
</tr>
<tr>
<td>Sep 21</td>
<td>1150.663</td>
<td>0.177</td>
<td>0.092</td>
</tr>
<tr>
<td>Oct 21</td>
<td>1191.645</td>
<td>0.160</td>
<td>0.073</td>
</tr>
<tr>
<td>Nov 21</td>
<td>1220.018</td>
<td>0.149</td>
<td>0.063</td>
</tr>
<tr>
<td>Dec 21</td>
<td>1232.628</td>
<td>0.142</td>
<td>0.057</td>
</tr>
</tbody>
</table>

It is to be noted however, that the solar parameters given in Table 1 are for the 21\textsuperscript{st} day of each month. For the present analysis, the ASHRAE clear-sky model is run for
every day in the year. Therefore, the values of the solar parameters (A, B, and C) for
days other than the 21st day in each month are obtained by linear interpolation.
Constants for ASHRAE equations for the average values of each month are given in
Table 2.

### III. Statistical Comparison of Solar Radiation

The most widely used statistical indicators are mean bias error (MBE), root mean
square error (RMSE), and t-statistics [11-17]. In the present study also, MBE, RMSE,
and t-statistics are calculated to assess the ASHRAE model performance.

In the following, we describe the three statistical parameters to evaluate a model’s
performance in estimating a value.

#### Table 2. Constants for ASHRAE equations for the average values of each month

<table>
<thead>
<tr>
<th>Months</th>
<th>A, Wm⁻²</th>
<th>B, dimensionless</th>
<th>C, dimensionless</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jan 17</td>
<td>1229.882</td>
<td>0.142</td>
<td>0.058</td>
</tr>
<tr>
<td>Feb 16</td>
<td>1216.255</td>
<td>0.144</td>
<td>0.060</td>
</tr>
<tr>
<td>Mar 16</td>
<td>1190.407</td>
<td>0.153</td>
<td>0.068</td>
</tr>
<tr>
<td>Apr 15</td>
<td>1144.663</td>
<td>0.175</td>
<td>0.092</td>
</tr>
<tr>
<td>May 15</td>
<td>1109.680</td>
<td>0.192</td>
<td>0.116</td>
</tr>
<tr>
<td>Jun 11</td>
<td>1092.697</td>
<td>0.202</td>
<td>0.130</td>
</tr>
<tr>
<td>Jul 17</td>
<td>1084.880</td>
<td>0.207</td>
<td>0.136</td>
</tr>
<tr>
<td>Aug 16</td>
<td>1102.986</td>
<td>0.202</td>
<td>0.124</td>
</tr>
<tr>
<td>Sep 15</td>
<td>1142.120</td>
<td>0.182</td>
<td>0.098</td>
</tr>
<tr>
<td>Oct 15</td>
<td>1183.449</td>
<td>0.164</td>
<td>0.077</td>
</tr>
<tr>
<td>Nov 14</td>
<td>1213.611</td>
<td>0.151</td>
<td>0.065</td>
</tr>
<tr>
<td>Dec 10</td>
<td>1228.004</td>
<td>0.145</td>
<td>0.059</td>
</tr>
</tbody>
</table>

The long term performance of a correlation in estimating a value is provided by
Mean Bias Error (MBE), which allows the comparison of the actual deviation between
the estimated and measured value of each term. A positive value gives the average
amount of overestimation in the estimated values and vice versa. The ideal value of
MBE is zero. It is defined as:

\[
MBE = \frac{1}{n} \sum_{i=1}^{n} (I_{\text{estimated}} - I_{\text{measured}})
\]  

where, \( I_{\text{estimated}} \) is the \( i^{th} \) estimated value of hourly solar radiation, \( I_{\text{measured}} \) is
the \( i^{th} \) measured value of hourly solar radiation, and \( n \) is the total number of

RMSE provides information on the short-term performance of an equation. The
smaller the value is, the better the equation’s performance. The value of RMSE is
always positive and ideally it should be zero [11].

