International Journal of Electrical Engineering.
ISSN 0974-2158 Volume 6, Number 2 (2013), pp. 151-165

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Optimal Regulator Design using Kalman's State Estimator for A Non Linear Multivariable Process

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Abstract

Designing controller for the Multi Input Multi Output (MIMO) process is difficult because of the changes in process dynamics and interactions between process variables. This paper presents the approach to design a Linear Quadratic Gaussian (LQG) Controller for a multivariable process with a transmission zero using Linear Quadratic Regulator (LQR) and Kalman's state estimation techniques. The performance of the proposed system is tested for reference tracking and disturbance rejection behaviour using simulation. Simulation results confirm the effectiveness of the proposed control methodology.

Keywords: Multivariable Control, Linear Quadratic Regulator, Kalman Filter, Linear Gaussian Compensator, Quadruple Tank System.

Introduction

The majority of the industrial processes are nonlinear and multivariable systems. There exist some complicated interactions between the measurement signals and control signals. Because of these interactions between input and output variables, it is very complex to design suitable controller for MIMO systems. Several control techniques are available to handle multivariable systems. Multivariable control problems are traditionally solved by centralized PID controllers to obtain the desired overall control function. To compensate the interactions between variables, decentralized controllers can be employed by designing suitable decoupler so that the MIMO system is decoupled into several SISO systems and can be controlled using simple feedback controllers. However, additional restrictions will be introduced in the feedback properties of system with decoupler [1] [2].

Non-minimum phase behaviour is one of the challenges in MIMO systems because it will lead to inverse response. It requires proper input output pairing of variables using Relative Gain Array (RGA) to handle the process with non-minimum phase response [3]. The objective of this paper is to suggest an optimal control methodology for a multivariable system to solve the problem of interactions with improved robustness.

Quadruple tank system [QTS] is used for this study because it is a laboratory process which is used to study the concepts of multivariable process because it can be configured in two different operating points known as minimum phase condition and non-minimum phase condition. i.e., one of its two multivariable zeros can be placed either in left half of s-plane [minimum phase condition] or right half of s-plane [non-minimum phase condition]. The multivariable zero dynamics of the system can be adjusted by simply changing a valve and it depends on the ratio of flow rates between the upper and lower tanks.

The control methodologies discussed here are based on state space approach and the decentralized PI controller is taken for comparison purpose. The controllers used are Linear Quadratic Regulator and Linear Quadratic Gaussian Compensator. In LQG controller design, plant is considered as a stochastic process or non-deterministic process, where the process is affected by the process noise and the measurement noise.

The steps for designing the LQG controller is given as follows: First the linearized mathematical model of the QTS in two different operating points are derived; Then the controllability and observability of the system is checked and then state feedback vector is obtained using LQR. Next a full state observer is developed with Kalman filter and finally by combining these two, a LQG controller is developed. The servo and regulator response of the QTS are obtained for both minimum and non-minimum phase operating points.

The concepts behind this study are organized as follows: Section II gives the description of QTS and the mathematical modeling of the system using state space analysis. The design of LQR is explained in Section III. The next Section briefs the design of Kalman filter technique. Section V explains the LQG compensator design, which is followed by Results and Discussions in Section VI. The conclusion is given in Section VII.

Process Description and Modeling

A quadruple tank apparatus which was proposed in literature [4], has been used in chemical engineering laboratories to illustrate the performance limitations for multivariable systems posed by ill-conditioning, right half plane transmission zeros and model uncertainties. The linear feedback controllers cannot be employed for this process because it has a time-varying movement of a right half plane transmission zero across the imaginary axis. The schematic diagram of the quadruple tank equipment is presented in figure 1.

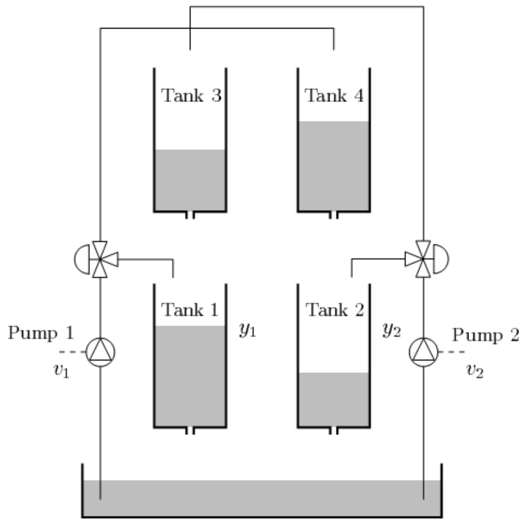

Figure 1: Schematic diagram of the quadruple tank process

The QTS consists of four interconnected tanks and two pumps. The process inputs are u_1 and u_2 (input voltages to the pumps) and the outputs are y_1 and y_2 (voltages from level measurement devices). The target is to control the level of the lower two tanks with inlet flow rates.

