Modeling and Simulation of Permanent Magnet Synchronous Motor using MATLAB

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Abstract:

This paper presents, the mathematical model of a field oriented controlled Permanent Magnet motor drive system which is developed & simulated in MATLAB-simulink. Today in many industries (especially in machine tool industry) the use of Permanent Magnet Synchronous Motor (PMSM) is increasing due to smaller size, less weight & low rotor loss compare to induction motor of same capacity. One of the important characteristics of Permanent Magnet Synchronous Motor is that they produce constant torque with sinusoidal stator current. The simulation model includes all the required component of the drive system. Due to this the calculation of current and voltages in different parts of the inverter and motor under transient and steady conditions are possible. A closed loop control system in the speed loop has been designed to operate in constant torque and flux weakening regions. The principle of speed control is verified by analyzing the variation in the results of speed & torque.

Keywords: PMSM motor, Field Oriented control, constant torque.

I. Introduction

Permanent Magnet Synchronous Motors (PMSM) are widely used in low and mid power applications such as computer peripheral equipment, robotics, adjustable speed drives and electric vehicles. The growth in the market of Permanent Magnet motor drives has demanded the need of simulation tools capable of handling motor drive simulations. Simulations have helped the process of developing new systems including motor drives, by reducing cost and time.

T. Sebastian [1] reviewed Permanent Magnet Synchronous Motor advancements and presented equivalent electric circuit models for such motors and compared
computed parameters with measured parameters. Experimental results on laboratory motors were also given. P. Pillay in [2] presented Permanent Magnet motor drives and classified them into two types such as Permanent Magnet Synchronous Motor drives (PMSM) and Brushless DC Motor (BDCM) drives. The PMSM has a sinusoidal back emf and requires sinusoidal stator currents to produce constant torque while the BDCM has a trapezoidal back emf and requires rectangular stator currents to produce constant torque. B. Cui [3] addresses the modeling and simulation of permanent magnet synchronous motor supplied from a six step continuous inverter based on state space method. The motor model was derived in the stationary reference frame and then in the rotor reference frame using Park transformation. The simulation results obtained showed that the method used for deciding initial conditions was very effective.

In this paper simulation of PMSM is done in MATLAB Software. The control of speed is implemented using speed control & torque control loops. The results of simulation are discussed. The result obtained validates the advantages of PMSM motor.

II. Basic Model of the System.
The motor drive consists of four main components, the PM motor, inverter, control unit and the position sensor. The components are connected as shown in Figure 1.

![Figure 1: Schematic of the Drive System](image)

The main components of the system are:
1. Permanent Magnet Synchronous Motor
2. Various Position Sensors

2.1 Optical Encoders
Optical encoder is very popular type of encoder in industry. This consists of a rotating disk, a light source, and a photo detector. The disk, is mounted on the rotating shaft, has coded patterns of opaque and transparent sectors. As the disk rotates, these patterns interrupt the light emitted onto the photo detector, generating a digital pulse
or output signal.

2.2 Incremental encoders
Incremental encoders have good precision and are simple to implement but they suffer from lack of information when the motor is at rest position and in order for precise position the motor must be stop at the starting point.

2.3 Absolute encoders
The absolute encoder exact position of the rotor with a precision directly related to the number of bits of the encoder. It can rotate indefinitely and even if the motor stops, the position can be measured or obtained.

2.4 Position Revolver
Position revolver is also known as rotary transformers. It works on the transformer principle in which the primary winding is placed on the rotor. Depending upon the rotor shaft angle the induced voltage at the two secondary windings of the transformer is shifted by 90°. The position can be calculated using two voltages.

3. Inverter
Voltage Source Inverters are devices that convert a DC voltage to AC voltage of variable frequency and magnitude. They are very commonly used in adjustable speed drives and are characterized by a well defined switched voltage wave form in the terminals.

4. Controller
The controller will generate the reference currents with the inverter within a range which is fixed by the width of the band gap. In this controller the desired current of a given phase is summed with the negative of the measured current. The error is fed to a comparator having a hysteresis band. When the error crosses the lower limit of the hysteresis band, the upper switch of the inverter leg is turned on. But when the current attempts to become less than the upper reference band, the bottom switch is turned on.

5. Load
For inverter power electronics switches are used. The switch is selected on the basis of its power capability and switching. Table 1 shown below illustrates power devices and their switching capabilities.