\[
RMSE = \left( \frac{1}{n} \sum_{i=1}^{n} (I_{\text{estimated}} - I_{\text{measured}}) \right)^{1/2}
\]  

where, \( I_{\text{estimated}} \) is the \( i^{th} \) estimated value of hourly solar radiation, \( I_{\text{measured}} \) is
the \( i^{th} \) measured value of hourly solar radiation, and \( n \) is the total number of
The $t$-statistic is defined as the following [13]:

$$ t = \left( \frac{1 \cdot \text{SIM} - \text{SIM}}{\text{RMSE}^2} \right)^{1/2} $$

(12)

The smaller the value of $t$ is, the better the performance of the model. In order to determine whether a model’s estimates are statistically significant, one simply has to determine a critical $t$ value obtainable from standard statistical tables, i.e., $t_{\alpha/2}$ at the level of significance and $(k - 1)$ degrees-of-freedom. For the model’s estimates to be judged statistically significant at the $(1 - \alpha)$ confidence level, the calculated $t$ value must be less than the critical $t$ value [13].

**IV. RESULTS AND DISCUSSION**

Table 3 provides the calculated values (using ASHRAE model) and measured values obtained from pyranometer on the hourly basis for clear sky solar radiation ($I$) on the horizontal surface for Aligarh.

Figure 1 shows the plot of estimated and measured values on a typical clear day of June 11th, 2014. It can be observed graphically that the measured and estimated values shown are close to each other.

![Figure 1. Estimated and measured values of hourly clear sky solar radiation ($I$, W/m$^2$) on the horizontal surface for 08th July, 2014](image)
Figure 2(a). Monthly average hourly global solar radiation using ASHRAE clear-sky model for the months of January to June

Table 3. Estimated and measured horizontal surface hourly clear sky global solar radiation values (I, kW/m²)

<table>
<thead>
<tr>
<th>Solar Time</th>
<th>ASHRAE (17th May)</th>
<th>Measured (17th May)</th>
<th>ASHRAE (11th June)</th>
<th>Measured (11th June)</th>
<th>ASHRAE (08th July)</th>
<th>Measured (08th July)</th>
</tr>
</thead>
<tbody>
<tr>
<td>06:30</td>
<td>0.2029</td>
<td>0.1567</td>
<td>0.2281</td>
<td>0.2275</td>
<td>0.2214</td>
<td>0.0778</td>
</tr>
<tr>
<td>07:30</td>
<td>0.4362</td>
<td>0.3945</td>
<td>0.4536</td>
<td>0.4163</td>
<td>0.4466</td>
<td>0.2319</td>
</tr>
<tr>
<td>08:30</td>
<td>0.6463</td>
<td>0.5920</td>
<td>0.6558</td>
<td>0.5831</td>
<td>0.6486</td>
<td>0.4270</td>
</tr>
<tr>
<td>09:30</td>
<td>0.8177</td>
<td>0.7288</td>
<td>0.8207</td>
<td>0.7505</td>
<td>0.8132</td>
<td>0.5919</td>
</tr>
<tr>
<td>10:30</td>
<td>0.9389</td>
<td>0.8937</td>
<td>0.9372</td>
<td>0.8457</td>
<td>0.9294</td>
<td>0.7030</td>
</tr>
<tr>
<td>11:30</td>
<td>1.0016</td>
<td>0.9714</td>
<td>0.9974</td>
<td>0.9188</td>
<td>0.9895</td>
<td>0.5706</td>
</tr>
<tr>
<td>12:30</td>
<td>1.0016</td>
<td>0.9885</td>
<td>0.9974</td>
<td>0.9560</td>
<td>0.9895</td>
<td>0.7437</td>
</tr>
<tr>
<td>13:30</td>
<td>0.9389</td>
<td>0.8591</td>
<td>0.9720</td>
<td>0.9191</td>
<td>0.9294</td>
<td>0.7096</td>
</tr>
<tr>
<td>14:30</td>
<td>0.8177</td>
<td>0.7731</td>
<td>0.8207</td>
<td>0.8668</td>
<td>0.8132</td>
<td>0.6231</td>
</tr>
<tr>
<td>15:30</td>
<td>0.6463</td>
<td>0.5150</td>
<td>0.6558</td>
<td>0.7113</td>
<td>0.6458</td>
<td>0.4942</td>
</tr>
<tr>
<td>16:30</td>
<td>0.4362</td>
<td>0.3240</td>
<td>0.4536</td>
<td>0.5173</td>
<td>0.4466</td>
<td>0.3186</td>
</tr>
<tr>
<td>17:30</td>
<td>0.2029</td>
<td>0.1660</td>
<td>0.2281</td>
<td>0.2278</td>
<td>0.2214</td>
<td>0.1625</td>
</tr>
</tbody>
</table>

