International Journal of Electronics and Communication Engineering.
ISSN 0974-2166 Volume 9, Number 2 (2016), pp. 97-104

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Effect of Multipath Propagation on the Noise Performance of Zero Crossing Digital Phase Locked Loop

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Abstract

The steady state performance of a zero crossing digital phase locked loop (ZCDPLL) in face of a signal accompanied with interference caused by multipath propagation and additive Gaussian noise has been investigated. In order to evaluate the noise performance of the loop, expressions for the steady state phase error variance have been derived in both cases of single interfering path and multiple interfering paths respectively.

Keywords: Multipath interference, ZCDPLL, Noise performance.

I. INTRODUCTION

The problem of same frequency interference on the reception of signal in the background of addictive Gaussian noise has drawn the attention of several workers. As such the performance of analog phase locked loop in the presence of interference and additive noise has been studied in the literature [1],[2]. In this paper the effect of same frequency interference caused by discrete multipath propagation of signals on the noise performance of a second order zero crossing digital phase locked loop (ZCDPLL) has been studied. This type of interference may appear in line-of-sight microwave transmission or in mobile radio communication because of reflection from the ground surface or from the surfaces of surrounding buildings. Two practical situations, for example interference caused by single interfering path and multiple interfering paths are considered separately. Approximate expressions of the phase error variance are analytically found out to measure the noise performance of the loop for the above two cases. The analytical results reduce to the well-known established result predicted by A. Weinberg and B. Liu [3] in the absence of interference and this supports the validity of the analysis.

II. STEADY STATE NOISE ANALYSIS OF ZCDPLL

(a) Single interfering path:

The ZCDPLL structure under consideration is shown in fig. 1. The input to the loop is considered as the sum of a sinusoid signal of amplitude unity, angular frequency $\omega\, rad/s$ and phase δ; a same frequency interference of relative amplitude α and relative phase θ and an additive Gaussian noise $n(t)$ of mean zero and varianceσ^2. The input$S(t)$ to the loop can therefore be written as

$$S(t) = \sin(\omega t + \delta) + \alpha \sin\big(\omega t + \delta + \theta(t)\big) + n(t) \tag{1}$$

For single interfering path α is a time invariant parameter but $\theta(t)$ is a random variable due to random path length fluctuations of the interference and distributed uniformly within –π to π. Clearly $\theta(t)$ and $n(t)$ are statistically independent. The input given by (1) can be written as

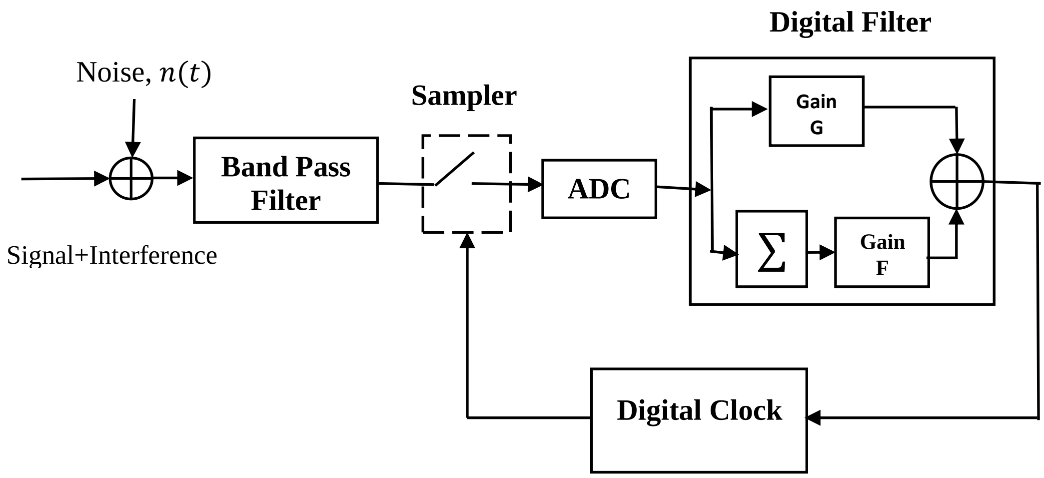

Fig. 1: Block diagram of ZCDPLL

$$S(t) = y(t)\sin\big(\omega t + \delta + \psi(t)\big) + n(t) \tag{2}$$

Where $y(t) = [1 + \alpha^2 + 2\alpha\cos\theta(t)]^{1/2}$ and $\psi(t) = tan^{-1}\frac{\alpha\sin\theta(t)}{1+\alpha\, cos\,\theta(t)}$

