Performance Analysis of Adaptive Filtering Algorithms for System Identification

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Abstract

The paper presents a comparative study of NLMS (Normalized Least Mean Square), NVSS (New Variable Step Size) LMS (Least Mean Square), RVSS (Robust Variable Step Size) LMS, TVLMS (Time Varying Least Mean Square) and IVSS (Improved Variable Step Size) LMS adaptive filter algorithms. Four performances criterion are utilized in this study: Minimum Mean Square Error (MSE), Convergence Speed, Algorithm Execution Time, and Tracking Capability. The comparisons of all algorithms are demonstrated using uncorrelated and correlated input data in both stationary and non-stationary environments. The Step Size Parameter (μ) in all algorithms is chosen to obtain the same exact value of Misadjustment (M) equal to 2% for white Gaussian input and 6% for correlated input in stationary environment. Simulation Plots are obtained by ensemble averaging of 200 independent simulation runs. The simulation results show that RVSS algorithm has fastest convergence speed and superior tracking capability. The algorithm execution time is lowest in case of IVSS algorithm in stationary environment. For non-stationary environment the performance of all algorithms is equivalent.

Keywords: Adaptive filter, MSE, Convergence Speed, Execution Time, Step Size, Tracking capability.

Introduction

An Adaptive filter is very generally defined as a filter whose characteristics can be modified to achieve some end or objective, and is usually assume to accomplish this modification (or "Adaptation") automatically, without the need for substantial intervention by the user. Adaptive filter algorithms have been very popular since last few decades and still it is very useful in many fields of image, speech and signal processing and communication [1].

The choice of one algorithm over other is determined by one or more factors like Convergence speed, Misadjustment, Robustness and Execution Time. Convergence speed is Number of iterations required in response to stationary inputs, to converge "close enough" to the optimum Wiener solution in the Mean-Square error (Mean Square value of the difference between the desired response and actual output) sense [1]. Misadjustment provides a quantitative measure of the amount by which the final values of mean square error, averaged over an ensemble of adaptive filters, deviates from the minimum mean square error produced by the Wiener filter. For an adaptive filter to be robust, small disturbances can only result in small estimation errors. Execution Time is the total time required for the execution of algorithm [4].

System Identification

The notion of a mathematical model is fundamental to science and engineering [2]. In the class of application dealing with identification, an adaptive filter is used to provide a linear model that represents the best fit (in some sense) to an unknown system. The unknown system and adaptive filter are driven by the same input. The unknown system output supplies the desired response for the adaptive filter. A block diagram of system identification setup is shown in Fig.1. The aim is to estimate the impulse response, h, of the unknown system. The adaptive filter adjusts its weights, w, using one of the LMS-like algorithms, to produce an output y(n) that is as close as possible to the plant output d(n). When MSE is minimized, the adaptive filter coefficients, w, are approximately equal to the unknown system coefficients, h. x(n) is the input signal for both unknown system and adaptive filter. The internal plant noise is represented as a additive noise n (n).
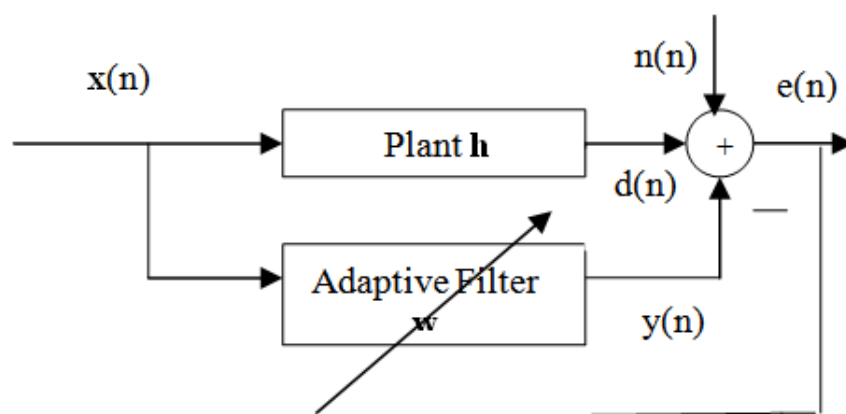

