Mathematical and Simulation Model of an SPWM Inverter

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Abstract

Inverter probably forms the only solution to an individual customer in need of high quality of power for critical loads. In this paper simulation study of inverter designed with 50 Hz transformers Sub topology has been reported. The results reported in the paper are based on the actual design of SPWM inverter and its simulation carried out based on mathematical model. Study of output voltage regulation with respect to variation in Battery voltage, sampling frequency and for different loads is carried out. It was observed that in the battery range 7.5 to 12.2V the regulation is within + 10% of expected output. However it is necessary to trip the battery voltage beyond 13V. The sampling frequency at 10 KHz and below provides substantially constant output near 230V which tends to increase for higher sampling frequencies. The load variation study indicates that the regulation is better in the range 450 to 540W.

Keywords: SPWM, Duty ratio (D), Mathematical Model, Vorms.

Introduction

An inverter converts the DC electricity from sources such as batteries, solar panels, or fuel cells to AC electricity. The electricity can be at any required voltage; in particular it can operate AC equipment designed for mains operation. The Battery voltage is maintained constant with the help of controlled charger in the presence of mains supply, however in the absence of mains decrease in the battery voltage tends to decrease the output voltage. Variation in load can also cause change in the output.
voltage. Constant output voltage inverters are used in the applications in notebook computers, web browsers, automotive and industrial instrumentation, and entertainment systems. A wide range of sine wave inverters are available. The topologies used by these inverters are also wide spread. The most common (popular) topology is high frequency switching in the primary using a Toroidal transformer with an output filter. Other popular options are the high frequency switching with a DC link in the secondary and CVT. Although all these methods seem to offer the same result, (i.e. is a sine wave output), their output waveform as well as the efficiency often get completely distorted due to certain critical loads. Typically, such loads are fluorescent lights with electronic ballasts, computers, televisions, microwave ovens and so on. The relevant work carried out is summarized below.

Ing. E. Ortjohann et al\(^{(1)}\) have reported work on simulation model for expandable hybrid power systems which investigates architecture of autonomous hybrid power system based on utilization of DC bus, DC/AC inverter, converter for power source. The control and management units show ability to prioritize the functionality of power supplies such that cheaper one gets highest priority. The architecture overcomes the limitations on expandability.

M. Ciobotaru et al\(^{(2)}\) have simulated a photovoltaic energy conversion system for inverter with its control system. Power system is simulated by using Matlab/ Simulink simulates the power system while PLECS toolbox simulates the control system. Both tools provide simplicity in circuit making and speed up the simulation process.

A. Rao et al\(^{(3)}\) have reported studies on the reduction technique of common mode EMI in single phase PWM inverters by means of active cancellation method. This study aims at balancing the switching pattern of the converter to reduce EMI.

Gawie J vd Merwe des\(^{(4)}\) have described the differences in design of sine wave inverter topologies with their advantages and disadvantages.

Henric Kragh \(^{(5)}\) presents studies on modeling, analysis and optimization of power electronic circuits for low cost drives.

The different inverter topologies in use are.

**Inverter employing constant voltage transformers**
Referred as Ferro resonant transformer inverter. However they suffer from the drawbacks as low efficiency (40 to 80%) on account of saturation associated with magnetic components, non-linear loads e.g. thyristor chopper leads to instability. Non-linear loads such as computers, fluorescent lights, etc. often cause waveform distortion.

**Inverter designs with 50 Hz transformers**
They are most common. These inverters have merits like relatively “clean” sine wave is generated, with very simple and limited control. Good reliability owing to isolation and electronic simplicity. But suffers from the demerits such as, the transformer should be overrated to compensate for eddy losses and high efficiency. There are three sub topologies of inverter design with 50 Hz transformer which are as below.

**Sub topology(1)**
This topology(1) has good number of merits as all MOSFETS (IDEAL switches) are
referred to ground thus simplifies the control and reduces the cost, there is always current flow path for inductive loads and the primary devices are protected by series line inductor. But only difficulty is it leads to skewing of transformer.

**Sub topology(2)**
The sub topology(2) has limited merits like current rating of inductor is smaller and the required value is higher. Also manufacturing of inverter is simpler. But suffers from disadvantages like the push-pull topology often leads to device destruction. So control and protection is required to compensate for this. Due to energy in the inductor, voltage overshoot on the MOSFET is high, therefore additional snubbers are required to protect against this. There is a very limited control over energy stored in the inductor. The MOSFET needs to be voltage overrated. This leads to unnecessary conduction losses.

**Sub topology(3)**
This Sub topology(3) has advantages like it has no overshoots on the MOSFETs and due to the flexibility of choice between Uni-polar and Bi-polar PWM control the switch frequency can be controlled and varied up to as high as 20 kHz. But it suffers from the demerits like, as MOSFETs are not referenced to ground the gate drive becomes more complex. Units bigger than 1KVA are not portable. Due to relative high impedance between primary and secondary (transformer & inductor), non-linear loads do cause voltage distortion on the output.

**Inverter design with High frequency isolated converter and separate secondary inverter control bridge**
Above topology has advantages such as the efficiency is not influenced by the non-linear load, direct control of the secondary bridge leads a better sine wave control. But it suffers from the demerits such as these inverters are more complex and expensive. Increased number of component count. Only for short period overload rating is possible. This can be increased but at a higher price. Additional EMI and RFI filtering components required, which increases the price.

