H-Infinity Controller Design for a Continuous Stirred Tank Reactor

Ashutosh Kumar Mishra[1], Vijander Singh[2], Asha Rani[3]

[1,2,3]*Instrumentation and Control Engineering Division*
Netaji Subhas Institute of Technology, University of Delhi
Sec-3, Dwarka, New Delhi, India

Abstract

The paper presents a method for H-infinity controller design for a system affected by parametric uncertainty. The proposed method is based on minimising infinity norm of sensitivity function of the nominal system. The design approach is verified by robust stabilization and control of an unstable and perturbed continuous stirred tank reactor (CSTR) with hydrolysis of Propylene Glycol. The control input is volumetric flow rate of coolant and the controlled output is the temperature of CSTR. The reactor has three uncertain parameters and an unstable process, is controlled with the help of proposed controller. The results obtained justify that designed H-infinity controller provides good tracking and disturbance rejection performances.

Keywords- Continuous stirred tank reactor (CSTR), H-infinity, infinity norm, linear fractional transformation (LFT), structured uncertainty.

1-INTRODUCTION:

CSTR is a very essential and important plant for chemical process industry. The distinguished property of CSTR is that it posses nonlinearity, potential safety problem and possibility to have multiple steady states. Moreover some of CSTRs parameters are not exactly known. These parameters are reaction rate constants, reaction enthalpies, heat transfer coefficients, etc [1, 2]. Hence, there is always some variation between mathematical model and true process. Because of all these facts CSTR is a very challenging plant from the control point of view and it is very important to design robust controllers for such uncertain and nonlinear plants like CSTR [3, 4].

   H-infinity controller design methods are used in control theory to synthesize controllers achieving stability with guaranteed performance. To use H-infinity method a control designer expresses the control problem as a mathematical optimization

problem and then evaluates the controller parameters, these methods were introduced into control theory in the late 1970's-early 1980's by Zames [5] and Zames and Francis [6]. The aim of this paper is to design H-infinity controller for stabilization of a CSTR which is open loop unstable.

2-PROCESS DESCRIPTION:

The process under consideration is a continuous stirred tank reactor for production of propylene glycol from hydrolysis of propylene oxide with sulphuric acid as catalyst [1, 7]. The reaction is given by

$$CH_2 - O - CH - CH_3 + H_2O \rightarrow CH_2OH - CH_2OH - CH_3 \ (1)$$
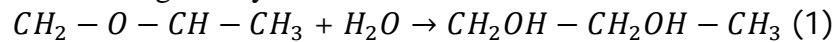
The simplified model of the CSTR system is governed by equation 3, 4 and 5. The mass balance of any component in reactor is given as

$$\frac{dC_A}{dt} = \frac{F}{V}(C_{Af} - C_A) - k_0 \exp\left(-\frac{E_a}{RT}\right)C_A \ (3)$$

The reactor energy balance and effect of cooling jacket dynamics is given as

$$\frac{dT}{dt} = \frac{F}{V}(T_f - T) + \frac{(-\Delta H)}{\rho C_p}k_0 \exp\left(-\frac{E_a}{RT}\right)C_A - \frac{UA}{V\rho C_p}(T - T_J) \ (4)$$

$$\frac{dT_j}{dt} = \frac{F_{js}}{V_j}(T_{jf} - T_j) + \frac{UA}{V_j\rho_j C_{pj}}(T - T_j) \ (5)$$

Where F is feed flow rate, C_A is molar concentration of feed, k_0 is the frequency factor, E_a is the activation energy, R is the ideal gas constant and T is the reactor temperature. ρ is the density, C_p is specific heat capacity, ΔH is the heat of reaction, U is the heat transfer coefficient, A is the heat transfer area, T_f is the feed temperature, and T_j is the jacket temperature. When the reactor is operated at 85% of the design volume and with a desired conversion of 50% (C_{As}=0.066 lbmol/ft^3), resulting in a reactor temperature of $101.1\,^OF$ and jacket temperature of $80\,^OF$ we obtain following state space model after linearising CSTR dynamics around steady state value. [8]

$$\dot{X} = AX + BU \ (6)$$
$$Y = CX + DU \ (7)$$

Where
$$X = [C_A - C_{As} \quad T - T_s \quad T_j - T_{js}]^T, U = F_{jf} - F_{jfs}, Y = [T - T_s \quad T_j - T_{js}]^T$$

and

$$A = \begin{bmatrix} -7.9909 & -0.013674 & 0 \\ 2922.9 & 4.55674 & 1.4582 \\ 0 & 4.7482 & -5.8977 \end{bmatrix}, B = \begin{bmatrix} 0 \\ 0 \\ -3.2558 \end{bmatrix}$$
$$C = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, D = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

