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Design and Analysis of Broadband Beamspace Adaptive Arrays using Fractional Fourier Transform

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Abstract

The beamwidth of a linear array depends on number of elements in the array and frequency of the input signal. At present designing of wideband antennas and beamformers became important in the fields of microphone arrays intended for teleconferencing, in transmitting or receiving spread spectrum signals, crip signals etc. A beamspace adaptive planar array for broadband beamforming is proposed based on the filter – and - sum beamforming technique and the required filters are implemented using fractional Fourier Transform. A detailed design method was provided for adaptive arrays and simulation results are provided for the proposed method. The results obtained shows that fractional Fourier Transform filter method is superior in interference rejection compared to conventional finite impulse response filter method.

Keywords: Antenna arrays; Linear arrays; Broadband antennas; Adaptive antennas; Constant beam width; Interference rejection; filter – and – sum beamformer; fractional Fourier Transform (FrFT).

Introduction

Arrays of broadband signals have been applied in sonar, radio, radar, acoustic imaging etc. These are often difficult to design because of highly frequency dependent array properties. Figure 1 shows the directivity pattern of a simple 21 element linear array. The figure shows, the mainlobe width decreases with increase in frequency. This causes, some signals to be received with distorted spectra, and also frequency – dependent null locations impair the ability to cancel broadband interference. Figure 2 shows the change in interference rejection capability of an adaptive array with input frequency.

In the past, broadband beamformers have been studied extensively and reported by many authors [4, 5, 7 – 11, 14, 16]. In early days tapped – delay – line circuits are used for beamforming networks and adaptive antenna arrays for broadband signals [16]. With the continued evolution of computing power, the traditional analog delay elements are being replaced with digital filters. In the literature many authors carried out investigations on the design and analysis of broadband beamformers. A class arrays with frequency invariant beam patterns [4, 5, 7, 14, 16, 12], in which a systematic method has been proposed by Ward et.al [16] and it can be applied to one – dimensional (1 – D), two dimensional (2 – D) and three – dimensional (3 – D) arrays. Thomas Chou [14] proposed a digital implementation of beamformer covering audio frequencies by using frequency nesting and filter – and – sum beamforming methods.

The technique to design broadband beamspace adaptive array antenna to suppress interference signals has been proposed by a number of authors [8 – 11]. Every design reported by authors had advantages one over the other and in common they require few adaptive weights compared to tapped – delay – line circuits. The implementation of broadband beamformer and broadband adaptive beamformer at radio frequencies by using filter – and – sum beamformer was described by Srinivasa rao et.al [12, 21].

From fractional Fourier Transform (FrFT) and related concepts [17, 18, 19], it has been seen that the properties and applications of continuous Fourier Transform (CtFT) is special case of FrFT. We have explained the usefulness of fractional Fourier Transform in the design of broadband beamformers with uniform spacing and advantages of it have been reported [12]. This paper gives the suitability of fractional Fourier Transform in the design of broadband beamspace adaptive array.

Beamforming Theory

For a linear array, the far field response for an input frequency ω and incident angle θ (measured relative to broadside) is given by [7]:

$$P(\omega,\theta) = \int_{-\infty}^{\infty} \exp\left(-j\frac{\omega \sin\theta}{c}x\right) D(x,\omega)\,dx \quad (1)$$

where c and θ are the propagation speed and angle of impinging signal and $D(x,\omega)$ is the frequency response with respect to the angular frequency ω and location x. Obviously, in general $P(\omega,\theta)$ is a function of both ω and θ, while for a frequency invariant beamformer, we require that the beam pattern $P(\omega,\theta)$ be independent of ω.

For a weighted linear array of $2N+1$ equally spaced omnidirectional elements, the far field response can be represented from the above equation (1):

$$P(\omega,\theta)=\sum_{n=-N}^{N}\exp\left(-j\frac{\omega\sin\theta}{c}x\right)D(x,\omega)=\sum_{n=-N}^{N}\exp(-jn\omega\tau_0\sin\theta)D(x,\omega) \qquad (2)$$

where, $\tau_0 = d/c$ is the interelement spacing divided by the speed of wave and $D(x,\omega)$ is the response of the filter connected to antenna element at x.