respectively denote the envelope and phase of the sum of signal and interference. Although the operation of ZCDPLL was described by A. Weinberg and B. Liu [3], however for good understanding of the loop operation in the presence of interference and noise the same is presented in brief. The input is sampled at its positive going zero crossing instants by a digital clock. The digital version of the sampled input is filtered by a digital filter consisting of a gain G in parallel with a summer and another gain F. The filtered output is used to control the period of the clock to move the loop towards the locked state. Let $X(l)$ be the l^{th} sampled value of the input. Then

$$X(l) = y\sin(\omega t + \delta + \psi) + n(l) \tag{3}$$

In all practical cases the time variation of $\theta(t)$ is very slow as compared to that of $n(t)$. As such the variations of $y(l)$ and $\psi(l)$ are also very slow and assumed quasi-stationary in a few successive random samples of $n(l)$. For these assumptions $y(l) \approx y$ and $\psi(l) \approx \psi$.

The open loop clock period $T(=\frac{2\pi}{\omega_0}, \omega_0$ being the open loop frequency of the clock in rad/s), the l^{th} control input $C(l)$ of the clock and the $(l+1)^{th}$ clock period $T(l+1)$ are related as

$$T(l+1) = T - C(l) \tag{4}$$

As $C(l)$ is the output of the digital filter therefore

$$C(l) = GX(l) + F\sum_{j=0}^{l} X(j) \tag{5}$$

Now, l^{th} clock period is the time interval between the l^{th} and $(l-1)^{th}$ sampling instant I,e

$$T(l) = t(l) - t(l-1) \tag{6}$$

Assuming $t(0)$ as zero, $t(l)$ can be written from (6) as

$$t(l) = \sum_{i=1}^{l} T(i) = lT - \sum_{i=0}^{l-1} C(i) \tag{7}$$

Substitution of (7) into (3) gives

$$X(l) = y\sin[\phi(l) + \psi] + n(l) \tag{8}$$

Where

$$\phi(l) = \frac{2\pi l \Delta\omega}{\omega_0} + \delta - \omega \sum_{i=0}^{l-1} C(i) \qquad (9)$$

is the loop phase error and $\Delta\omega = \omega - \omega_0$ is the open loop frequency error in rad/s.

The difference equation of ϕ can be obtained from (9) as

$$\phi(l+1) - 2\phi(l) + \phi(l-1) = \omega[C(l-1) - C(l)] \qquad (10)$$

Substitution of (5) and (8) successively in (10) gives

$$\phi(l+1) - 2\phi(l) + \phi(l-1) = G_1[y(\sin(\phi(l-1) + \psi) + n(l-1)]$$

$$-G_1(1 + \frac{G_2}{G_1})[y(\sin(\phi(l) + \psi) + n(l)] \qquad (11)$$

Where $G_1 = \omega G$ and $G_2 = \omega F$

For large signal to noise power ratio (SNR) and small interference to signal power ratio (ISR) at the input, ϕ samples are generally small and $\sin(\phi)$ *and* $\cos(\phi)$may be replaced by ϕ and 1 respectively. Replacing $\sin(\phi)$ *and* $\cos(\phi)$ by their linearized values and changing the variables from (y, ψ) to (α, θ) back, (11) reduces to

$$\phi(l+1) - a\phi(l) = b\, \phi(l-1) + G_1 n(l-1) - rG_1 n(l) + k \qquad (12)$$

Where $r = 1 + \frac{G_2}{G_1}$, $a = 2 - G_1 r(1 + \alpha cos\theta), b = G_1(1 + \alpha cos\theta) - 1,$

$k = G_1 \alpha(1 - r)\sin\theta$

Keeping the quasi stationary variable θ constant and, following the statistical analysis given in the literature [4] by the same author, the mean and mean squared value of ϕ can be obtained from (12) after substitution of r, a, b and k as

$$\overline{\phi}(\alpha, \theta) = \frac{-\alpha \sin\theta}{1 + \alpha\, cos\,\theta} \qquad (13)$$

and

$$\bar{\phi}^2(\alpha, \theta) = \frac{1}{2R}\left[\frac{x(1 + \alpha\, cos\theta) + d}{1 - x(1 + \alpha\, cos\theta)}\right]\frac{1}{(1 + \alpha\, cos\theta)^2} + \frac{\alpha^2 sin^2\theta}{(1 + \alpha\, cos\theta)^2} \qquad (14)$$

Where $x = \frac{2G_1 + G_2}{4}$, $d = \frac{G_2}{2G_1}$ *and* $R = \frac{1}{2\sigma^2}$

Here over bar denotes the statistical average and R is the input SNR.