<table>
<thead>
<tr>
<th>Device</th>
<th>Power Capability</th>
<th>Switching Speed</th>
</tr>
</thead>
<tbody>
<tr>
<td>BJT</td>
<td>Medium</td>
<td>Medium</td>
</tr>
<tr>
<td>GTO</td>
<td>High</td>
<td>Low</td>
</tr>
<tr>
<td>IGBT</td>
<td>Medium</td>
<td>Medium</td>
</tr>
<tr>
<td>MOSFET</td>
<td>Low</td>
<td>High</td>
</tr>
<tr>
<td>THYRISTOR</td>
<td>High</td>
<td>Low</td>
</tr>
</tbody>
</table>
Current regulators for AC drives are complex because an AC current regulator must control both the amplitude and phase of the stator current. The AC drive current regulator forms the inner loop of the overall motion controller. As such, it must have the widest bandwidth in the system and must have zero or nearly zero steady-state error. Both current source inverters (CSI) and voltage source inverters (VSI) can be operated in controlled current modes. The current source inverter is a natural current supply and can readily be adapted to controlled current operation. The voltage source inverter requires more complexity in the current regulator but offers much higher bandwidth and elimination of current harmonics as compared to the CSI and is almost exclusively used for motion control applications.

III Equivalent Circuit of Permanent Magnet Synchronous Motor

Equivalent circuits of the motors are used for study and simulation of motors. From the d-q modeling of the motor using the stator voltage equations the equivalent circuit of the motor can be derived. Assuming rotor d axis flux from the permanent magnets is represented by a constant current source as described in the following equation $\lambda_f = L_{dm}i_f$, illustrated in the below Figure 2.

![Figure 2: Permanent Magnet Synchronous motor Electric Circuit without Damper Windings](image)

Detailed modeling of PM motor drive system is required for proper simulation of the system. The d-q model has been developed on rotor reference frame as shown in figure 3. At any time $t$, the rotating rotor d-axis makes and angle $\theta_r$ with the fixed stator phase axis and rotating stator mmf makes an angle $\alpha$ with the rotor d-axis. Stator mmf rotates at the same speed as that of the rotor.
The model of PMSM without damper winding has been developed on rotor reference frame using the following assumptions:
1. Saturation is neglected
2. The induced EMF is sinusoidal.
3. Eddy currents and hysteresis losses are negligible.
4. There are no field current dynamics.

Voltage equations are given by
\[ V_q = R_s i_q + \omega_r \lambda_d + \rho \dot{\lambda}_q \] ............................................ (1)
\[ V_d = R_s i_d - \omega_r \lambda_q + \rho \dot{\lambda}_d \] ............................................ (2)

Flux Linkages are given by
\[ \lambda_q = L_q i_q \] ............................................................. (3)
\[ \lambda_d = L_d i_d + \lambda_f \] ................................................... (4)

Arranging equations in matrix form
\[
\begin{bmatrix}
V_q \\
V_d
\end{bmatrix} =
\begin{bmatrix}
R_s + \rho L_q & \omega_r L_d \\
-\omega_r L_q & R_s + \rho L_d
\end{bmatrix} +
\begin{bmatrix}
i_q \\
i_d
\end{bmatrix} +
\begin{bmatrix}
\omega_r \lambda_f \\
\rho \lambda_f
\end{bmatrix}
\] ............................................................ (5)

The developed torque motor is being given by
\[ T_e = \frac{3}{2} \times \frac{p}{2} (\lambda_d \lambda_q - \lambda_q \lambda_d) \] ............................................................. (6)

The mechanical Torque equation is
\[ T_e = T_L + B \omega_m + J \frac{d\omega_m}{dt} \] ............................................................. (7)

Solving for the rotor mechanical speed from equation
\[ \omega_m = \int (T_e - T_L - \frac{B \omega_m}{dt}) dt \] ............................................................. (8)
Park Transformation Converting the phase voltages variables \( V_{abc} \) to \( V_{dqo} \)

Variables in rotor reference frame the following equations are obtained

\[
\begin{bmatrix}
V_q \\
V_d \\
V_o
\end{bmatrix} = \frac{2}{\sqrt{3}} \begin{bmatrix}
\cos \theta_r \cos (\theta_r - 120) & \cos (\theta_r + 120) \\
\sin \theta_r \sin (\theta_r - 120) & \sin (\theta_r + 120) \\
1/2 & 1/2 & 1/2
\end{bmatrix}
\begin{bmatrix}
V_a \\
V_b \\
V_c
\end{bmatrix}
\]