3-CONTROLLER DESIGN:

Model uncertainties of the reactor follow from the fact that the three physical parameter $\Delta H, k_0$ and U are not known exactly. However it is assumed that their values are within certain known intervals and can be given as [9],

$$\Delta H = \overline{\Delta H}(1 + p_{\Delta H}\delta_{\Delta H}), k_0 = \overline{k_0}(1 + p_{k_0}\delta_{k_0}), U = \overline{U}(1 + p_U\delta_U)$$

Where$\overline{\Delta H}$, $\overline{k_0}$ and $\overline{U}$ are the nominal values of respective parameters given in Table 1. $p_{\Delta H}$, p_{k_0} and p_U and $\delta_{\Delta H}$, δ_{k_0} and δ_U represent possible perturbation in these parameters. In the present case, $p_{\Delta H} = 0.02$ $p_{k_0} = 0.09$, $p_U = 0.06$ and $-1 \le \delta_{\Delta H}, \delta_{k_0}, \delta_U \le 1$, which represents 2%, 9% and 6% uncertainties in ΔH, k_0 and U respectively. Taking all uncertainties into account the state space representation of CSTR is given as

$$G_{CSTR} = \begin{bmatrix} A & B_1 & B_2 \\ C_1 & D_{11} & D_{12} \\ C_2 & D_{21} & D_{22} \end{bmatrix}$$

Where

$$A = \begin{bmatrix} -7.9909 & -0.013674 & 0 \\ 2922.9 & 4.55674 & 1.4582 \\ 0 & 4.7482 & -5.8977 \end{bmatrix},$$

$$B_1 = \begin{bmatrix} 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.09 & 0.00 & 0.09 & 0.00 \\ 4.35 & 0.01 & 0.00 & 65.9 & 0.01 & 0.00 & 0.00 & 0.06 & 0.00 & 0.06 \\ 0.00 & 0.00 & 0.06 & 0.00 & 0.00 & 0.06 & 0.00 & 0.00 & 0.00 & 0.00 \end{bmatrix},$$

$$B_2 = \begin{bmatrix} 0.00 \\ 0.00 \\ -3.2558 \end{bmatrix},$$

$$C_1 = \begin{bmatrix} 0.00 & 0.00 & 0.00 & 3.99 & 732 & 0.00 & -3.99 & 0.00 & 0.00 & 0.00 \\ 0.21 & 10.0 & 0.00 & 0.00 & 0.00 & 5.58 & 0.00 & 0.00 & -0.27 & -1.46 \\ 0.00 & 0.00 & -5.58 & 0.00 & 0.00 & 0.00 & 0.00 & 1.46 & 0.00 & 0.00 \end{bmatrix}^T$$

$$D_{11=} \begin{bmatrix} 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 \\ 4.35 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 \\ 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 \\ 0.00 & 0.00 & 0.00 & 65.9 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 \\ 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 \\ 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 \\ 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 \\ 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 \\ 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 \\ 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 \end{bmatrix}, D_{12} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$C_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}^T, D_{21} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, D_{22} = 0$$

The uncertain behaviour of original system can be described by an upper LFT representation [10] with diagonal uncertainty matrix $\Delta= diag(\delta_{\Delta H}, \delta_{k_0}, \delta_U)$ which is given as

$$y = F_U(G_{CSTR}, \Delta)u$$

Δ is an unknown matrix, which is called uncertainty matrix. Such type of representation of uncertainty is called structured uncertainty [11].