Inter null beamwidth of a uniformly excited linear array is given by [4]:

$$\theta_{BW} = 2\sin^{-1}(2\pi/M\omega\tau_0)\approx 4\pi/M\omega\tau_0 \qquad (3)$$

where, $M = 2N+1$. This expression clearly indicates that the beamwidth of an array was inversely proportional to frequency. It implies that an increase in either the number of elements or interelement spacing results in a decrease in the beamwidth as well.

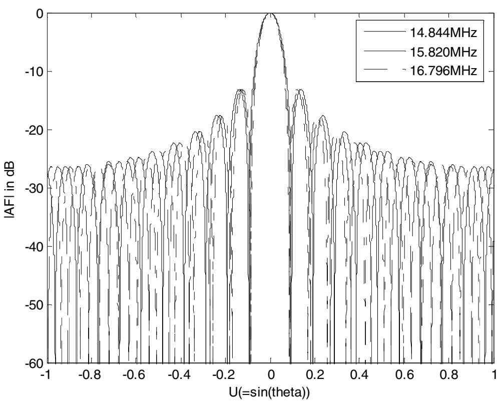

Figure 1: Change in plane wave response of a narrowband linear array with input signal frequency

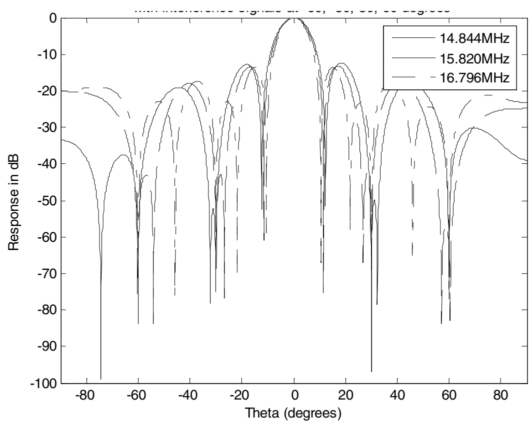

Figure 2: Response of a narrowband adaptive array at interference signals [- 60º, - 30º, 30º, 60º]

Adaptive Array Antenna for Broadband Signals

Proposed Structure of the Beamspace Adaptive Array Antenna for Broadband Signals

Figure 3 shows the proposed beamspace adaptive array antenna for broadband signals. In this structure, a digital multibeam network that can pass the broadband signal and adaptive weights follow the beam selector to reject the interference signals.

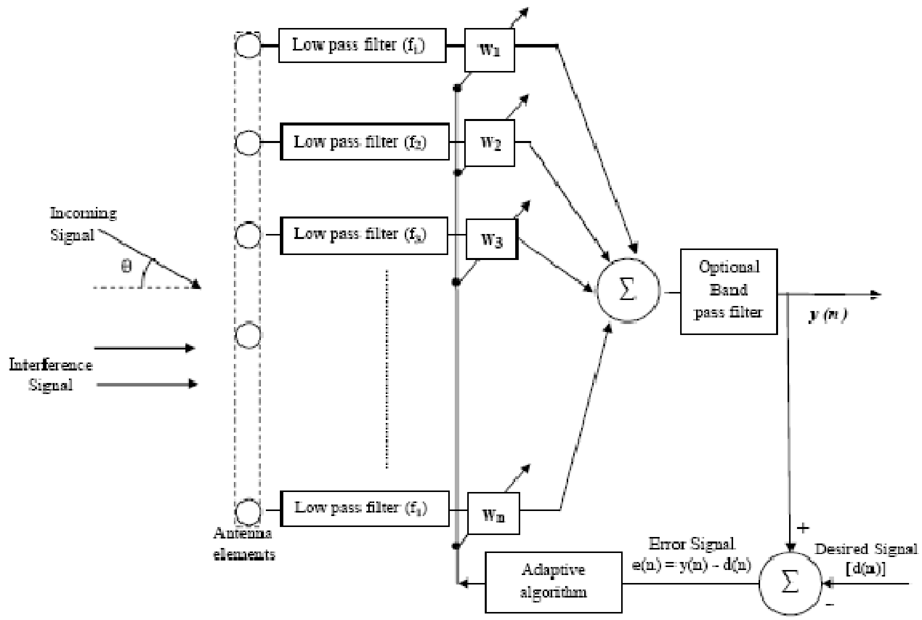

Figure 3: Proposed structure of the beamspace adaptive array antenna for broadband signals

