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Binary Orthogonal Code Generation for Multi-User Communication using n-bit Gray and Inverse Gray Codes

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Abstract

This paper presents a technique for the generation of Binary Orthogonal Codes using Gray and Inverse Gray codes. Orthogonal codes and Pseudo random Noise codes are used as spreading sequences in multi-user communications. The Spreading codes are also referred to as spreading sequences. Walsh Codes, Gold Codes, Maximal Length sequences, Barker Codes and Kasami codes are commonly used orthogonal codes. In this paper orthogonal codes obtained using Gray code and its inverse are presented. An n-bit Cyclic Gray Code is a circular list of all 2^n bit strings such that successive code words differ in only one bit position. An 'n' bit Inverse Gray Code , is defined exactly opposite to Gray code, it is a circular list of all 2^n bit strings of length 'n' each, such that successive code words differ in (n-1) bit positions. Algorithms discussed in this paper for the generation of Gray and Inverse Gray codes result in n! combinations. The method explained in this paper enables us to generate six and eight length Binary Orthogonal Codes and also Codes of lengths of their even multiples. The auto and cross correlation properties of these codes along with 8,16-length Walsh Codes are also presented in the current work.

Keywords: Gray Code, Inverse Gray Code, Binary Orthogonal Code.

Introduction

This paper presents a technique for the construction of Binary Orthogonal Codes using Gray and Inverse Gray codes. The Spreading sequences and orthogonal codes that are popularly used are Walsh Codes, Gold Codes, Maximal Length sequences, Barker Codes and Kasami codes. Orthogonal codes should possess high auto correlation and low cross correlation properties. An n-bit Gray code is a list of all 2^n bit strings such that successive code words differ in only one bit position. If the first and last code words also differ in one bit position then the resultant code is called cyclic. Gray codes have the adjacency property which makes the hamming distance between adjacent code words always equal to 1. Inverse Gray codes, as defined in [4], on the contrary, exhibit maximum possible hamming distance (n-1) between the two successive code words. A very commonly used method of generating n-bit Gray code from binary is by performing bit-wise XOR operation of two successive bits. The encoding method in [6] to generate Inverse Gray code is similar to the Gray code generation technique mentioned above. In this an n-bit Inverse Gray code is derived from the binary representation $[b_1, b_2, b_3 \ldots. b_n]$ of an integer 'h' from a set of all the integers $\{0,1,2,\ldots..(2^n-1)\}$. If 'n' is even $b_0 = 0$ else $b_0 = b_1$.Inverse Gray code is obtained by performing the bit-wise XOR or XNOR of b_{i-1} & b_i depending on whether 'h' is even or odd respectively. Adopting the encoding technique described above for odd values of 'n' results in a code with the code words in the bottom half of the list similar to the code words in the top half of the list. So, the Inverse Gray code for 'n' odd is obtained by complementing the Most Significant Bit (MSB) of bottom half.

Inverse Gray Code generation algorithm is obtained by suitably modifying the Binary Cyclic Gray Code generation technique in [2]. A total of n! combinations of Inverse Gray codes can be generated for any integer 'n'. For even and odd values of 'n' there is a small difference in the generation procedure. Inverse Gray codes combined with Robust Symmetrical Number System (RSNS) find applications in the area of error detection and correction [6]. Previous work on the generation of orthogonal codes [3] discusses exhaustive search scheme wherein to construct N-length binary orthogonal codes all the binary representations of integers from 1to 2^{N-1} are checked for orthogonality and this method is proved to be tedious as the length of the code increases.

The paper is organized in the following manner: Section II briefly discusses the Binary Cyclic Gray Code generation algorithm in [2]. In Section –III the Inverse Gray Code generation algorithm is discussed. The proposed technique for the construction of Binary Orthogonal Code sets from n-bit Gray and Inverse Gray codes is explained in Section-IV. Section-V discusses the auto and cross correlation properties of the generated orthogonal codes and finally, Section-VI concludes the paper with the limitations and future scope of work.