Figure 1: System identification

The performance of an algorithm for system identification can be measured in the terms of its misadjustment M, which is a normalized mean square error defined as the ratio of the steady state excess mean-square error (EMSE) to the minimum MSE [4].

$$M = \frac{EMSE_{ss}}{MSE_{min}} \qquad (1)$$

The MSE at the nth iteration is given by:

$$EMSE(n) = MSE(n) - MSE_{min}, \qquad (2)$$

Where

$$MSE(n) = E[\,|e(n)\,|^2\,] \qquad (3)$$

However, the MSE in (3) is approximately estimated by averaging $|e(n)|^2$ over J independent trials of the experiment. Thus, (3) can be estimated as:

$$MSE(n) = \frac{1}{J}\sum_{n=1}^{J}|e(\underline{n})|^2 \qquad (4)$$

From (2), we can write:

$$EMSE_{ss} = MSE_{ss} - MSE_{min} \qquad (5)$$

The value of MSE_{min} obtained when the coefficients of the unknown system and the filter match, is equal to irreducible noise variance σ_n^2 for zero mean noise n.

Adaptive Filtering Algorithms
The Adaptive NLMS Algorithm
The adaptive NLMS algorithm takes the following form:

$$w(n+1)=w(n)+(\mu\, e(n)x(n)/\,(\varepsilon + x^T(n)x(n))) \qquad (6)$$

$$y(n)=w^T(n)x(n) \qquad (7)$$

$$e(n)=d(n)-y(n) \qquad (8)$$

where $w(n)=[w_0(n)\ w_1(n)\ldots\ldots w_N(n)]^T$ (N+1 being the filter length) is the weight vector, μ is the convergence parameter(sometimes referred to as step size),e(n) is the error, d(n) is the desired output, y(n) is the filter output, ε is a constant prevents division by a very small number of data norm, $x(n)=[x(n)\ x(n-1)\ldots\ldots .x(n-N+1)]^T$ is input vector [1].

The New Variable Step Size LMS (NVSS-LMS) Algorithm
The NVSS-LMS algorithm takes the following form[3]:

$$w(n+1)=w(n)+ \mu x(n)e(n)/(1+\mu\,\|e(n)\,\|^2) \qquad (9)$$

$$y(n)=w^T(n)x(n) \qquad (10)$$

$$e(n)=d(n)-y(n) \qquad (11)$$

Where $\|e(n)\|^2 = \sum\limits_{i=0}^{n-1} e^2(n-i)$

The parameters μ in the algorithm is appropriately chosen to achieve the best trade off between convergence speed and low final MSE [3].

A modified version of NVSS (MNVSS) algorithm that is suited for nonstationary environment is given by:

$$w(n+1)=w(n)+ \mu x(n)e(n)/(1+\mu\ e^2(n)) \tag{12}$$

where $e^2(n)$ is the square of the instantaneous error value at the nth iteration. This modified version is also known as MNVSS (Modified New Variable Step Size) algorithm.

Robust Variable Step Size LMS Algorithm

In Robust Variable Step Size (RVSS) Least Mean Square (LMS) algorithm the step size is dependent on both data and error normalization. With an appropriate choice of the value of the fixed step size and the ratio between error and data normalization in this algorithm, a trade-off between speed of convergence and misadjustment can be achieved. In this algorithm step size varies according to a nonlinear function of the norm of the error vector and the input to the adaptive filter [5].

The NVSS-LMS algorithm takes the following form:

$$w(n+1) = w(n) + \frac{\mu\ \|e_L(n)\|^2\ x(n)\ e(n)}{\alpha\ \|e(n)\|^2 + (1-\alpha)\|x(n)\|^2} \tag{13}$$

$$y(n)=w^T(n)\ x(n) \tag{14}$$

$$e(n)=d(n)-y(n) \tag{15}$$

Where $\|e(n)\|^2 = \sum\limits_{i=0}^{n-1} |e(n-i)|^2 \tag{16}$

$$\|e_L(n)\|^2 = \sum\limits_{i=0}^{L-1} |e(n-i)|^2 \tag{17}$$

where n is the iteration number, w is an N×1 vector of adaptive filter weights, x is an N×1 filter input vector, μ is an iteration-dependent step size, The parameters α, μ and L in the algorithm are appropriately chosen to achieve the best trade-off between convergence speed and low final mean square error.