The corresponding monthly average hourly solar radiation results estimated using the ASHRAE clear-sky model with the original set of coefficients are presented in
Figure 2(a) for the months of January to June and in Figure 2(b) for the months of July to December.

The values of the monthly averaged daily global, diffuse, beam and extraterrestrial radiation on horizontal surfaces for Aligarh are presented in Figure 3. The values of the clear day monthly average daily radiation ($H_c$) are calculated for days giving the average of each month offered by Klein [18].

As can be observed from Figure 3, the radiation values obtained from the ASHRAE equation are all maximum in June. The maximum value of the monthly average daily clear sky global radiation is about 30 MJ/m$^2$; monthly average daily diffuse solar radiation is about 4.5 MJ/m$^2$; monthly average daily beam radiation is about 24.9 MJ/m$^2$ calculated in June for the Aligarh location in India. This value is nearly the same with a measured value for June 11th 2014, as shown in Table 3.

The value of statistical comparators MBE, RMSE, and $t$-statistic are given in Table 4 for the three days used for validation. As observed from Table 4, the ASHRAE equation yield good results in terms of MBE, RMSE, and $t$-statistics.

Table 4. Values of calculated MBE, RMSE, and $t$-statistic for three clear days

<table>
<thead>
<tr>
<th>Day</th>
<th>MBE</th>
<th>RMSE</th>
<th>$t$-statistics</th>
</tr>
</thead>
<tbody>
<tr>
<td>17 May, 2014</td>
<td>0.2195</td>
<td>0.2288</td>
<td>1.374</td>
</tr>
<tr>
<td>11 June, 2014</td>
<td>0.2045</td>
<td>0.0560</td>
<td>1.297</td>
</tr>
<tr>
<td>08 July, 2014</td>
<td>0.1920</td>
<td>0.2001</td>
<td>1.977</td>
</tr>
</tbody>
</table>

Figure 2(b). Monthly average hourly global solar radiation (I) using ASHRAE clear-sky for the months of July-December
V. CONCLUSION
Based on the analysis presented, it is recommended that the ASHRAE clear-sky model can be used for estimation of clear sky solar radiation in Aligarh (India). Solar radiation incident on different surfaces, solar collectors, solar PV-panels etc., can be easily analyzed using the ASHRAE model. Also, the components of global solar radiation (beam and diffuse) can be calculated separately. Further, clear sky radiation can also form the basis for evaluation of various sky conditions.

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The authors would like to thank Prof. Naiem Akhtar, Department of Mechanical Engineering, Aligarh Muslim University, Aligarh (India) for providing meteorological data and support for the pyranometer and related equipments.

NOMENCLATURE
$H_c$ Monthly average daily clear day radiation (MJ/m$^2$)
$H_e$ Monthly average daily extraterrestrial radiation (MJ/m$^2$)
$H_b$ Monthly average daily beam radiation (MJ/m$^2$)
$H_d$ Monthly average daily diffuse radiation (MJ/m$^2$)
$I$ Hourly clear sky global radiation (W/m$^2$)
$I_{b}$ Hourly clear sky beam radiation (W/m$^2$)
$I_{d}$ Hourly clear sky diffuse radiation (W/m$^2$)
$\theta_z$ Zenith angle, the between the vertical and the line to the sun (°)
$\phi$ Latitude of site (°)
$\delta$ Declination angle (°)
$\omega$ Hour angle (°)
Estimation of Clear-Sky Solar Radiation Using ASHRAE Model

Figure 3. Monthly averaged daily global (H), diffuse (H_d), beam (H_b), and extraterrestrial radiation (H_0) on horizontal surfaces in Aligarh

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