Algorithm to generate Binary Cyclic Gray Codes n - bit Cyclic Gray code (radix r=2), M = 2^n

Let an n-bit Cyclic Gray code be needed. Let $(P_1, P_2, P_3, \ldots\ldots\ldots P_n)$ be a permutation of $(1,2,3,\ldots.n)$. The $M = 2^n$ integers $(0, 1, 2, \ldots\ldots, (2^n-1))$can be arranged in the following indexed indicial sets.

$Q_0 = 2^0 \{1, 3, 5\ldots\ldots.\}$

$Q_1 = 2^1 \{1, 3, 5\ldots\ldots.\}$

:

:
$Q_{n-1} = 2^{n-1}$

Then, for any integer value of 'n', starting with the row of all zeros as a zeroeth row, the i^{th} row is obtained from the $(i-1)^{th}$ row by replacing the p_j^{th} bit by its successor, if it is in Q_{j-1}.

Let us consider the construction of a 3-bit binary Gray code. All the integers, i.e., $\{0, 1, 2, 3,\ldots.(2^3 - 1)\}$ are arranged in the form of indicial sets as shown below:

$Q_0 = 2^0 \{1, 3, 5,7\} = 1,3,5,7$
$Q_1 = 2^1 \{1, 3\} = 2,6$
$Q_2 = 2^2 \{1\} = 4$

As stated earlier, let $(P_1, P_2, P_3 \ldots\ldots P_j \ldots\ldots P_n)$ be a permutation of $(1,2,3,\ldots.. j,\ldots.n)$. Since we are considering a 3-bit case, consider the permutation {2,3,1}. Hence, $P_1 = 2$; $P_2 = 3$; $P_3 = 1$. The first code word is (0 0 0) which is the zeroeth row of the code. To obtain 1st row, we have to change $P_j^{\ th}$ bit if '1' is in Q_{j-1}.

Here, 1 is in Q_0. Therefore, P_1 bit is to be changed and P_1=2, hence the code is (0 1 0). Similarly, since '2' is in Q_1, P_2 bit (i.e. 3rd bit) is to be changed, hence the code is (1 1 0). The resulting code obtained by continuing this procedure is tabulated in Table I.

Table I: 3-bit Cyclic Gray Code with permutation {2, 3, 1}

i^{th} row	P_j	Bit to be changed	3-bit Binary Gray Code		
			3	2	1
0	-	-	0	0	0
1	P_1	2	0	1	0
2	P_2	3	1	1	0
3	P_1	2	1	0	0
4	P_3	1	1	0	1
5	P_1	2	1	1	1
6	P_2	3	0	1	1
7	P_1	2	0	0	1

A total of n! Gray codes can be generated using the above technique for any integer value of 'n' and all these Gray codes are cyclic.

Generation of Binary Inverse Gray Codes n - bit Inverse Gray code (radix r=2), M = 2^n.

Let an n-bit Inverse Gray code be needed. Let $(P_1, P_2, P_3, \ldots\ldots\ldots.P_n)$ be a permutation of $(1,2,3,\ldots.n)$. The 2^n integers $(0, 1, 2, \ldots\ldots, (2^n-1))$can be arranged in the following indexed indicial sets.

$Q_0 = 2^0$ {1, 3, 5…….}
$Q_1 = 2^1$ {1, 3, 5…….}
:
:
$Q_{n-1} = 2^{n-1}$

Then, for ‘n’ even, starting with the row of all zeros as a zeroeth row, the i^{th} row is obtained from the $(i-1)^{th}$ row by replacing all other bits except the p_j^{th} bit by its successor, if it is in Q_{j-1}. And for ‘n’ odd, the above procedure is used to obtain all the rows except M/2 th row. For M/2 th row, all the bits have to changed irrespective of where it falls within the indicial sets.