The Time Varying LMS (TV-LMS) Algorithm
The TVLMS algorithm works in the same manner as conventional LMS algorithm, except for a time dependent convergence factor μ_n. To determine μ_n it is necessary to find the optimal μ_0.

$$\mu_n = \mu_0 \text{ x } \alpha_n$$

where α_n is a decaying factor and is given by

$$\alpha_n = C^{(1/(1+anb))}$$

where C, a, b are positive constants that will determine the magnitude and rate of decrease for α_n. According to the above law, C has to be a positive number larger than 1. When C=1, α_n will be equal to 1 and this algorithm will be same as conventional LMS algorithm. An outline of this algorithm is as follows [6]:

$$w(n+1)=w(n)+\mu_n e(n)x(n) \tag{13}$$

$$y(n)=w^T(n)x(n) \tag{14}$$

$$e(n)=d(n)-y(n) \tag{15}$$

$$\mu_n = \mu_0 \text{ x } \alpha_n \tag{16}$$

$$\alpha_n = C^{(1/(1+anb))} \tag{17}$$

Improved Variable Step Size (IVSS) LMS Algorithm
The adaptive IVSS LMS algorithm takes the following form:

$$w(n+1) = w(n) - 2\mu e(n)x(n)$$

$$\mu(n) = \beta[1 - \exp(-\alpha|e(n)x(n)|)]$$

$$y(n)=w^T(n)x(n)$$

$$e(n)=d(n)-y(n)$$

where α and β are positive constants and the parameters μ, α and β in the algorithm are appropriately chosen to achieve the best trade off between convergence speed and low final MSE [7].

Simulation Results
The simulation is done using MATLAB and in all simulations of system identification the length of the unknown System impulse response is assumed to be N=4. The internal unknown system noise n(n) is assumed to be white Gaussian with mean equals zero and variance equals 0.09 or -10.46 dB. The Step Size Parameter (μ) in all Algorithms is chosen to obtain the same exact value of Misadjustment (M). The value of M is estimated by averaging excess MSE over iteration number (n) after the algorithm has reached steady state. Simulation plots are obtained by ensemble averaging of 200 independent simulations runs.

The simulation plots in stationary environment for different inputs are as follows:

White Gaussian Input

In this case, the adaptive filter and unknown system are both excited by zero-mean white Gaussian signal of unit variance. The impulse response of the unknown system is assumed to be h = [1 0.7 0.5 -0.2]. The Step Size Parameter (μ) in all algorithms is chosen to obtain the same exact value of Misadjustment (M) equal to 2%. The value of step size parameter obtained for the NLMS, NVSS, RVSS, TVLMS and IVSS LMS algorithms are 0.0201, 0.1200, 0.1150, 0.0077 and 0.01 (β) as shown in Fig. 2, Fig. 3, Fig. 4, and Fig. 5 respectively. Fig.6 shows the MSE curves of different algorithms. The Execution Time of NLMS, NVSS, RVSS, TVLMS, and IVSS LMS algorithms are: 33.6875, 32.5469, 35.2813, 32.4219, 32.1719 seconds and number of iterations required are: 500, 108, 90, 214 and 333 respectively.
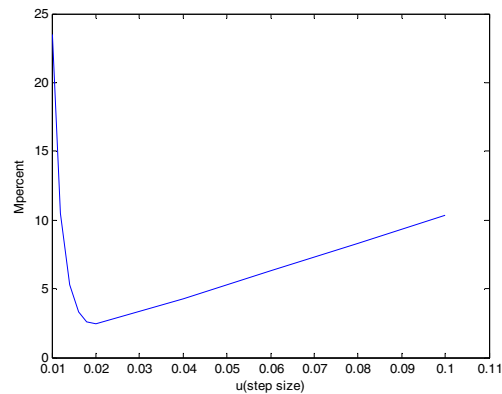
Figure 2: Misadjsutment curve for different values of step size parameter for NLMS algorithm (white input case).
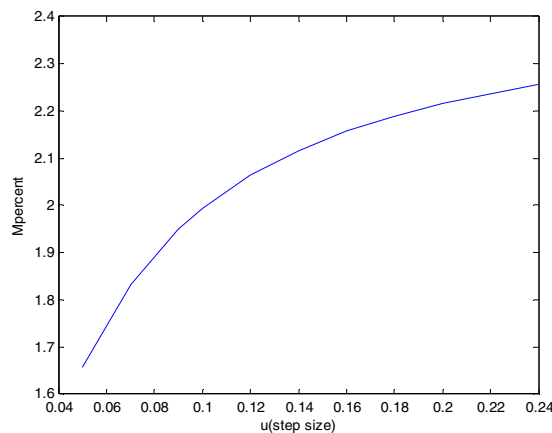