Convert \( V_{dqo} \) to \( V_{abc} \)

\[
\begin{bmatrix}
V_a \\
V_b \\
V_c
\end{bmatrix} = \begin{bmatrix}
\cos \theta_r & \sin \theta_r & 1 \\
\cos (\theta_r - 120) \sin (\theta_r - 120) & \sin (\theta_r + 120) & 1 \\
\cos (\theta_r + 120) \sin (\theta_r + 120) & \sin (\theta_r - 120) & 1
\end{bmatrix}
\begin{bmatrix}
V_d \\
V_o
\end{bmatrix}
\]

Control of PM motors is performed using field oriented control for the operation of synchronous motor as a dc motor. The stator windings of the motor are fed by an inverter that generates a variable frequency variable voltage. Instead of controlling the inverter frequency independently, the frequency and phase of the output wave are controlled using a position sensor.

Considering the currents as inputs, the three currents are

\[
i_a = I_m \sin (\omega_r t + \alpha) \]
\[
i_b = I_m \sin (\omega_r t + \alpha - \frac{2\pi}{3}) \]
\[
i_c = I_m \sin (\omega_r t + \alpha + \frac{2\pi}{3})
\]

Writing equations in the matrix form

\[
\begin{bmatrix}
i_a \\
i_b \\
i_c
\end{bmatrix} = \begin{bmatrix}
\cos (\omega_r t + \alpha) \\
\cos (\omega_r t + \alpha - \frac{2\pi}{3}) \\
\cos (\omega_r t + \alpha + \frac{2\pi}{3})
\end{bmatrix}
\begin{bmatrix}
I_m
\end{bmatrix}
\]

Constant torque control strategy is derived from field oriented control, where the maximum possible torque is desired at all times like the dc motor. This is performed by making the torque producing current \( i_q \) equal to the supply current \( I_m \). That results in selecting the ‘\( \alpha \)’ angle to be 90º degrees according to equation.

Flux weakening is the process of reducing the flux in the d axis direction of the motor which results in an increased speed range. The motor drive is operated with rated flux linkages up to a speed where the ratio between the induced emf and stator frequency (V/f) is maintained constant. After the base frequency, the V/f ratio is reduced due to the limit of the inverter dc voltage source which is fixed. The weakening of the field flux is required for operation above the base frequency. This reduces the V/f ratio. This operation results in a reduction of the torque proportional to a change in the frequency and the motor operates in the constant power region.

The realization process of equivalent flux-weakening control is as follows:

1) Measuring rotor position and speed \( \omega_r \) from a sensor which is set in motor rotation axis.
2) The motor at the flux weakening region with a speed loop \( T_e \) is obtained from the
PI controller.

3) Calculate $i_q$ using equation
$$i_q = \frac{T_e}{(\frac{3}{2})P_2\lambda_f} \quad \text{......................................} \quad (15)$$

4) Calculate $i_d$ using equation
$$i_d = \frac{(\lambda_d - \lambda_f)}{L_d} \quad \text{......................................} \quad (16)$$

5) Calculate $\alpha$ using equation
$$\alpha = Tan^{-1}(\frac{i_q}{i_d}) \quad \text{......................................} \quad (17)$$

6) Using $\alpha$ and rotor position the controller will generate the reference currents as per equation
$$[i_a] = \begin{bmatrix} \cos(\omega_r t + \alpha) \\ \cos(\omega_r t + \alpha - \frac{2\pi}{3}) \\ \cos(\omega_r t + \alpha + \frac{2\pi}{3}) \end{bmatrix} [I_m] \quad \text{......................................} \quad (18)$$

7) Then the current controller makes uses of the reference signals to control the inverter for the desired output currents.

8) The load torque is adjust to the maximum available torque for the reference speed
$$T_L = T_e(\text{rated}) \frac{\omega_{\text{rated}}}{\omega_r} \quad \text{......................................} \quad (19)$$

**IV Drive System in Matlab / Simulink**

Figure 4 shown below is a Simulink model of PMSM drive system consisting of PMSM mechanical model, step load, relay, sub-system blocks, Multiplexers (MUX), De-multiplexers (DEMUX) and different Scopes for screening the results.