Inverse Gray code for ‘n’ even

Let us consider the construction of a 4-bit Inverse Gray code. All the integers, i.e., {0, 1, 2, 3,…..$(2^4 - 1)$} are arranged in the form of indicial sets as given below:

$Q_0 = 2^0$ {1,3,5,7,9,11,13,15}= 1,3,5,7,9,11,13,15
$Q_1 = 2^1$ {1,3,5,7} = 2,6,10,14
$Q_2 = 2^2$ {1,3} = 4,12
$Q_3 = 2^3$ {1} = 8

As stated earlier, let (P_1, P_2, P_3 …… P_j ……P_n) be a permutation of (1,2,3,….. j,….n). Since we are considering a 4-bit case, consider the permutation {1,2,3,4}. Hence, $P_1 = 1$; $P_2 = 2$; $P_3 = 3$; $P_4 = 4$. The first code word is (0 0 0 0) which is the zeroeth row of the code. To obtain 1st row, we have to change all other bits except P_j th bit if ‘1’ is in Q_{j-1}. Here, 1 is in Q_0. Therefore, retaining P_1 bit as it is all other bits are to be changed, since $P_1 = 1$ 1st is retained and 2nd, 3rd and 4th bits are changed. Hence the resultant codeword of the 1st row is (1 1 1 0). Similarly, to obtain 2nd row since ‘2’ is in Q_1, P_2 bit is to be unchanged, hence the code is (0 0 1 1). Table II shows the resulting code obtained by continuing this procedure.

Table II: 4-bit Inverse Gray Code with permutation {1, 2, 3, 4}

i th row	P_j	Bits to be Changed	4-bit Inverse Gray code 4 3 2 1
0	-	-	0 0 0 0
1	P_1	2,3,4	1 1 1 0
2	P_2	1,3,4	0 0 1 1
3	P_1	2,3,4	1 1 0 1
4	P_3	1,2,4	0 1 1 0
5	P_1	2,3,4	1 0 0 0
6	P_2	1,3,4	0 1 0 1
7	P_1	2,3,4	1 0 1 1
8	P_4	1,2,3	1 1 0 0
9	P_1	2,3,4	0 0 1 0

10	P_2	1,3,4	1 1 1 1
11	P_1	2,3,4	0 0 0 1
12	P_3	1,2,4	1 0 1 0
13	P_1	2,3,4	0 1 0 0
14	P_2	1,3,4	1 0 0 1
15	P_1	2,3,4	0 1 1 1

Using the above algorithm, 4! i.e. 24 possible combinations of 4-bit binary inverse gray codes can be generated.

Inverse Gray code for n- odd

Let us consider the construction of a 3-bit binary Inverse Gray code. All the integers, i.e., {0, 1, 2,3,…..($2^3 - 1$)} are arranged in the form of indicial sets as given below:

$Q_0 = 2^0 \{1, 3, 5,7\} = 1,3,5,7$

$Q_1 = 2^1 \{1, 3\} = 2,6$

$Q_2 = 2^2 \{1\} = 4$

Since we are considering a 3-bit case, consider the permutation {2,3,1}. Hence, $P_1 = 2$; $P_2 = 3$; $P_3 = 1$. The first code word is (0 0 0) which is the zeroeth row of the code. To obtain 1st row, we have to change P_j th bit if '1' is in Q_{j-1}. Here, 1 is in Q_0. Therefore, all the other bits except P_1 are to be changed, hence the code is (1 0 1). Similarly, since '2' is in Q_1, all the other bits except P_2 bit are to be changed, hence the code is (1 1 0). This procedure is to be repeated for all the rows except M/2 th row (in this case it is 4th row).To obtain 4th row all the 3-bits are to be changed. The resulting code obtained by continuing this procedure is given in Table III.