Figure 3: Misadjsutment curve for different values of step size parameter for NVSS algorithm (white input case).
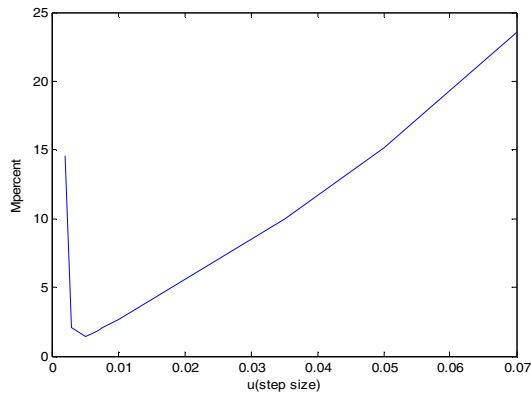
Figure 4: Misadjsutment curve for different values of step size parameter for TVLMS algorithm (white input case).
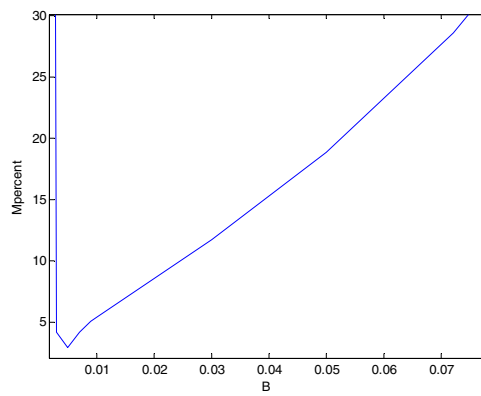

Figure 5: Misadjsutment curve for different values of β for IVSSS algorithm (white input case).
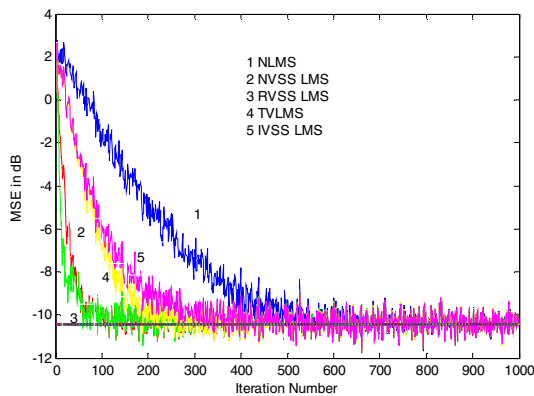

Figure 6: MSE curves for NLMS, NVSS, RVSS, TVLMS and IVSS algorithms (white input case).

Correlated Input

This simulation repeats the previous one with the exception that both the adaptive filter and unknown system are now excited by a correlated signal generated by the first order difference equation: $x(n) = 0.9x(n-1) + g(n)$, [2] where $g(n)$ is a zero-mean white Gaussian noise process of unit variance that is independent with the plant internal noise. The impulse response of the unknown system is assumed to be h = [1 0.7 0.5 -0.2]. This choice of the low pass filter coefficients results in a highly colored input signal with large eigenvalue spread, which makes convergence more difficult.

