Equivalent Circuit Model of Square Patch FSS Using Vector Fitting

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Abstract

The analysis and equivalent circuit modeling of square patch FSS adopting an efficient vector-fitting is proposed. The simulations of microstructure are performed with CST Microwave Studio on single-substrate for different physical parameters, angle of incidences and polarizations. Then circuit model is extracted and developed using vector-fitting tool and implemented in SPICE (Simulation Program with Integrated Circuit Emphasis) simulator for both time and frequency analyses. ADS SPICE generator is used for validating the proposed circuit model. The developed model is within 1% of average deviation against the results obtained from the EM simulation and traditional circuit model.

Keywords: Equivalent circuit model; Lumped components; Square patch FSS; Vector-fitting technique

Introduction

The equivalent circuit model approach to the design of FSSs is very popular because of the ease with which it can be understood. The evaluation of transmission and reflection properties of FSSs become simple and accurate with the help of circuit approach. Moreover, the approximate analysis, based on the parallel between real structure and a lumped-RLC-network counterpart is also useful to understand the working principles of FSSs [1, 6].

In this paper, we present the analysis and equivalent circuit modeling of square patch FSS [1, 6-7], as shown in Fig. 1, with resonant unit cells. For modeling purpose, vector fitting (VF) technique [2] is used for determination of poles and residues from simulated *S*-parameters obtained from full-wave simulation tool CST Microwave Studio [3]. Then the SPICE-compatible equivalent circuit model [4] of frequency-

domain responses approximated by rational functions is developed. The transmission and reflection properties evaluated through VF based circuit approach are within 1% of average deviation against reference data.



Fig. 1: (a) Layout of Square patch FSS and (b) its equivalent circuit.

Equivalent Circuit Modeling using Vector-Fitting

Given f(s) as a frequency response of the rational function approximation in VF [2], it can be expressed as follows:

$$f(s) = \sum_{n=1}^{N} \frac{c_n}{s - a_n} + d + sh$$
(1)

The residues c_n and poles a_n are either real quantities or come in complex conjugate pairs, while d and h are real. It gives the fractional terms that directly lead to fixed forms of R, L and C.

In this section, circuit representations for complex pairs obtained from VF and synthesis approach are presented briefly for the generation of SPICE [4] compatible equivalent circuits enabling both time and frequency analyses of cross and circular ring FSSs from three-dimensional models. The detailed methodology and expressions can be obtained from [4].

Real Poles

Given a pair of a pole and residue extracted by a VF procedure, the corresponding couple of R and L parameters are

$$L = \frac{1}{c_{RL}} \qquad R = -a_{RL}L = -\frac{a_{RL}}{c_{RL}}$$
(2)

Series RLC Complex Pole Pair

 R_s, L_s, C_s and $f_{add}(s)$ parameters from the equivalent circuit can be evaluated as

$$R_{s} = -\left(\frac{a_{1} + a_{2}}{c_{1} + c_{2}}\right) L_{s} = \frac{1}{(c_{1} + c_{2})} C_{s} = \left(\frac{c_{1} + c_{2}}{a_{1}a_{2}}\right) f_{add}(s) = p \left(\frac{1}{s^{2} + s\frac{R_{s}}{L_{s}} + \frac{1}{L_{s}C_{s}}}\right)$$
(3)



(a)



Fig. 2: (a) Equivalent *RL* circuit for real pole synthesis; and (b) Equivalent series *RLC* circuit for complex pole pair synthesis.



Fig. 3: Equivalent Π circuit.

The following synthesis approach has been adopted to achieve the SPICE-compatible equivalent circuit of square FSS microstructures:

Step 1) S-parameter extraction by means of simulation of FSS microstructure in CST Microwave Studio;

Step 2) Evaluation of ABCD parameters;

Step 3) Building of Π equivalent circuit as shown in Fig.3;

Step 4) Residues and poles extraction of admittances \overline{Y}_A , \overline{Y}_B and \overline{Y} ;

Step 5) Synthesis of SPICE-compatible equivalent circuit [4].

The VF technique [2] has been applied to extract poles and residues of admittances \overline{Y}_A , \overline{Y}_B and \overline{Y} , which have been synthesized in the equivalent circuit, as shown in Fig.2(a)-2(b), of FSS microstructure and simulated in a ADS SPICE [5] environment enabling both time and frequency analyses.



Fig. 4: Plane-wave reflection and transmission response for Square patch FSS.

Parametric Study and Model Validation

This section is dedicated to the validation of the presented equivalent circuit model for square patch FSS. Firstly, the microstructure is studied and investigated on singlesubstrate for different physical parameters using CST Microwave Studio [3]. The effects of oblique incidence and TE/TM polarization are also studied. The FSS design is simulated on FR4 substrate with permittivity $\varepsilon_r = 4.4$, loss tangent tan $\delta = 0.025$ and substrate thickness h = 1 mm. The geometrical parameters of the designed square patch FSS are w = 4 mm, d = 2 mm and L = 8 mm.

Fig.4 shows the parametric study of square patch FSS at normal and oblique incidences. For the sake of brevity, few results are shown here. Fig.4(a) and 4(b) show magnitude and phase of reflection and transmission characteristics respectively for different patch sizes. It can be observed that by increasing patch size there is shift in resonance towards higher frequency. By increasing cell size there is considerable shift in resonance peak towards lower frequency. By increasing substrate thickness, there is shift in resonance towards lower frequency.Fig.4(c) and 4(d) show the effect of oblique incidence and TE/TM polarization on magnitude of reflection and transmission characteristics of square patch FSS.

The reflection and transmission coefficients have been fitted by using 12 poles (two real poles and five complex pairs) [6]. The equivalent circuit has been designed and simulated in ADS SPICE generator using circuit for real and complex pole pair synthesis as shown in Fig.2. Fig.5 shows the compared results of magnitude and phase of reflection and transmission characteristics at normal incidence obtained from the developed circuit model using VF tool, circuit model available from literature and CST simulations for square patch FSS. An excellent agreement has been observed between SPICE simulation results with the distributed equivalent circuits obtained using the VF technique and 3D-EM simulation results for the FSS.

Thus, the proposed synthesis allows a satisfactory approximation of the square patch FSS being the percentage errors on magnitude and phase of the order of 1%. While the traditional equivalent circuits, shown in Fig.1, have the percentage errors on magnitude and phase of the order of 2%. In future work, this work will be extended to multilayer substrates.



Fig. 5. Square Patch FSS-(a) Magnitude and (b) Phase.

Conclusions

The present work reports the development of vector-fitting based equivalent circuit model of square patch FSS, with resonant unit cells. The simulations are performed with full wave simulation tool CST Microwave Studio on single-substrate for

different physical parameters, angle of incidences along with the effect of polarization. The VF tool is employed to extract equivalent circuit from *S*-parameters of FSS microstructure to use in circuit simulators. The developed circuit model has been verified using ADS SPICE generator and has usefulness both in frequency and time-domain analyses. The present model is within 1% of average deviation w.r.t. EM simulator and traditional circuit model.

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