Implementation of low pass filter using FrFT

The continuous Time Fourier transforms (CtFT), which is defined by the following pair [19]:

$$F(\omega)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} f(t)\exp(-j\omega t)dt \leftrightarrow$$
$$f(t)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} F(\omega)\exp(j\omega t)d\omega \qquad (4)$$

The CtFT reflects to the assumption that the signal of interest has stationary frequency content. However, signal representations using intermediate "angularly coupled axes" hold some promise for analyzing signals with time – frequency coupling, e.g., linear – Frequency Modulation. Angular transform of the CtFT, which is called as the angular Fourier transforms or Fractional Fourier Transform (AFT or FrFT), which is controlled by a single continuous angular parameter α. So FrFT can be represented as the rotation of signal in Time-Frequency plane [19] as shown in Figure 4.

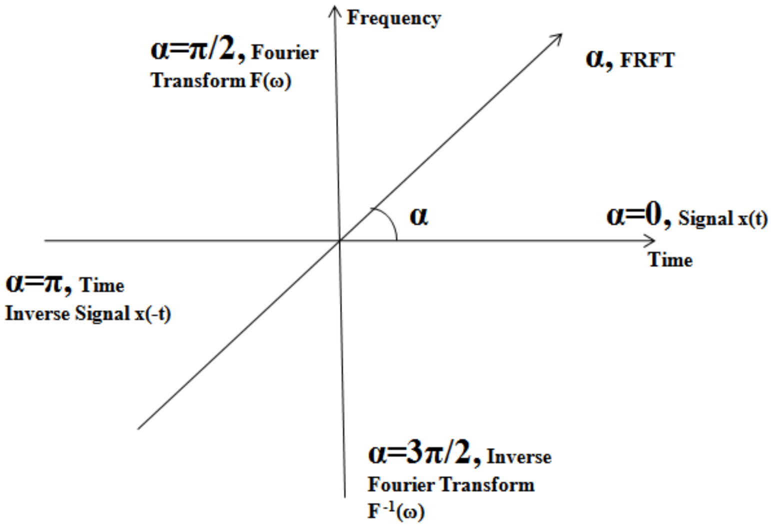

Figure 4: Rotation of signal in Time Frequency plane with an angle α

The generalization of the CtFT is obtained if we consider a rotation through an arbitrary angle α in the (t,ω)plane. Thus, the FrFT of a signal $f(t)$, can be expressed as [12]:

$$F^a[f(t_a)] = \int_{-\infty}^{\infty} K_a(t_a,t) f(t) dt$$

$$K_a(t_a,t) = K_\phi \exp\left[j\pi\left(t_a^2 \cot\phi - 2t_a t \operatorname{cosec}\phi + t^2 \cot\phi\right)\right] \quad (5)$$

$$K_\phi = \exp[-j(\pi \operatorname{sgn}(\phi)/4 - \phi/2)] / [\sin\phi]^{0.5} \quad (6)$$

where $\phi = a\pi/2$.

The Kernel function $K_a(t_a,t)$ has the following spectral expansion:

$$K_a(t_a,t) = \sum_{k=0}^{\infty} \psi_k(t_a) \exp\left(-j\frac{\pi}{2}ka\right)\psi_k(t) \quad (7)$$

where $\psi_k(t)$denotes k th Hermite – Gaussian function, and t_a denotes the variable in the ath – order *Fractional Fourier Domain*. The kth order Hermite – Gaussian function is defined as $(k = 0, 1, 2, \cdots\cdots)$:

$$\psi_k(t) = \frac{2^{1/4}}{\sqrt{2^k k!}} H_k\left(\sqrt{2\pi}\, t\right)\exp\left(-\pi t^2\right) \quad (8)$$

where H_k denotes k th order Hermite polynomial having k real zeros.

In the equation (8), $\exp\left(-j\frac{\pi}{2}ka\right)$ represents the a th power of the eigenvalues. When $a=1$, the FrFT reduces to the ordinary Fourier transform, where t_1 denotes the frequency – domain variable.