The output of each pump is split into two by using a three- way valve. Thus, each pump output goes to two tanks, one lower and another upper which are diagonally opposite and the ratio of the split up is controlled by the position of the valve. With the change in position of the two valves, the system can be appropriately placed either in the minimum phase or in the non-minimum phase.
Let the parameter $\boldsymbol{\gamma}$ be determined by how the valves are set. If $\boldsymbol{\gamma}_1$ is the ratio of flow to the first tank, then (1 - $\boldsymbol{\gamma}_1$) will be the flow to the fourth tank. Similarly if $\boldsymbol{\gamma}_2$ is the ratio of flow to the second tank, then (1 - $\boldsymbol{\gamma}_2$) will be the flow to the third tank. The voltage applied to Pump 'i' is V_i and the corresponding flow rate is K_iV_i. The parameters $\boldsymbol{\gamma_1}$ $\boldsymbol{\gamma_2}$ **Ɛ [0, 1]** are determined from how the valves are set prior to an experiment. The flow to tank '1' is $\boldsymbol{\gamma}_1K_1V_1$ and the flow to tank '4' is (1- $\boldsymbol{\gamma}_1$) K_1V_1 and similarly for Tank '2' and Tank '3'. The acceleration of gravity is denoted as 'g'. The measured level signals are $y_1 = k_ch_1$ and $y_2= k_ch_2$.

The state space equations of the four tank system are given in equation (1) and the state space model after linearization is given by equation (2).

$$\frac{dh_1}{dt} = -\frac{a_1}{A_1}\sqrt{2gh_1} + \frac{a_3}{A_1}\sqrt{2gh_3} + \frac{\gamma_1 k_1}{A_1}u_1$$
$$\frac{dh_2}{dt} = -\frac{a_2}{A_2}\sqrt{2gh_2} + \frac{a_4}{A_2}\sqrt{2gh_4} + \frac{\gamma_2 k_2}{A_2}u_2$$

$$\frac{dh_3}{dt} = -\frac{a_3}{A_3}\sqrt{2gh_3} + \frac{(1-\gamma_2)k_2}{A_3}u_2$$

$$\frac{dh_4}{dt} = -\frac{a_4}{A_4}\sqrt{2gh_4} + \frac{(1-\gamma_1)k_1}{A_4}u_1 \qquad (1)$$

$$\frac{dX}{dt} = \begin{bmatrix} -\frac{1}{T_1} & 0 & \frac{A_3}{A_1T_3} & 0 \\ 0 & -\frac{1}{T_2} & 0 & \frac{A_4}{A_2T_4} \\ 0 & 0 & -\frac{1}{T_3} & 0 \\ 0 & 0 & 0 & -\frac{1}{T_4} \end{bmatrix} X + \begin{bmatrix} \frac{\gamma_1 k_1}{A_1} & 0 \\ 0 & \frac{\gamma_2 k_2}{A_2} \\ 0 & \frac{(1-\gamma_2)k_2}{A_3} \\ \frac{(1-\gamma_1)k_1}{A_4} & 0 \end{bmatrix} U$$

$$Y = \begin{bmatrix} k_c & 0 & 0 & 0 \\ 0 & k_c & 0 & 0 \end{bmatrix} X \qquad (2)$$

The parameter values and steady state operating points of the process are assumed as per the system given in literature [4]. The transfer function matrices are given in equations (3) and (4) for minimum phase and non-minimum phase operating points respectively.

$$G\_(s) = \begin{bmatrix} \frac{2.6}{1+62s} & \frac{1.5}{(1+23s)(1+62s)} \\ \frac{1.4}{(1+30s)(1+90s)} & \frac{2.5}{1+90s} \end{bmatrix} \qquad (3)$$

$$G_+(s) = \begin{bmatrix} \frac{1.5}{1+63s} & \frac{2.5}{(1+39s)(1+63s)} \\ \frac{2.5}{(1+56s)(1+91s)} & \frac{1.6}{1+91s} \end{bmatrix} \qquad (4)$$

The transfer matrix G has two zeros, one of them is always in the left half of s-plane, but the other can be located either in left half or right half of s-plane. So, the system is in minimum phase, if the values of γ_1 *and* γ_2 satisfy the condition $0 < \gamma_1 + \gamma_2 < 1$ and in non-minimum phase, if the values of γ_1 *and* γ_2 satisfy the condition $1 < \gamma_1 + \gamma_2 < 2$.

Linear Quadratic Regulator

Linear quadratic regulator provides an optimal control law for a linear system with a quadratic performance index. The main objective of this controller is to minimize the deviation of the level of the lower tanks. The control configuration of LQR is presented in fig. 2.

The 'cost function' is often defined as a sum of the deviations of key measurements from their desired values. In effect, this algorithm therefore finds the controller settings that minimize the undesired deviations. Often the magnitude of the control action itself is included in this sum so as to keep the energy expended by the control action itself being limited. In the particular case of a quadratic performance index, combining the square of the error and square of the actuation, the solution to the optimal control problem is a feedback control, where the measurements used for the feedback are all of the state variables [5].