The simulation plots of Misadjustment for different values of step size parameter (μ) for all algorithms are shown in Fig.7, Fig.8, Fig.9 and Fig.10. These curves are used to find the step size (μ) for which Misadjustment is 6%. The value of step size parameter (μ) for NLMS, NVSS, RVSS, TVLMS and IVSS algorithms obtained are 0.03, 0.013, 0.0187 0.0029 and 0.002 (β) respectively. Fig.11 shows MSE curves of different algorithms. The Execution Time of NLMS, NVSS, RVSS, TVLMS, and IVSS LMS algorithms are: 34.4688, 35.5000, 37.6406, 34.1094, and 33.4063 (sec) and the number of iterations required are: 550, 425, 286, 444 and 483.
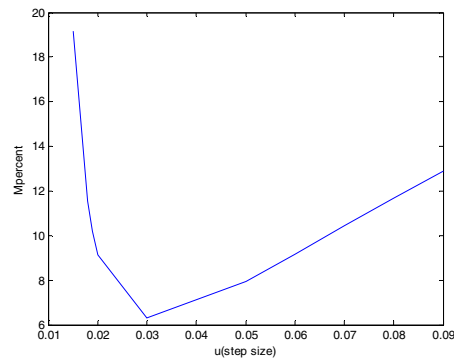

Figure 7: Misadjsutment curve for different values of step size parameter for NLMS algorithm (correlated input case).
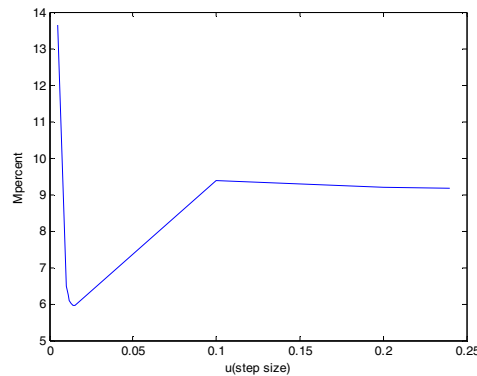

Figure 8: Misadjsutment curve for different values of step size parameter for NVSS algorithm (correlated input case).
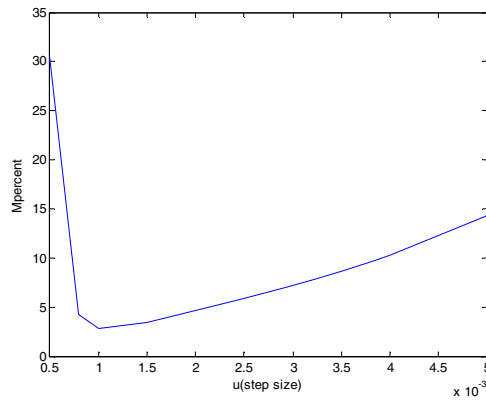
Figure 9: Misadjsutment curve for different values of step size parameter for TVLMS algorithm (correlated input case).
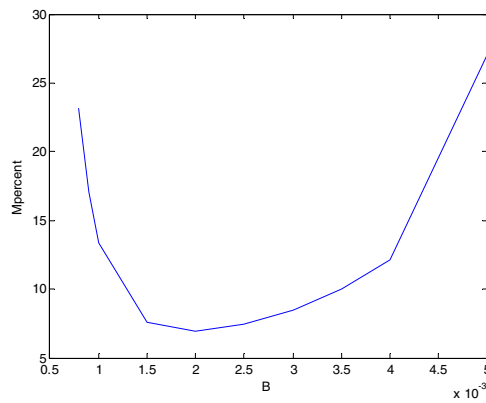

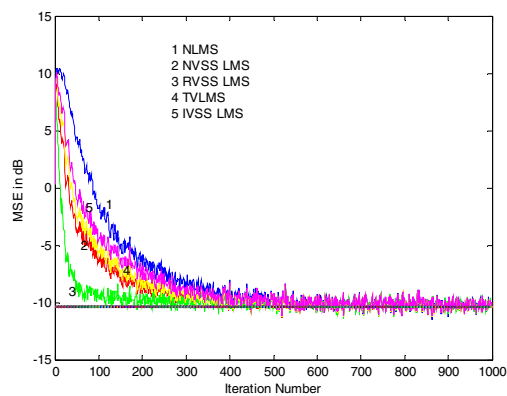**Figure 10:** Misadjsutment curve for different values of β for IVSS algorithm (correlated input case).