Table III: 3-bit Inverse Gray Code with Permutation {2, 3, 1}

i^{th} row	P_j	Bits to be changed	3-bit Gray Code 3	2	1
0	-	-	0	0	0
1	P_1	1,3	1	0	1
2	P_2	1,2	1	1	0
3	P_1	1,3	0	1	1
4	All	1,2,3	1	0	0
5	P_1	1,3	0	0	1
6	P_2	1,2	0	1	0
7	P_1	1,3	1	1	1

Similarly n! Inverse Gray Codes can be generated for any integer 'n' using n! permutations.

Algorithm to Generate Binary Orthogonal Codes using n-bit Gray and Inverse Gray Codes (n=3 or 4)

Two sequences are said to be Orthogonal when the Cross-correlation (inner product) between them is zero. With this construction procedure 2^n orthogonal code words of length 2n can be obtained. This algorithm can be applied to n=3 or 4 only.

Step 1: Generate n-bit Gray code using the algorithm discussed in Section II with any permutation.

Step 2: Using the same permutation generate n-bit Inverse Gray Code using the algorithm in Section III.

Step 3: Append Inverse Gray Code to the Gray Code to result in 2n-length GIG Code (Gray Inverse Gray).

Step 4: Each row of this GIG Code is an Binary Orthogonal Code word of length 2n.

6-length Binary Orthogonal Code Construction using 3-bit Gray and Inverse Gray Codes

A 6-bit (3+3) GIG Code of $8(2^3)$ code words is constructed by appending 3-bit Gray Code with 3-bit Inverse Gray Code generated using the algorithms of section II & III for a given permutation. Each row of this GIG Code is a 6-length Binary Orthogonal Code word. Table IV & V give the 6-length Orthogonal Code sets generated using the permutation {1, 3, 2} & {1,2,3}.

Table – IV

3-bit Gray Code	3-bit Inverse Gray Code	6-bit GIG code	6-length Orthogonal Code Set I (in decimal notation)
000	000	000000	0
001	110	001110	14
101	101	101101	45
100	011	100011	35
110	100	110100	52
111	010	111010	58
011	001	011001	25
010	111	010111	23

Table –V

3-bit Gray Code	3-bit Inverse Gray Code	6-bit GIG code	6-length Orthogonal Code Set II (in decimal notation)
000	000	000000	0
001	110	001110	14
011	011	011011	27
010	101	010101	21
110	010	110010	50
111	100	111100	60
101	001	101001	41
100	111	100111	39

With n=3, this construction technique results in two different Binary Orthogonal Codes.

Binary Orthogonal Code of length 12 can be obtained using the following relationship

$$C_{12} = \begin{bmatrix} C_6 C_6 \\ C_6 \overline{C_6} \end{bmatrix}$$

8-length Binary Orthogonal Code Construction using 4-bit Gray and Inverse Gray Codes

Similarly, 8-length Binary Orthogonal Code can be generated using the algorithm explained above. Table-VI gives the 8-length Orthogonal Code generated with the permutation {1, 2, 3, 4}. From the Table VI it can be observed that the bottom-half of the code words are logical inverses of top-half. All the 4! Permutations result in same Binary Orthogonal Code set.

Table – VI

4-bit Gray Code	4-bit Inverse Gray Code	8-bit GIG Code	8-length Binary Orthogonal Code (in decimal notation)
0000	0000	00000000	0
0001	1110	00011110	30
0011	0011	00110011	51
0010	1101	00101101	45
0110	0110	01100110	102
0111	1000	01111000	120
0101	0101	01010101	85
0100	1011	01001011	75
1100	1100	11001100	204
1101	0010	11010010	210
1111	1111	11111111	255
1110	0001	11100001	225
1010	1010	10101010	170
1011	0100	10110100	180
1001	1001	10011001	153
1000	0111	10000111	135