3.1. DESIGN REQUIRMENT OF CLOSED-LOOP SYSTEM:

The design objective for the CSTR system in this study is to design a controller K which ensures the robust stability and performance of the closed-loop system. Robust stability is achieved if the closed-loop system for all $G = F_U(G_{CSTR}, \Delta)$, satisfies the performance criterion [12]

$$\left\| \begin{matrix} W_p(I + GK)^{-1} \\ W_u(I + GK)^{-1} \end{matrix} \right\|_\infty < 1 \quad (8)$$

where W_p is the performance weighting function, W_u is control weighting function and $\|.\|_\infty$ is infinity norm. The block diagram of closed loop system showing the feedback structure is given in figure 3.
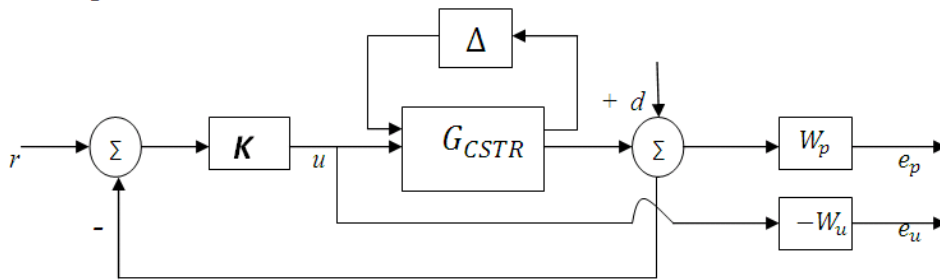

Fig. 3. The block diagram of closed loop system

4. RESULTS AND DISCUSSION:

The H-infinity controller is synthesised with the help of MATLAB version R2011 (7.13.0.564). The obtained performance weighting function W_p is given as

$$W_p = .95 \frac{s + 6}{s^2 + 4.26s + 28.3}$$

The control weighting function W_u is simply obtained as $W_u = 10^{-2}$. The PID controller for comparative analysis is tuned with the help of Ziegler Nichols and Tyreus Luyben method. The tuning parameters are given in table1. Fig.4 shows the comparison of H-infinity controller to the PID controllers.

Table 1. Tuning parameters of PID controller

	Kp	Ki	Kd
Ziegler Nichols	-16.264	-3.597	-2.092
Tyreus Luyben	-9.46	-2.52	-2.13

The robustness of the controller is verified by performing set point tracking and disturbance rejection analysis. The set point is varied from 101.5 ^0F to 111 ^0F at 9 second and from 111 ^0F to 91^0F at 18 second. It is indicated from the results (fig.4) that H-infinity controller is better than the designed PID controllers. The disturbance of two units is introduced at 10th second and the results are shown in fig.5. The synthesised controller achieves infinity norm within specified range and it also stabilises the unstable perturbed CSTR.
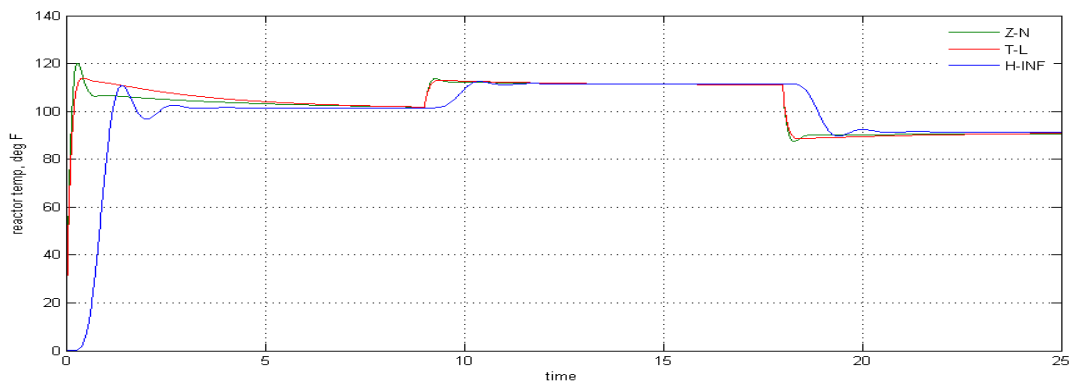