**Inverter design with more than one series transformer**
This topology has advantages like it is easy to configure as a bi-directional inverter. Output voltage regulation is good. But suffers from disadvantages such as this inverter is complex to manufacture. It is heavy and bulky. It’s control strategy is complex and it’s output wave is not pure sine wave. Here sub topology (1) of Inverter designs with 50 Hz transformers is selected because it is cost effective, the control circuit is less complex and it is not mandatory to provide electrical isolation. This topology therefore is more suitable for all ranges of inverters and UPS systems. We have developed a mathematical and simulation model. The circuit diagram of SPWM inverter is shown in fig.1 circuit diagram.
Functioning
In this circuit IC 555 is used to generate square wave signal of frequency 10KHz. A 50Hz square signal is derived from this 10 KHz signal and it is converted into sinusoidal signal with the help of an integrator and a diode clipper circuit. The dc shift of sinusoidal signal is varied according to the changes in the output voltage and output current (a function of load). This is used to control the pulse width of a square wave generated by IC 555. This SPWM (Sinusoidal Pulse Width Modulation) signal is used to switch the MOSFETS present in the push pull centre tapped inverter. An inductor is added in secondary to remove high frequency components.

Mathematical Model
The fig.2 shows the waveform representation of SPWM generation In this method of modulation the width of each pulse is varied proportional to the amplitude of a sine-wave evaluated at the centre of the same pulse. By comparing a sinusoidal reference signal with a triangular carrier wave of frequency (fc), the gating signals are generated. The frequency of reference signal(fr), determine the inverter output frequency (fo) and it’s peak amplitude Vr controls the modulation index, M, and then in turn the rms output voltage(Vo).The modulation index(M) mathematically expressed as
\[ M = \frac{V_r}{V_c} \]  

Here Vr is the peak amplitude of reference signal and Vc is the peak amplitude of carrier signal. The number of pulses per half cycle depends upon on the carrier frequency. Within the constraint that two MOSFETS cannot conduct at the same time. By varying the modulation index (M), the rms output voltage can be varied. It can be observed that the area of each pulse corresponds approximately to the area under the sine-wave between the adjacent midpoints of off periods on the gating signals. If Pm is the width of the mth pulse then the rms value of output voltage can be mathematically xpressed as
This SPWM is used as an exciting signal to the power switches present in the inverter section.

**Figure 2:** Sinusoidal pulse width modulation (SPWM).

**MATLAB/Simulink environment**

to effectively design a power control system and accurately predict its performance, designers must understand the behavior of the entire system in which the control system resides. MATLAB and Simulink form the core environment for Model-Based Design for creating accurate, models of physical system behavior. The block diagram of simulation model is as shown in fig.3. The graphical, block-diagram paradigm of the MATLAB/Simulink environment lets the user drag-and-drop predefined modeling elements, connect them together, and create models of dynamic systems. These dynamic systems can be continuous-time, multi-rate discrete-time, or any combination of the three.

**Figure 3:** Block diagram of simulation model.
Simulation Model
The details of simulation model are as shown in fig.4. The pulse generator block (sampling frequency) with negative bias voltage gives the bidirectional 50% duty cycled square waveform. It is converted into triangular signal by an integrator. The amplitude of this triangular waveform is amplified by a gain block. The sine wave with 50Hz frequency is derived from signal generator block. This sine wave is subtracted from triangular wave and result is compared with zero. If result is less than or equal to zero then output of comparator block is set to one otherwise zero. Thus SPWM is generated. This SPWM is splitted into two separate 180 degree out of phase waveforms with a NOT and AND gates phase splitter circuit. This drives the MOSFETs (ideal switches) connected to the primary terminals of centre tapped transformer (12V to 230V step up mode 600VA) alternately biased by Battery voltage(Vb). The secondary of transformer is coupled to an LC filter and load. The voltage feed back is derived from the rectifier and attenuator circuit. The error is generated by comparing this fed back dc voltage with set point. The result is added with sinusoidal wave. This varies the duty cycle so as to keep the output voltage constant.

The different submodels of the PWM inverter are as shown in fig.05 to fig 09.

Figure 4: Simulation Model of an inverter.

Figure 5: Simulation model to generate triangular wave.

Figure 6: Simulation model to generate PWM wave.
Figure 7: Simulation model of phase splitter.

Figure 8: Simulation model of Inverter section.

Figure 9: Simulation model of Feed Back section.

Results
From graph shown in fig.10. It is seen that over the range of battery voltage from 7.5V to 13.6V the Vo shows increase in magnitude. However in the range of battery voltage 7.5V to 13.2V it gives good regulation within + 10% variation of expected
output for a battery voltage between 7.5V to 13V. However there is a drastic change in Vo when the battery voltage goes beyond 13V. Thus the battery should be tripped during charging over 13V. From fig. 11 it is seen that for sampling frequency below 10KHz the Vorms remains substantially constant near 230V. For higher sampling frequency the Vorms increases, however it remains almost constant near 262V.

Fig. 12 shows the variation of Vo against load. Initially it is high then decreases from 438V at 350W load and there after it remains fairly regulated. However it indicates desired regulation for the loads between 450W to 540W.

The whole simulation results are as shown in fig. 13.

**Figure 10:** Vo (V) V/S Battery Voltage (V).

Battery Voltage = 12V Load = 500W

**Figure 11:** Vo (V) V/S Sampling Frequency (Hz).

Battery Voltage = 12V Sampling Frequency = 10 KHz
Conclusion
The simulation study of inverter indicates that the regulation offered is better in the battery range 7.5 to 13.2V with sampling frequency below 10 KHz for load variation 450 to 550W offers case study under taken. However there are overshoots in the output variations as a function of battery voltage which forces the battery voltage to be tripped down.

References