Equation (14) can be written in another form as

$$\bar{\phi}^2(\alpha,\theta) = \frac{1}{2R}\left[\frac{x+d}{1-x}\right]\left(1+\frac{x\alpha\cos\theta}{x+d}\right)\left(1-\frac{x\alpha\cos\theta}{1-x}\right)^{-1}(1+\alpha\cos\theta)^{-2}$$
$$+\alpha^2 sin^2\theta(1+\alpha\, cos\theta)^{-2} \quad (15)$$

For optimum values of $G_1(\approx 0.8) and\ G_2(\approx 0.35)$ [4] and smaller values of ISR $(\alpha^2 \ll 1)$, (15) can be simplified after binomial expansion of $(1-\frac{x\alpha\cos\theta}{1-x})^{-1}$ and $(1+\alpha\cos\theta)^{-2}$ up to second order term as

$$\bar{\phi}^2(\alpha,\theta) = \frac{M}{2R}[1 + A\alpha cos\theta + B\alpha^2 cos^2\theta + C\alpha^3 cos^3\theta + D\alpha^4 cos^4\theta]$$
$$+[\alpha^2 sin^2\theta - 2\alpha^3 sin^2\theta cos\theta + 3\alpha^4 sin^2\theta cos^2\theta] \quad (16)$$

Where

$$\text{A} = \left(\frac{d+1}{x+d}\right)\left(\frac{x}{1-x}\right) - 2\,; \quad B = 3 - 2\left(\frac{d+1}{x+d}\right)\left(\frac{x}{1-x}\right) + \left(\frac{d+1}{x+d}\right)\left(\frac{x}{1-x}\right)^2;$$
$$C = \left(\frac{d+1}{x+d}\right)\left(\frac{x}{1-x}\right)\left(3-\frac{2x}{1-x}\right); \quad D = 3\left(\frac{d+1}{x+d}\right)\left(\frac{x}{1-x}\right)^2; M = \frac{x+d}{1-x}$$

Averaging of $\bar{\phi}(\alpha,\theta)$ and $\bar{\phi}^2(\alpha,\theta)$ over all possible values of θ between –π to π, the mean and mean square values of ϕ as a function of α can be obtained from (13) and (16) as

$$\bar{\phi}(\alpha) = 0 \quad (17)$$

and

$$\bar{\phi}^2(\alpha) = \sigma_\phi^2(\alpha) = \frac{M}{2R}\left[1 + \left(\frac{R}{M}+\frac{B}{2}\right)\alpha^2 + \frac{3}{8}\left(\frac{2R}{M}+D\right)\alpha^4\right] \quad (18)$$

Here $\sigma_\phi^2(\alpha)$ is the variance of ϕ and it is equal to its mean square value, since $\bar{\phi}(\alpha)$ is zero.

In the absence of interference (18) reduces to

$$\bar{\phi}^2(0) = \sigma_\phi^2(0) = \frac{M}{2R} \quad (19)$$

This result is identical with that obtained by A. Weinberg and B.liu [3].

Equation (18) is plotted in fig.2 as the functions of α and R (SNR) for $G_1 = 0.8\ and\ G_2 = 0.35$.