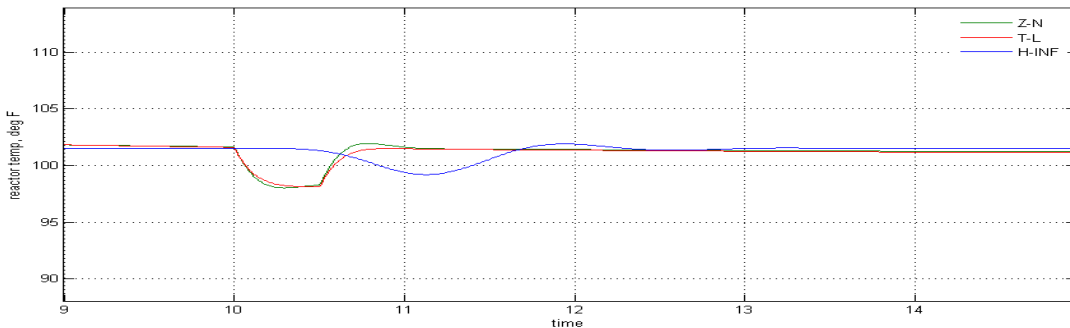**Fig. 4.**Set point tracking for designed controllers

Fig.5. Disturbance rejection analysis using designed controllers

5. CONCLUSION:

In this paper, design of robust H-infinity controller to control unstable and uncertain CSTR is presented. The aim is to control the unstable reactor at its operating point in the presence of uncertainties and disturbance. The design of H-infinity controller is based on minimising the infinity norm of the sensitivity function to obtain the desired performance criterion. The designed controller is also tested for set point tracking and disturbance rejection. The synthesised controller ensures good disturbance attenuation and good transient response and achieved norm is within specified limits. Hence it is concluded that H-infinity controller is a robust and more efficient controller as compared to the PID controllers.

REFERENCES-

[1]   A. Molnar, J. Markos, L. Jekmensky, "Accuracy of mathematical model with regard to safety analysis of chemical reactors", *chemical papers, vol. 56 no. 6, pp. 357-361, 2002.*

[2]   R. Waschkr, S. Pushpavanam, A. Kienle," Multiple steady states in two phase reactors under boiling conditions ", *Chemical Engineering Science, vol. 58, no. 11, pp. 2203-2214, 2003*

[3]   J. Gao, H. M. Budman," Design of robust gain-scheduled PI controllers for nonlinear processes ", *Journal of Process Control, vol. 15, no. 7, pp. 807-817, 2005.*

[4]   Y. Rao, J. Bhat, M. Chidambaram, K. Madhavan," Robust controllers for nonlinear chemical processes", *Computer & Chemical Engineering, vol. 16, no. 1, pp. 61-65, 1992.*

[5]   G. Zames," Feedback and optimal sensitivity: Model reference transformations, multiplicative seminorms and approximate inverses", *IEEE Transactions on Automatic Control, AC-26, pp. 301-320, 1981.*

[6]   G. Zames, B. A. Francis," Feedback, minimax sensitivity, and optimal robustness", *IEEE Transactions on Automatic Control, AC-28, pp. 586-600, 1983.*

[7]   M. Bakasova, D. Puna, P. Postal, K. Vanekova," Control of system with parametric uncertainties using a Robust PI controller", *AT & P Journal plus, no.2, pp. 84-87, 2007.*

[8]   B. Wayne Bequette, Process Modelling, Design, and Simulation, Prentice Hall of India, 2006.

[9]   M.K.H. Fan, A. L. Tits, J. C. Doyle, " Robustness in the presence of mixed parametric uncertainty and un-modelled dynamics", *IEEE Transactions on Automatic Control, AC-36, pp. 25-38, 1991.*

[10]   R. M. Redheffer," On a certain linear fractional transformation", *Journal of Mathematics and Physics, 39, pp. 269-286, 1960.*

[11]   J.C. Doyle, "Analysis of feedback systems with structured uncertainties", *IEE Proceedings-part-D, 129.pp. 242-250, 1982.*

[12]   D. W. Gu, P. Hr. Petkov, M. M. Konstantinov, Robust Control Design with MATLAB, Springer-Verlog London Limited, 2005.