In this feedback control, each of the state variables is multiplied by a gain and the results are summed to get a single actuation value. The result of the LQR formulation is the set of gains, based on the relative weighting of the error and actuation in the performance index.

Consider the process model in state space given by the following equation:

$$\dot{X} = AX + BU$$
$$Y = CX \tag{5}$$

In LQR, for the above system of equation (5) with non zero initial state, the input signal u(t), which drives the system back to the zero state can be found in optimal manner by minimizing the cost function,

$$J_r = \int_0^{\infty} [x(t)^T\, Qx(t) + \, u(t)^T Ru(t)]\, dt \tag{6}$$

The matrices Q and R are the weight matrices. Where Q is a n x n positive semi definite matrix and R is a p x m positive definite matrix, with the restriction that the system is observable. The optimal solution for any initial state is

$$u(t) = -Kx(t) \tag{7}$$

where K = $R^{-1}\, B^T P$ and
$P = P^T \geq 0$ is the unique positive semi definite solution of the algebraic Riccati equation,

$$A^T P + PA - PBR^{-1}B^T P + Q = 0 \tag{8}$$

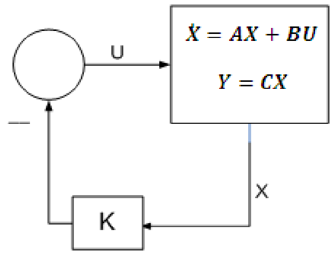

Figure 2: Control configuration of LQR

To design the LQR controller, the first step is to select the weight matrices Q and R. The value of the R matrix weighs inputs more than the states while the value of Q matrix weighs the state more than the inputs. Then the state feedback vector, K can be computed and the closed loop system responses can be found by simulation [6] [7].
This controller guaranties reference input tracking and reduces disturbance effects in closed loop system and eliminates them at steady state condition. Adjusting the parameter ‘K’ can improve the closed loop system transient performance, but this system does not exhibit good performance for parameter variations. Indeed, nonlinear behaviour, plant disturbance, sensor noise and model errors will invariably lead to deviation from the true states unless precautions are taken during the observer design. Also, the measured noise and process noise can disturb the model characteristics. Then it is required to design a controller which can overcome these problems. Consequently, Kalman filter can be combined with LQR to improve its performance.

Kalman Filter Design

The Kalman filter is described as a set of mathematical equations that provides an efficient computational scheme to estimate the state of a process, in a way that minimizes the mean of the squared error. State estimation is the process of extracting a best estimate of a variable from a number of measurements that contain noise. This filter is very powerful in several aspects, that it supports estimations of past, present and even future states and it can do so even when the precise model of the system is not known [8] [9].

Kalman filter is a fundamental tool for analyzing and solving a broad class of estimation problems. The Kalman filter operates by propagating the mean and covariance of the state through time. This filter is derived using the following steps: The mathematical description of a dynamic system whose states are to be estimated is obtained using state space modeling. The set of equations that describe how the mean and the covariance of the state propagate with time are derived. Then the dynamic system that describes the propagation of the state mean and covariance is implemented as equations. These equations form the basis for the derivation of the Kalman filter because the mean of the state is the Kalman filter estimate of the state and the covariance of the state is the covariance of the Kalman filter state estimate. The mean and covariance of the state is updated every time when the measurement is taken [10] [11].

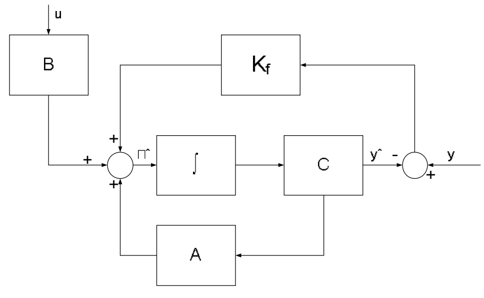

Figure 3: Structure of Kalman filter

Kalman filter has the structure of an ordinary state estimator or observer, as shown in Fig.3. with

$$\dot{\hat{x}} = A\hat{x} + Bu + K_f(y - C\hat{x}) \tag{9}$$

The optimal choice of K_f, which minimizes the expectation operator, $E\{[x - \widehat{x}]^T[x - \widehat{x}]\}$, is given by

$$K_f = P_k C^T R^{-1} \tag{10}$$

where $P_k = P_k{}^T \geq 0$ is the unique positive semi definite solution of the algebraic Riccatti equation, given as equation (10)

$$P_k A^T + AP_k - P_k CR^{-1}CB^T P_k + Q = 0 \tag{11}$$

Linear Gaussian Compensator

The Kalman filter together with Linear Quadratic Regulator is called Linear Quadratic Gaussian compensator. In practical approach, separation principle [5] is used to design this controller. It means the regulator and observer is designed separately and put them together to form a compensator for the plant whose state vector was immeasurable. Here the optimal estimator (Kalman Filter) and optimal regulator (LQR) are combined to form an optimal compensator (LQG). The control structure for LQG is shown in fig. 4.