Binary Orthogonal Code of length 16 can be obtained using the following relationship

$$C_{16} = \begin{bmatrix} C_8 C_8 \\ C_8 \overline{C_8} \end{bmatrix}$$

Binary Orthogonal Code sets of lengths 12 & 16 are given in Table VII. Similarly, Binary Orthogonal Codes of any even multiples of lengths 6 & 8 can be generated recursively using the following relationship

$$C_{2M} = \begin{bmatrix} C_M C_M \\ C_M \overline{C_M} \end{bmatrix}$$

Table – VII

C_{12} Set I	C_{12} Set II	C_{16}
0	0	0
910	910	7710
2925	1755	13107
2275	1365	11565
3380	3250	26214
3770	3900	30840
1625	2665	21845
1495	2535	19275
63	63	255
945	945	7905
2898	1764	13260
2268	1386	11730
3339	3213	26265
3717	3843	30855
1638	2646	21930
1512	2520	19380

Table-VIII: Integer representation of 8 and 16-length Walsh Code sets

8-length Walsh Code set	16-length Walsh Code set
0	0
15	4080
60	15420
102	26214
85	21930
90	23205
105	26985
51	13107
	255
	3855
	15555
	26265
	21845
	23130
	27030
	13260

Auto & Cross Correlation Metrics

The performance of different spreading sequences is evaluated by Mean Square Aperiodic Auto Correlation (MSAAC), Mean Square Aperiodic Cross Correlation (MSACC) and Figure Of Merit (FOM) [1][5]. If $c_i(k)$ and $c_i(k+\tau)$ represent the non-delayed and delayed versions of a code word then 'τ' is the number of units by which a code word is delayed. 'N' is the length of a code word c_i.

The discrete Aperiodic Correlation is defined as

$$r_{i,j}(\tau) = \frac{1}{N}\sum_{\tau=1-N}^{N-1} c_i(k)c_j(k+\tau)$$

The Mean Square Aperiodic Auto Correlation (MSAAC) for a Code set containing M sequences is given by

$$R_{AC} = \frac{1}{M}\sum_{i=1}^{M}\sum_{\tau=1-N,\tau\neq 0}^{N-1}\left|r_{i,i}(\tau)\right|^2$$

Mean Square Aperiodic Cross Correlation

$$\text{(MSACC)}\; R_{CC} = \frac{1}{M(M-1)}\sum_{i=1}^{M}\sum_{j=1,j\neq i}^{M}\sum_{\tau=1-N}^{N-1}\left|r_{i,j}(\tau)\right|^2$$

Figure Of Merit (FOM)

$$F = \frac{r_{i,i}^2(0)}{\sum_{\tau\neq 0}\left|r_{i,i}(\tau)\right|^2} = \frac{N^2}{2.\sum_{\tau=1}^{N-1}\left|r_{i,i}(\tau)\right|^2}$$

Table – IX

Parameter	8-length Walsh Code	8-length Binary orthogonal code set	6-length Binary orthogonal code set I	6-length Binary orthogonal code set II
MSAAC	2.375	2.125	1.0556	1.2778
MSACC	0.6607	0.6964	0.8492	0.8175
FOM	0.4211	0.4706	0.9474	0.7826

Table –X

Parameter	16-length Walsh Code	16-length Binary orthogonal code set	12-length Binary orthogonal code set I	12-length Binary orthogonal code set II
MSAAC	4.0625	3.6875	2.0833	2.4167
MSACC	0.7292	0.7542	0.8611	0.8389
FOM	0.2462	0.2712	0.48	0.4138

Conclusion

The main contribution in this paper is the technique that allows to construct '2^n' Binary Orthogonal Code words of length 2n. In this paper usage of proposed technique is limited to n = 3 & 4. As there is a change in the construction procedure of Inverse Gray Codes for even and odd values of 'n', n=3 resulted in two different Binary Orthogonal Code sets whereas only one Code set is obtained with n=4. Auto and Cross correlation properties reveal that these codes are competent with Walsh Codes. Possibility of Binary Orthogonal Code generation using any value of 'n' is to be investigated.

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