Here, we have adopted the design low pass filter using FrFT based on procedure of tunable FIR filters. A FIR digital filter operation is a linear convolution of the finite duration impulse response with the input signal sequence $x(n)$. The impulse response $h(n)$, of Kaiser Window is given as [20]:

$$h(n) = h_d(n)w(n) \tag{9}$$

where $h_d(n)$ is the desired or ideal impulse response, and $w(n)$ is the Kaiser Window sequence. Since multiplication in the time domain corresponds to convolution in the frequency domain which can be expressed as a complex convolution operation given as:

$$H(\omega) = \frac{1}{2\pi}\int_{-\pi}^{\pi} W(\lambda)H_d(\omega-\lambda)d\lambda \tag{10}$$

where, $H(\omega)$ is the frequency response of filter, $H_d(\omega)$ is desired or ideal frequency response of lowpass filter, and $W(\omega)$ is frequency response of Kaiser Window. From the above equation, the transition bandwidth of $H(\omega)$ is proportional to mainlobe width of Kaiser Window $W(\omega)$. Figure 5 shows the variation of mainlobe width with order of FrFT. It is observed that as the FrFT order is reduced the main lobe width of FrFT Kaiser Window shrinks. This feature has been used here to tune the transition bandwidth of lowpass filter.

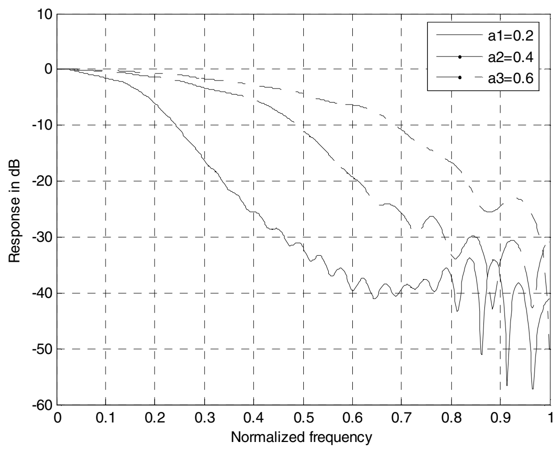

Figure 5: Variation in Kaiser Window response with order of FrFT.

Design Method

The following procedure was used to implement filters used in adaptive array:

Step 1: The linear array geometry has to be determined. Inter – element spacing must be at most $d = c/4f_0$ to avoid aliasing. Where c is speed of wave propagation and f_0 is lower frequency of operation required for antenna array.

Step 2: A set of frequency values can be calculated as $f_1, f_2, f_3 \cdots\cdots f_K$ such that $f_1 = f_0$, $f_K = 2f_0$, where $K = (N+1)/2$ and the remaining values are uniformly distributed in the range $[f_0, 2f_0]$. So, the cutoff frequencies of filters connected to antenna elements on the array in the order $f_1, f_2, f_3 \cdots\cdots f_K, \cdots\cdots f_3, f_2, f_1$.

Step 3: For each element, for the frequency response constraints determined in step 2; required low pass filters are implemented by using fractional Fourier Transform.

Step 4: When a Least Mean Square (LMS) algorithm is applied to the beamspace adaptive array antenna to update the array weights $w_1, w_2, w_3 \cdots\cdots w_N$. Adaptive weights are updated according to the following equations:

$$\begin{aligned} &\text{Output} \quad && y[n] = w^H[n]\,x[n] \\ &\text{Error} \quad && e[n] = d[n] - y[n] \\ &\text{Weight} \quad && w[n+1] = w[n] + \mu\, x[n]\varepsilon^*[n] \end{aligned} \tag{11}$$

where μ is a gain constant and control the rate of adaptation. The adaptive algorithm is not limited to the LMS, but in the paper LMS algorithm was used to generate computer simulations.

Simulations

Conditions of Simulation

We assume that antenna array was uniformly spaced and each antenna element has an omnidirectional pattern and no mutual coupling. Table 1 gives basic design parameters used for designing of the antenna. Table 2, gives the details of radio environment used in computer simulation.