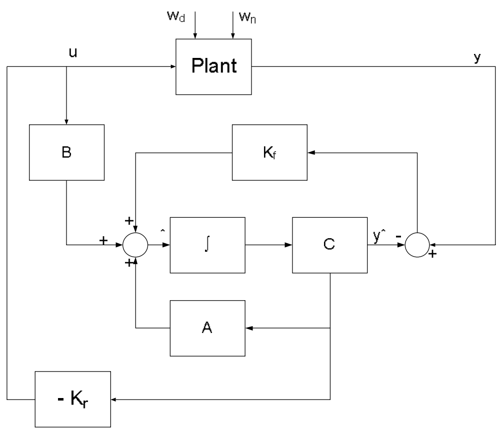

Figure 4: The LQG Controller Structure

The LQG controller is designed based upon a linear plant, a quadratic objective function and an assumption of white noise that has a normal or Gaussian probability distribution. The process model with measurement and disturbance noises is given by equation (12).

$$\dot{X} = AX + BU + w_d$$
$$Y = CX + w_n \tag{12}$$

where w_d and w_n are the process noise (disturbance) and measurement noise inputs respectively, moreover w_d and w_n are white noise processes with covariance as given in (13).

$$E\{w_d(t)w_d\,(t)^T\} = Q\delta(t-\tau)$$
$$E\{w_n(t)w_n\,(t)^T\} = R\delta(t-\tau) \quad (13)$$

where $\delta(t-\tau)$ is a delta function.

The LQG control problem is to find the optimal control u(t) which minimizes the cost function

$$J_r = E\{\lim_{T\to\infty}\frac{1}{T}\int_0^T[x(t)^T\,Qx(t) + u(t)^T Ru(t)]\,dt\} \quad (14)$$

where Q and R are appropriately chosen constant weight matrices such that $Q = Q^T \geq 0\; and\; R = R^T > 0$.

The solution to the LQG problem is known as the Separation Theorem. It consists of first determining the optimal control to a deterministic linear quadratic regulator problem. The solution to this problem can be written in terms of the simple state feedback law

$$u(t) = -K_r x(t) \quad (15)$$

where K_r is a constant matrix which is easy to compute and is clearly independent of Q and R, the statistical properties of the plant noise. The next step is to find an optimal estimate $\hat{x}$ of the state x, so that

$E\{[x-\widehat{x}]^T[x-\widehat{x}]\}$ is minimized. The optimal state estimate is given by a Kalman filter and is independent of Q and R. The required solution to the LQG problem is then found by replacing x by $\hat{x}$, to give

$$u(t) = -K_r\hat{x}(t) \quad (16)$$

The detailed description of LQG design is as follows. First, an optimal regulator is designed for a linear plant assuming full-state feedback and based on quadratic objective function. The regulator is designed to generate a control input, u(t), based upon the measured state vector, x(t). Also, a Kalman filter is designed for the plant assuming a known control input u(t), a measured output y(t) and white noises w_d and w_n with known power spectral densities [5].

The Kalman filter is used to provide an optimal estimate of the state vector, $\hat{x}$(t). Combine the separately designed optimal regulator and Kalman filter into an optimal compensator, which generates the input vector u(t), based upon the estimated state-vector $\hat{x}$(t), rather than actual state vector x(t) and the measured output vector y(t). The closed loop system performance can be obtained by suitably selecting the optimal regulator weight matrices and the Kalman filter‘s spectral noise densities [9].

Results and Discussions

The servo and regulatory responses for Quadruple Tank System in both minimum phase and non-minimum phase operating conditions using LQG controller are obtained through simulation using MATLAB and it is given in fig. 7 to fig. 10. It gives improved performance for set point tracking and disturbance rejection when compared to PI controller whose responses are given in fig.5 and fig.6.

The estimation capability of Kalman filter is also shown in fig. 11. In fig. 12 and fig.13, the LQG regulator responses for different values of Q and R weight matrices are compared.

It has been observed from the graphs of fig.7 to fig.10 that the tracking and regulator responses of LQG have less settling time and free from steady state error and overshoots when compared to PI controller. The comparison of these parameters is given in Table I. The estimated values of levels using Kalman Filter is closer to true values than the measured values. This can be verified from fig.11. Also, from fig.12 and fig.13, it can be seen that the regulation of the dynamic variable is improved with large values of Q matrix and smaller values of R matrix.