Figure 11: MSE curves for NLMS, NVSS, RVSS, TVLMS and IVSS algorithms (correlated input case).

Abrupt Change in the Plant Parameters

This is the same as the previous case but all the elements of h are multiplied by (-1) [3] at iteration number 500 to make an abrupt change. Fig.12 shows MSE curves of different algorithms.
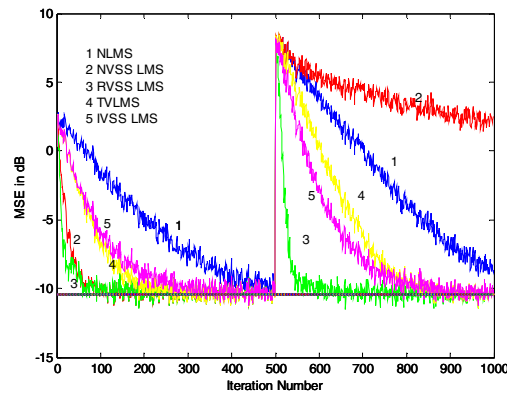

Figure 12: MSE curves for NLMS, NVSS, RVSS, TVLMS and IVSS algorithms for an abrupt change in plant parameters.

Non-stationary Environment

The adaptive filter in this case is used to model a time varying system whose impulse response is generated by: $h(n+1) = h(n) + g(n)$, [1] where $g(n)$ is white Gaussian noise with zero mean and variance equals 0.0001. The same level of misadjustment is achieved in the five algorithms with $\mu_{NLMS} = 0.4500$, $\mu_{NVSS} = 0.1500$, $\mu_{RVSS} = 0.1160$, $\mu_{TVLMS} = 0.0900$ and $\mu_{IVSS} = 0.07$ (β) Fig.13 shows MSE curves of different algorithms. From this figure it is clear that the minimum level of MSE obtained (-8 dB) for this degree of environment is larger than the irreducible noise variance equal to 0.09 (-10.46 dB), which is an expected result in non-stationary environment.
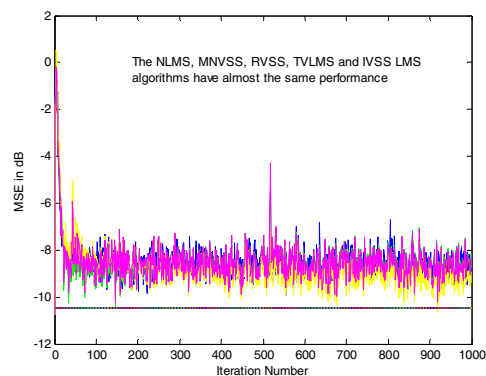

Figure 13: MSE curves for NLMS, NVSS, RVSS, TVLMS and IVSS algorithms (non-stationary case).

Conclusion

Simulation plots showed that RVSS LMS algorithm has fastest convergence speed for white Gaussian and correlated inputs in stationary environment as shown in Fig.2 and Fig.3. The Execution Time of NLMS, NVSS, RVSS, TVLMS, and IVSS LMS algorithms are: 33.6875, 32.5469, 35.2813, 32.4219, 32.1719 seconds for white Gaussian input and 34.4688, 35.5000, 37.6406, 34.1094, 33.4063 (sec) for correlated input respectively. It shows that execution time of IVSS LMS is lowest for both the cases. Fig.4 showed that RVSS LMS algorithm shows superior tracking capability when subjected to an abrupt disturbance. For non-stationary environment the performance of all algorithms is equivalent as shown in Fig.5. In some applications in communication industry, convergence speed and execution time is an issue of consideration. When execution time is vital to the application, IVSS algorithm will be a better choice than the other algorithms while RVSS algorithm will be better, when convergence speed is considered.

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