Table 1: Basic design parameters

Number of Antenna elements	15
Gain constant μ	0.00015
Sampling frequency f_c	200 MHz
Type of filter	FrFT Low pass filter
Order of the filter	100

Table 2: Radio environment used in computer simulation

	Direction of arrival		SNR/SINR	Kind of signal
Desired signals				
$f_1 = 14.844\ MHz$	90º		--	Sinusoidal
$f_2 = 15.820\ MHz$	90º		--	Sinusoidal
$f_3 = 16.796\ MHz$	90º		--	Sinusoidal
Interference signals				
	Case1	Case 2		
Interference 1 (I_1)	- 60º	- 75º	- 10 dB	Gaussian
Interference 2 (I_2)	- 30º	- 45º	- 10 dB	Gaussian
Interference 3 (I_3)	30º	45º	- 10 dB	Gaussian
Interference 4 (I_4)	60º	60º	- 10 dB	Gaussian
Noise	--		- 10 dB	Gaussian

Results

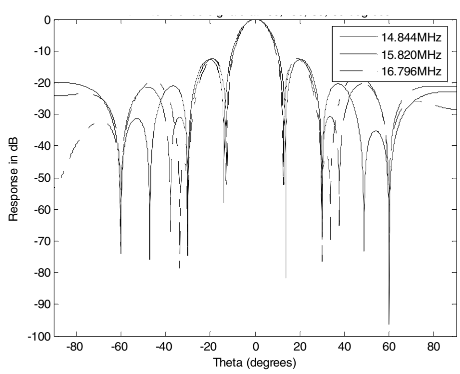

Figure 6: Response of a broadband adaptive array at interference signals [- 60º, - 30º, 30º, 60º] with FIR filters

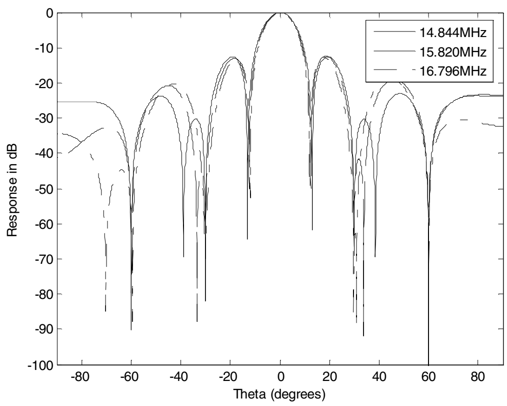

Figure 7: Response of a broadband adaptive array at interference signals [- 60º, - 30º, 30º, 60º] with FrFT filters

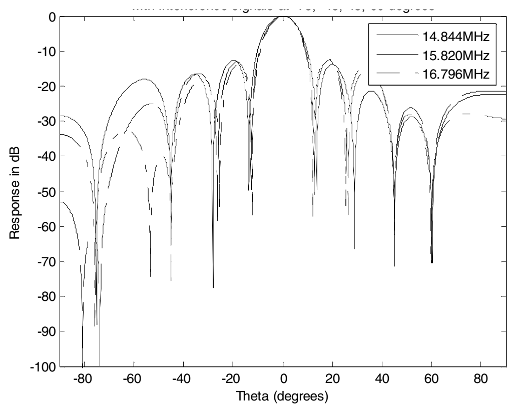

Figure 8: Response of a broadband adaptive array at interference signals [- 75º, - 45º, 45º, 60º] with FIR filters

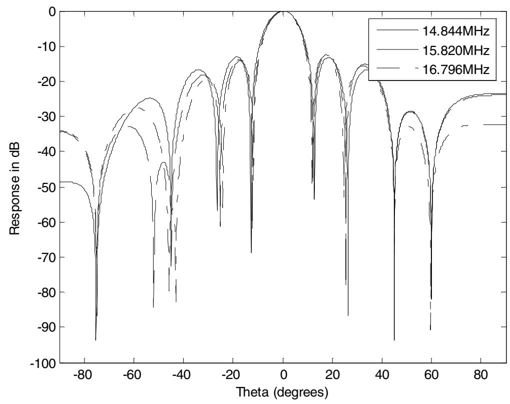

Figure 9: Response of a broadband adaptive array at interference signals [- 75º, - 45º, 45º, 60º] with FrFT filters

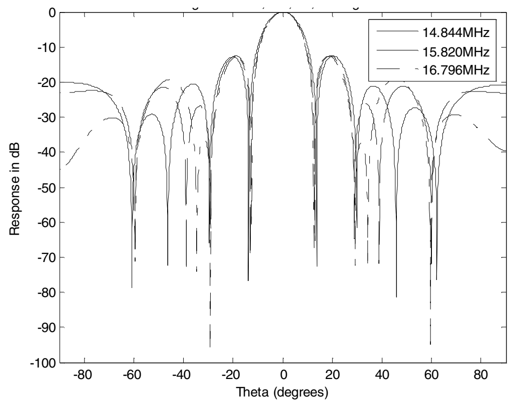

Figure 10: Response of a broadband adaptive array at interference signals [- 60º, - 30º, 30º, 60º] with FIR filters for iterations 100