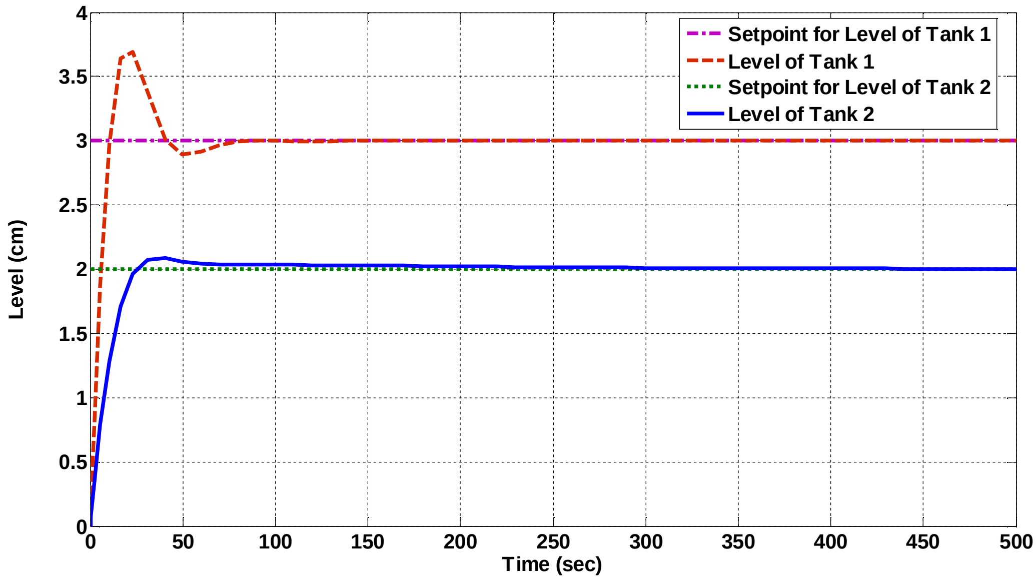

Figure 5: Servo response using PI controller for QTS in Min Phase

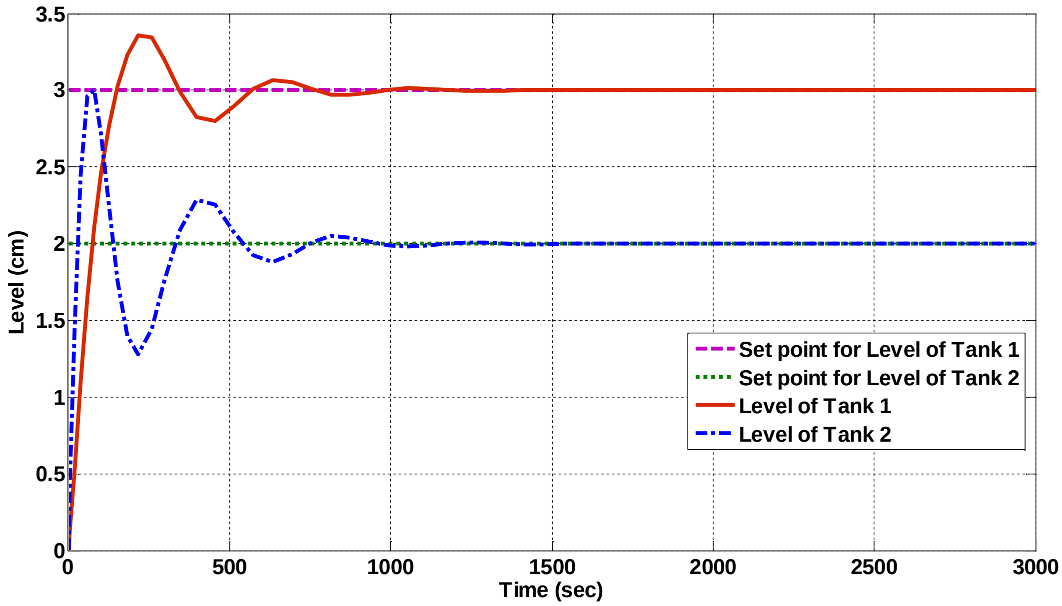

Figure 6: Servo response using PI controller for QTS in Non-Min Phase

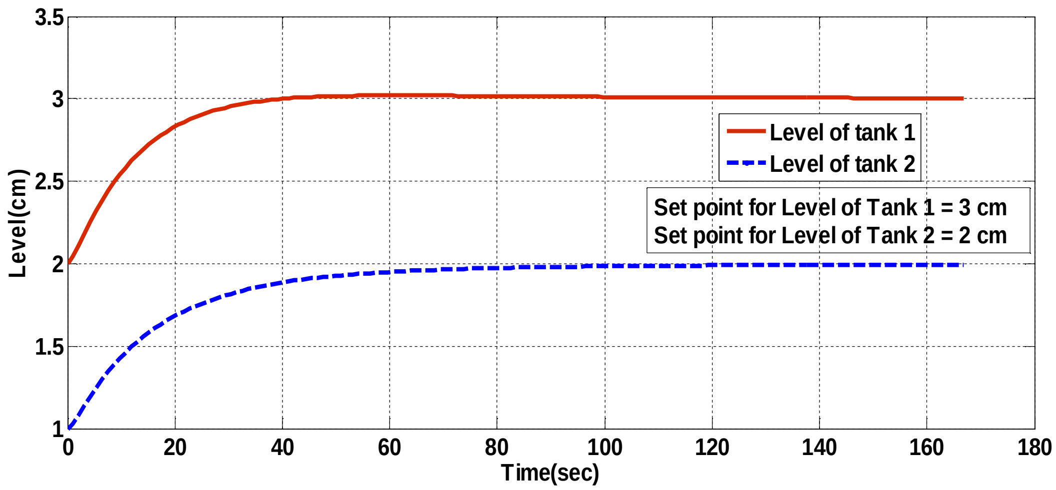

Figure 7: Servo response using LQG for QTS in Min Phase

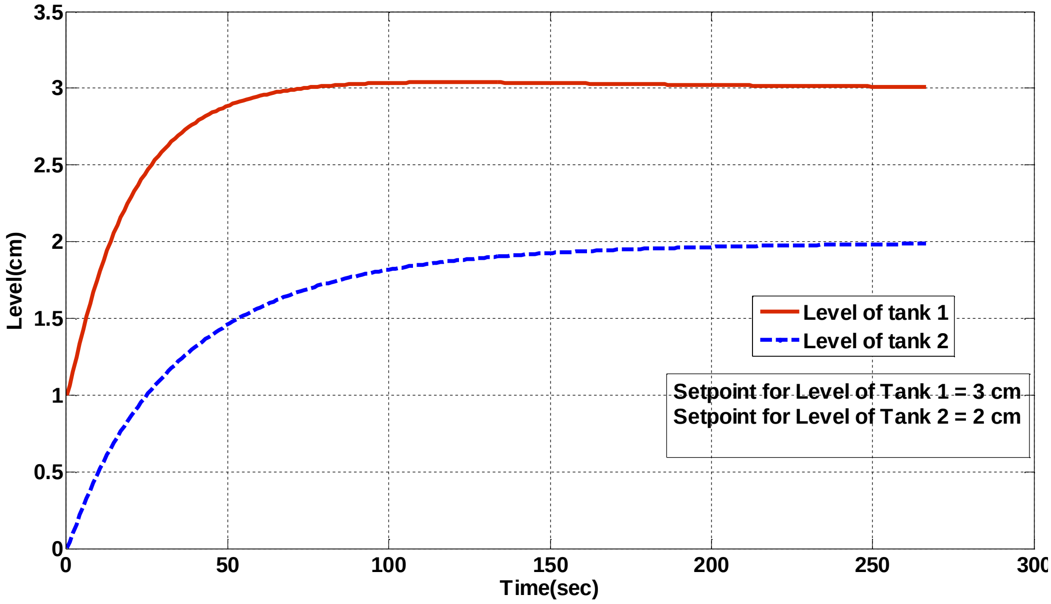

Figure 8: Servo response using LQG for QTS in Non-Min Phase

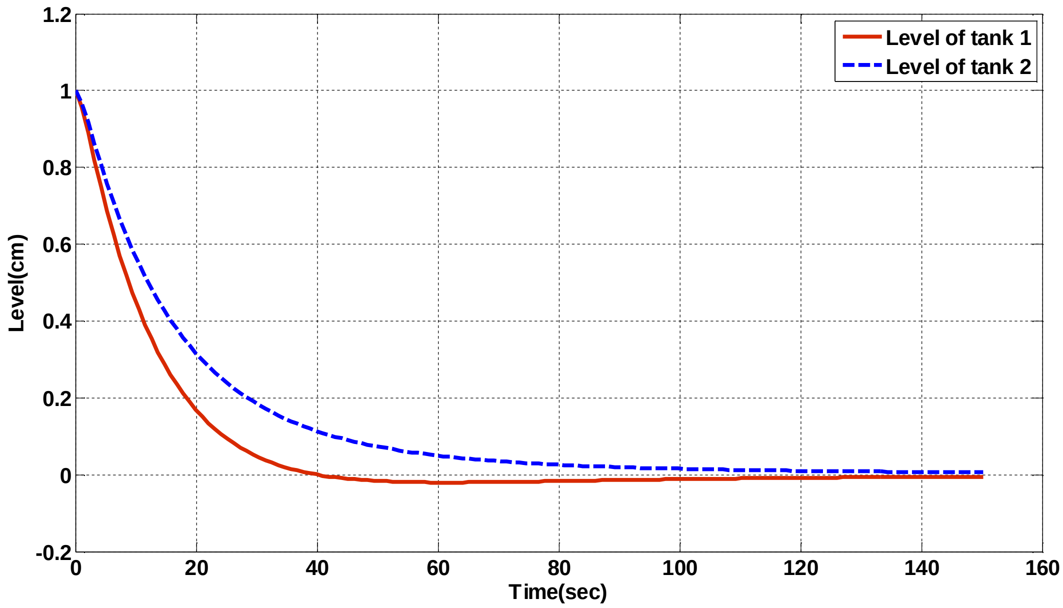

Figure 9: Regulator response using LQG for QTS in Min Phase

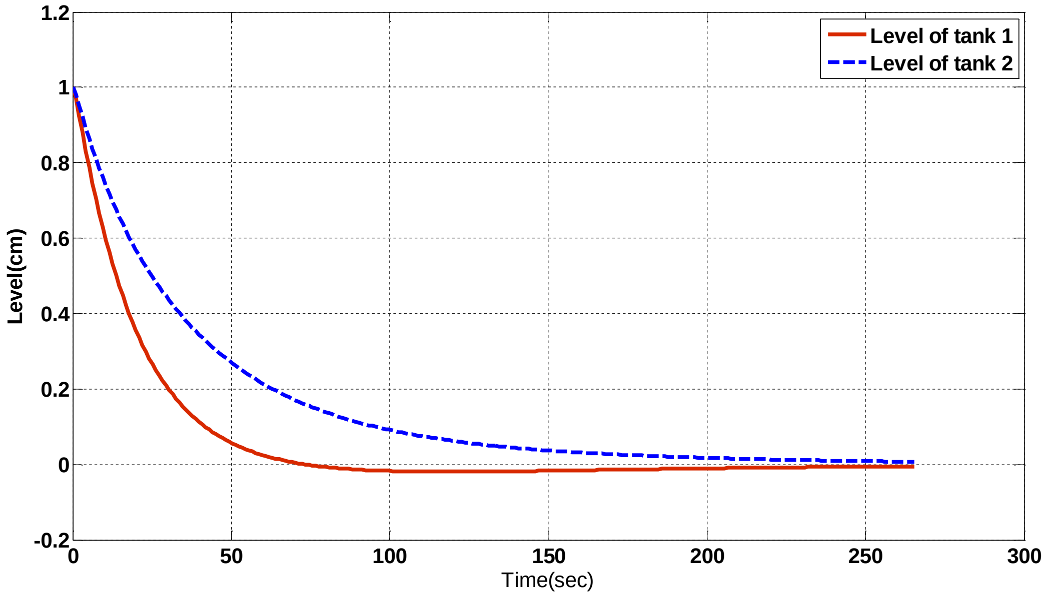

Figure 10: Regulator response using LQG for QTS in Non-Min Phase