The Constant block generates a real or complex constant value. Also, the block can generate either a sample-based or frame-based signal, depending on the setting of the Sampling mode. The current reference and the output of speed reference block are compared. The input from speed reference is taken through PI controller, where the PI controller is the Subsystem block which represents a subsystem of the system that contains it. The Subsystem block can represent a virtual subsystem or a non-virtual subsystem. The heart of the system is Permanent magnet Synchronous Motor model block is a Subsystem to which d-axis & q-axis voltages are given. The output of the PMSM Mechanical model which are d-axis & q-axis current along with torque produced are connected to the various Scopes after converting to abc model, which displays inputs signals with respect to simulation time. The function of relay is to switch output between two constants, whereas step load is used to Generate step function and also provides a step between two definable levels at a specified time.
Figure 4: Permanent Magnet Synchronous Motor Drive System in Simulink

Figure 5 shows the d and q axis motor circuits which are built using simulink elements. In this model it is considered that this two axis PMSM stator winding have equal turns per phase. The rotor flux can be assumed to be concentrated along the d axis while there is zero flux along the q axis. Further it is assumed that the machine core losses are considered to be negligible. Also, rotor flux is assumed to be constant at a given operating point. Various parameters are taken with the help of constant available in simulink elements for armature resistance, number of poles, d-axis and q-axis inductance to develop the mathematical model. The values of these motor constants are as mentioned in Table 2.

Figure 5: d-axis and q-axis circuit

Table 2: Interior Permanent Magnet Motor Parameters

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Name</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>J</td>
<td>Motor Inertia</td>
<td>8x10^-4 kg m^2</td>
</tr>
<tr>
<td>L_d</td>
<td>d-axis Inductance</td>
<td>8.5 mH</td>
</tr>
<tr>
<td>L_q</td>
<td>q-axis Inductance</td>
<td>8.5 mH</td>
</tr>
<tr>
<td>R_a</td>
<td>Armature Resistance</td>
<td>2.875Ω</td>
</tr>
<tr>
<td>P</td>
<td>Number of Poles</td>
<td>8</td>
</tr>
<tr>
<td>λ</td>
<td>PM Flux</td>
<td>0.175wb</td>
</tr>
</tbody>
</table>
V Simulation Result & Discussion

The proposed method built in simulink for a PMSM drive system has been tested with the two current control methods, Hysteresis and PI controller, at the constant torque and flux-weakening regions of operation. The motor is operated with constant torque up to its rated speed and beyond that rated speed flux-weakening mode is adopted. Figure 6 shows the three phase currents drawn by the motor as a result of the hysteresis current control. The currents are obtained using Park’s reverse transformation. It is clear that the current is non sinusoidal at the starting and becomes sinusoidal when the motor reaches the controller command speed at steady state.

![Figure 6: Currents with Hysteresis Control at 200 rad/s](image)

Figure 6: Currents with Hysteresis Control at 200 rad/s

Figure 7 shows $i_q$ and $i_d$ components of current with respect to time and it is observed that, initially the magnitude of $i_q$ component of the current is high and reaches to stable condition after a specific time period and remain steady with respect to time. Since field oriented control is used the value of $i_d$ is zero.

![Figure 7: Currents with Hysteresis Control at 200 rad/s](image)

Figure 7: Currents with Hysteresis Control at 200 rad/s
Figure 8 shows a variation of the speed with time. Speed increases linearly from 0 to 200 rad/sec and remains steady after 0.02 seconds. The steady state speed is the same as that of the commanded reference speed which validates the simulation result.

![Figure 8: Developed speed with Hysteresis Control at 200 rad/s](image)

Figure 9 shows the developed torque of the motor. During initial time i.e. from 0 to 0.01 seconds, starting torque is twice the steady state value. After 0.03 seconds Torque will remain steady and the value of this steady torque observed from the graph of Torque Vs Speed is 1.12 Nm. After 0.023 sec it is observed that the actual torque follows the reference torque. The torque density is also observed to be good.

![Figure 9: Motor Electrical Reference & Actual torque with Hysteresis Control at 200 rad/s](image)
VI. Conclusion
The mathematical model of Permanent Magnet motor drive system using field oriented control is developed. A speed controller has been designed successfully for closed loop operation of the Speed Control of Permanent Magnet Synchronous Motor so that the motor runs at the commanded or reference speed. The fast speed response validates the advantages of Permanent Magnet Synchronous motor. Reference and actual torque produce by the motor.

VII. REFERENCES