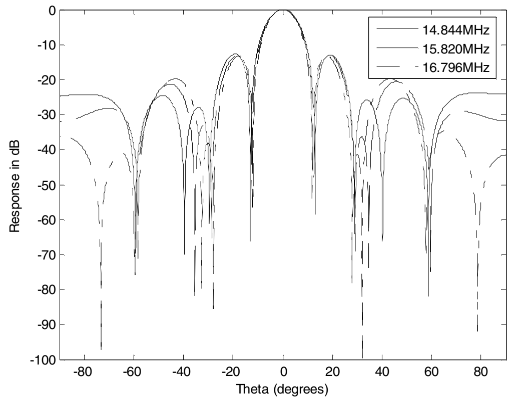

Figure 11: Response of a broadband adaptive array at interference signals [- 60º, -30º, 30º, 60º] with FrFT filters for iterations 100

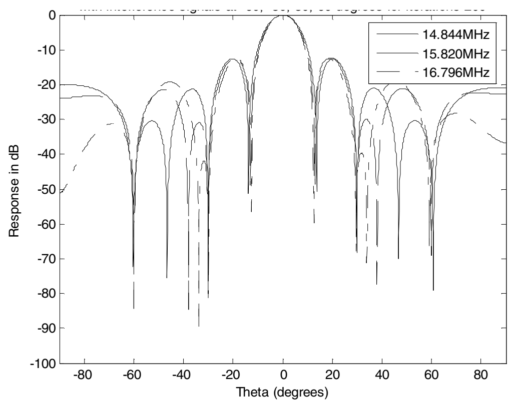

Figure 12: Response of a broadband adaptive array at interference signals [- 60º, -30º, 30º, 60º] with FIR filters for iterations 300

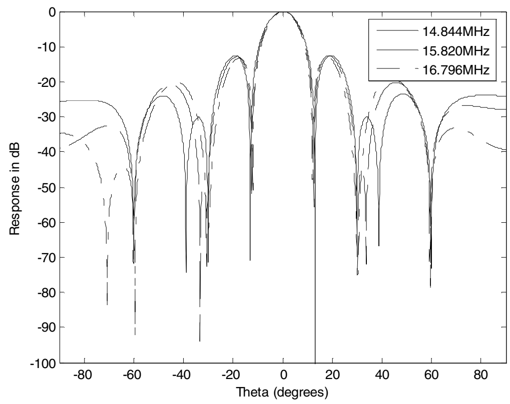

Figure 13: Response of a broadband adaptive array at interference signals [- 60º, -30º, 30º, 60º] with FrFT filters for iterations 300

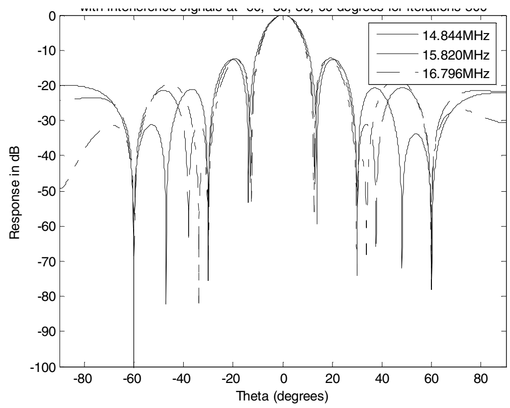

Figure 14: Response of a broadband adaptive array at interference signals [- 60º, -30º, 30º, 60º] with FIR filters for iterations 500

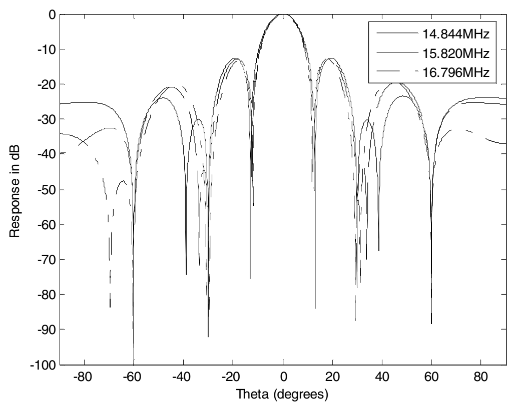

Figure 15: Response of a broadband adaptive array at interference signals [- 60º, -30º, 30º, 60º] with FrFT filters for iterations 500