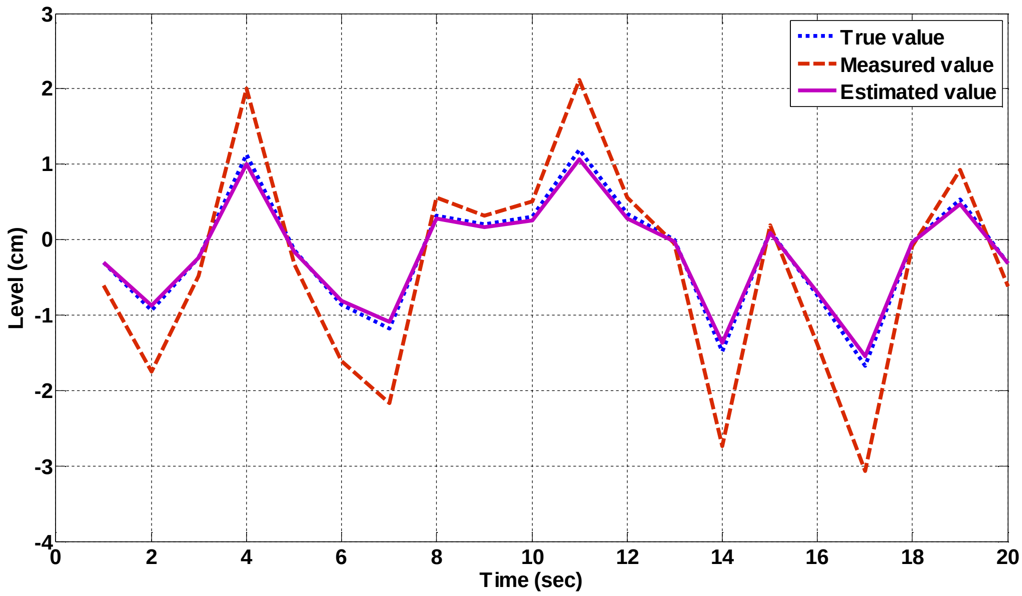

Figure 11: Kalman filter estimation for Level Measurement

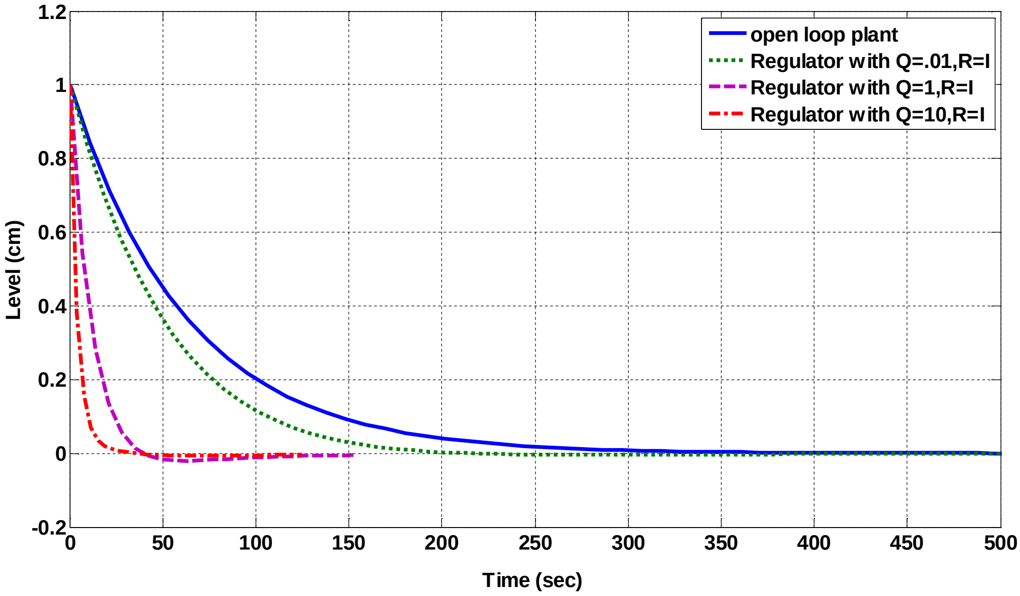

Figure 12: Regulator responses for different values of Q matrix

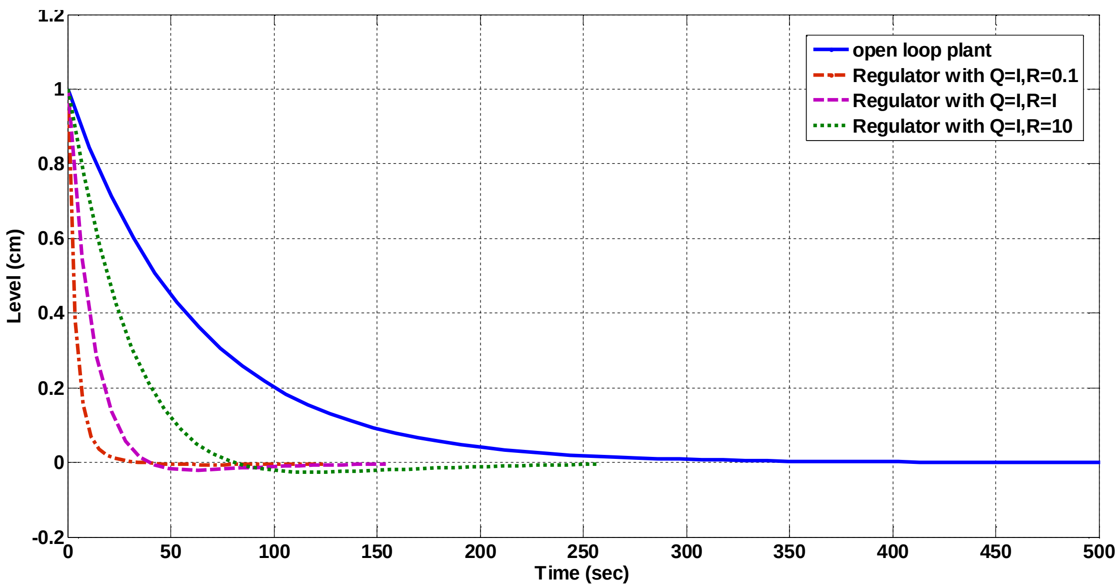

Figure 13: Regulator responses for different values of R matrix

Table 1: Quantitative Comparison of Performance

Controller	Parameters	Minimum Phase		Non-Minimum Phase	
		Level 1	***Level 2***	***Level 1***	***Level 2***
PI Controller	Settling Time	150 sec	250 sec	1380sec	1380sec
	Peak Overshoot	23.33%	5%	12%	50%
	Steady State Error	Nil	2%	Nil	Nil
LQG Controller	Settling Time	80 sec	100 sec	180 sec	220 sec
	Peak Overshoot	Nil	Nil	Nil	Nil
	Steady State Error	Nil	Nil	Nil	Nil

Conclusion

Kalman filter is a recursive filter which is efficiently used in various applications. This paper brings down Kalman filter to process control application for estimating the levels of a quadruple tank system. Kalman filter gives an accurate estimation of the states of the QTS from a stochastic process. LQR provides the optimal control of the process. Conventional controllers like PI and pole placement methods are seems to be very difficult to control the quadruple tank system while in non-minimum phase, i.e. a zero in the right half side of s-plane.

But the optimal controllers like LQR and LQG provides an optimal control with better settling times and the most important factor is that it can be used in both minimum phase and non-minimum phase almost equally effective. Moreover, in this methodology, the need for decoupler to minimize interactions between loops is not required and the loop interactions are managed by the controller itself. The performance of the control system can further be improved by proper selection of weight matrices Q and R using advanced optimization techniques.

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