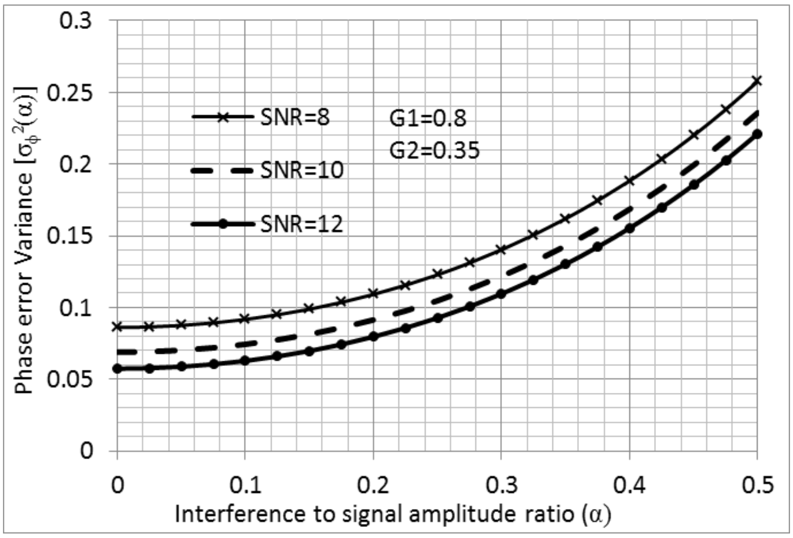

Fig.2: Variation of Phase error variance with interference to signal amplitude ratio for different SNR

It is observed from fig.2 that the phase error variance increases with the increase of interference amplitude α from its value at α =0 indicating the degradation of noise performance in the presence of interference. In order to maintain $\sigma_{\phi}^{2}(0)$ in the presence of interference, one way is to increase the value of input SNR. The percentage increase of input SNR required to maintain $\sigma_{\phi}^{2}(0)$ in the presence of interference may be estimated as follows.

$$\sigma_{\phi}^{2}(R,0) = \sigma_{\phi}^{2}(R + \Delta R, \alpha) \approx \sigma_{\phi}^{2}(R, \alpha) + \frac{\delta \sigma_{\phi}^{2}(R,\alpha)}{\delta R}\Delta \mathrm{R}. \tag{20}$$

Where $\sigma_{\phi}^{2}(R, \alpha)$ is given by (18). Substitution of (18) in (20) gives

$$\frac{\Delta R}{R} = \frac{\left[\left(\frac{R}{M}+\frac{B}{2}\right)\alpha^{2}+\frac{3}{8}\left(\frac{2R}{M}+D\right)\alpha^{4}\right]}{\left[1+\left(\frac{B}{2}\right)\alpha^{2}+\left(\frac{3}{8}D\right)\alpha^{4}\right]} \tag{21}$$

For α =0.2, R=10, $G_1 = 0.8$ *and* $G_2 = 0.35$ the percentage increase in R is found out as $\frac{\Delta R}{R} = 32\%$.

b) Multiple interfering paths:

It is more difficult to derive a general expression of phase error variance for any number of interfering paths greater than one. However for large number of interfering paths both the in-phase and quadrature components of the interference approach

Gaussian distribution and hence the resultant phase is uniformly distributed as before and the resultant envelope is Rayleigh distributed given by

$$p(\alpha) = \frac{\alpha}{\sigma_\alpha^2} e^{\frac{-\alpha^2}{2\sigma_\alpha^2}} (\alpha \geq 0) \tag{22}$$

where $\sigma_\alpha^2 = \frac{1}{2}\sum_{i=1}^{N} \alpha_i^2$

Then for $R \gg 1$ and σ_α^2<<1, averaging of $\sigma_\phi^2(\alpha)$ over α gives

$$\sigma_\phi^2(\sigma_\alpha) = \int_0^\infty \sigma_\phi^2(\alpha) p(\alpha) d\alpha \tag{23}$$

Substitution of (18) and (22) in (23) and integration results

$$\sigma_\phi^2(\sigma_\alpha) = \frac{M}{2R}\left[1 + \left(\frac{2R}{M} + B\right)\sigma_\alpha^2 + 3\left(\frac{2R}{M} + D\right)\sigma_\alpha^4\right] \tag{24}$$

From (24), it is also observed that phase error variance increases with the increase of σ_α .

III. CONCLUSION

The noise performance of a second order ZCDPLL in the presence of interference caused by multipath propagation of signals is analytically studied. The performance of the loop is determined by deriving the loop phase error variance as a function of ISR and SNR under certain realistic assumptions. From the analytical results it is observed that phase error variance increases with the increase of ISR indicating degradation of noise performance in the presence of interference. It is also observed that degradation effect is more pronounced for higher SNR values even if the interference power is small .A large amount of signal power is therefore lost due to multipath propagation.

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