Table 3: Response of broadband adaptive antenna (designed with FIR filters) for interference signals

	Interference rejection in dB					
	Case1			Case 2		
	f_1	f_2	f_3	f_1	f_2	f_3
Interference 1 (I_1)	-74.13	-67.73	-61.01	-81.65	-67.93	-61.11
Interference 2 (I_2)	-71.85	-70.95	-57.2	-76.28	-64.3	-68.51
Interference 3 (I_3)	-68.84	-62.92	-76.33	-61.97	-60.75	-76.2
Interference 4 (I_4)	-66.22	-63.95	-74.01	-55.21	-75.55	-68.62

Table 4: Response of broadband adaptive antenna (designed with FrFT filters) for interference signals

	Interference rejection in dB					
	Case1			Case 2		
	f_1	f_2	f_3	f_1	f_2	f_3
Interference 1 (I_1)	-90.11	-60.28	-57.51	-82.33	-71.08	-70.21
Interference 2 (I_2)	-82.06	-68.51	-64.97	-71.21	-63.21	-57.82
Interference 3 (I_3)	-69.18	-66.53	-66.59	-93.81	-65.88	-64.99
Interference 4 (I_4)	-107.26	-66.26	-67.8	-81.92	-70.15	-60.24

Table 5: Change in broadband adaptive antenna (designed with FIR filters) response with respect to no. of iterations

	100 iterations			300 iterations			500 iterations		
Rejection Maximum at (degrees)	f_1	f_2	f_3	f_1	f_2	f_3	f_1	f_2	f_3
Interference 1 (I_1)	-61.1	-60.0	-59.9	-60.2	-60.0	-60.0	-60.0	-59.9	-59.9
Interference 2 (I_2)	-29.8	-29.4	-28.4	-30.0	-30.0	-30.0	-30.0	-30.0	-29.9
Interference 3 (I_3)	30.1	29.2	28.8	30.1	30.0	29.7	30.0	30.1	29.9
Interference 4 (I_4)	60.2	58.7	59.7	60.3	59.7	59.9	60.1	59.9	59.8

Table 6: Change in broadband adaptive antenna (designed with FrFT filters) response with respect to no. of iterations

	100 iterations			300 iterations			500 iterations		
Rejection Maximum at (degrees)	f_1	f_2	f_3	f_1	f_2	f_3	f_1	f_2	f_3
Interference 1 (I_1)	-59.4	-58.2	-59.7	-59.9	-59.9	-59.6	-60.0	-59.9	-60.4
Interference 2 (I_2)	-29.5	-28.5	-27.7	-30.0	-30.6	-30.0	-30.0	-30.0	-30.4
Interference 3 (I_3)	28.9	29.2	28.1	29.9	30.2	30.2	30.0	29.9	29.3
Interference 4 (I_4)	58.6	59.6	57.8	59.9	59.3	59.6	59.9	59.9	59.9

Conclusions

In this paper, a new wide-band adaptive array processing structure was presented with the filter – and – sum beam forming method using fractional Fourier Transform. We demonstrated by computer simulation the possibility of suppressing wideband interference signals as well as a much faster convergence speed, of our proposed method. Design examples considered in Table 2 and results presented in Figure 6 to 9, shows that it can achieve a satisfactory interference rejection and frequency invariant response over the frequency range of interest. From Table 3 and 4 and from Figure 6 to 9, it is observed that FrFT filter method offers better interference rejection compared to FIR filter method. This advantage is compensated by slow convergence of FrFT method, as shown in Table 5 and 6 and from Figure 10 to 15.

References

[1] Applebaum Sidney P. (1976): Adaptive Arrays, IEEE Transactions on Antennas and Propagation, 24(5), pp. 585 – 598.

[2] Er M. H. and Cantoni A. (1985): On an Adaptive Antenna Array Under Directional Constraint, IEEE Transactions on Acoustics, Speech, and Signal Processing, 33(4), pp. 1326 – 1328.

[3] Gabriel William F. (1992): Adaptive Processing Array Systems, Proceedings of the IEEE, 80(1), pp. 152 – 162.

[4] Goodwin. M. M., Elko G. W. (1993): Constant beamwidth beamforming, in Proc. IEEE International Conference on Acoustics, Speech, and Signal Processing, Minneapolis, MN, USA, 1, pp.169 – 172.

[5] John H. Doles, III and Frank D. Benedict (1988): Broad-Band Array Design Using the Asymptotic Theory of Unequally Spaced Arrays, IEEE Transactions on Antennas and Propagation, 36(1), pp. 27 – 33.

[6] Lamont Frost III O. (1972): An Algorithm for Linearly Constrained Adaptive Array Processing, Proceedings of the IEEE, 60(8), pp. 926 – 935.

[7] Liu W. and Weiss S. (2004): A new class of broadband arrays with frequency invariant beam patterns, in Proc. IEEE International Conference on Acoustics, Speech, and Signal processing, Montreal, Canada, 2, pp. 185 – 188.

[8] Liu W., Wu R. B. and Langley R. (2006): Analysis and a Novel Design of the Beamspace Broadband Adaptive Array, Progress In Electromagnetics Research Symposium, USA, pp. 368 – 373.

[9] Liu Wei, Wu Renbiao, and Langley Richard J. (2007): Design and Analysis of Broadband Beamspace Adaptive Arrays, IEEE Transactions on Antennas and Propagation, 55(12), pp. 3413 – 3420.

[10] Sekiguchi MiuraTakashi, Ryu and Karasawa Yoshio (1996): Beamspace Adaptive Array Antenna for Broadband Signals, Proceedings of ISAP, CHIBA, JAPAN, pp. 761 – 764.

[11] Sekiguchi T akashi and KarasawaYoshio (2000): Wideband Beamspace Adaptive Array Utilizing FIR Fan Filters for Multibeam Forming, IEEE Transactions on Signal Processing, 48(1), pp. 277 – 2840.

[12] Srinivasa rao A. S., Mallikarjuna rao P., Muralidhar P. V. and Nayak S. K. (2010): Frequency Invariant beampatterns Using Fractional Fourier Transform, International J.of Multidispl. Research & Advcs. in Engg. (IJMRAE), 2(II), pp.123-134.

[13] Takao Kazuaki, Fujita Masaharu and Nishi Takashi (1976): An Adaptive Antenna array under Directional Constraint, IEEE Transactions on Antennas and Propagation, 24(5), pp. 662 – 669.

[14] Thomas Chou (1995): Frequency – Independent Beamformer with low response error, Proc. IEEE International Conference on Acoustics, Speech, and Signal Processing, Detroit, USA, 5, pp. 2995 – 2998.

[15] Widrow B., Mantey P. E., Griffiths L. J., and Goode B. B. (1967): Adaptive Antenna Systems, Proceedings of the IEEE, 55(12), pp. 2143 – 2159.

[16] Ward Darren B., Kennedy Rodney A. and Williamson Robert C. (1995): Theory and design of broadband sensor arrays with frequency invariant far – field beam patterns, J. Acoust. Soc. Am., 97, pp. 1023 – 1034.

[17] Almeida L.B. (1994): The Fractional Fourier transform and time-frequency representation, *IEEE Transactions on Signal Processing*, 42(11), pp. 3084 – 3091.

[18] Candan C. and Alper Kutay M. and Ozaktas H. M. (2000): The Discrete Fractional Fourier Transform, IEEE Transactions on Signal Processing, 48(5), pp. 1329 – 1337.

[19] Santhanam B. and Mcclellan J.H.(1996): The Discrete Rotational Fourier transform, IEEE Transactions on Signal Processing, 44(4), pp. 994 – 998.

[20] Sharma S. N., Saxena R. and Saxena S. C.(2006): Sharpening the response of an FIR filter using Fractional Fourier Transform, J. of Indian Inst. Sci., 86, pp. 163 – 168.

[21] Srinivasa rao A. S., Mallikarjuna rao P., Jaya E., and Ashokkumar V. (2011): An alternative approach to design and analysis of broadband beamspace adaptive arrays, International J. of Engineering Science and Technology, 3(9), pp.7029 – 7036.