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Multirate Signal Processing: Graphical Representation & Comparison of Decimation & Interpolation Identities using MATLAB

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Abstract

In this paper, the decimation and interpolation techniques of multirate signal processing are presented. Decimation technique is used for decreasing the sampling rate and interpolation technique is used for increasing the sampling rate. The decimation and interpolation have the six most important identities in the multirate signal processing. Identities, first to third are decimation identities and fourth to sixth are interpolation identities. All the six identities are described and verified by the some graphical results with the help of MATLAB software.

Keywords: Multirate, Decimation, Interpolation, Sampling.

Introduction

Modern communication standards [1] such as WiMAX and UMTS-LTE relay on multiple signaling bandwidths and modulation schemes to provide the user with a variety of services. Presently, these services require high data rate communication utilizing complex modulation schemes for video and file transfer applications, while at other times the user simply desires to establish a simple phone call using a voice application. This flexibility implies that the radio has to be capable of processing signals at various data rates. Multirate digital signal processing [2] techniques provide the tools that enable the SDR to process the data signals with varying bandwidths. Interpolation can be used to increase the data rate of a given signal. Decimation is concerned with lowering the data rate of a given signal to obtain a new signal with a smaller data rate. Filtering and fractional delay processing are also powerful tools that are used by the SDR to alter the data rate of a signal [3].

Basics of Sample rate conversion

The sampled sequence x(n) obtained by sampling [4] the analog signal $x_a(t)$ at the sampling rate of F_s. The sampling rate conversion is the process of obtaining a new sampling sequence y(m) of $x_a(t)$ directly from x(n). The newly obtained samples of y(m) are equivalent to the sampled values of $x_a(t)$ generated at the new sampling rate of F_s'. Decimation and interpolation by integer values, both operations are considered fundamental to multirate signal digital signal processing.

Decimation

Decimation [1,5] or down-sampling by an integer is the process of decreasing the sampling rate of a given sequence {x(n)} by an arbitrary factor M. The new sequence {y(m)} can be expressed as a function of the old sequence x(n) as will be shown shortly. A signal flow representation of the down-sampling process is shown in Figure 1. The variable m denotes the variable of the new sequence sampled at the new sampling rate. The down-sampling operation can be thought of as a two-step process. First, consider the original sequence {x(n)} depicted in Figure 2(a) for M=3. Multiplying {x(n)} by the sequence:

$$\varphi_M(m) = \begin{cases} 1 & \text{for } m = Mn, \quad M \in N^+ \\ 0 & \text{otherwise} \end{cases} \tag{1}$$

results in a new sequence depicted in Figure 2(b). Discarding the zeros as expressed in (1) results in a new sequence at the reduced sampling rate as shown in Figure 2(c). This new sequence, in theory, is made up of every M^{th} sample of {x(n)}.

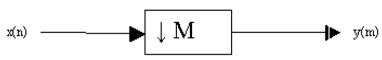


Figure 1: A down-sampling block diagram

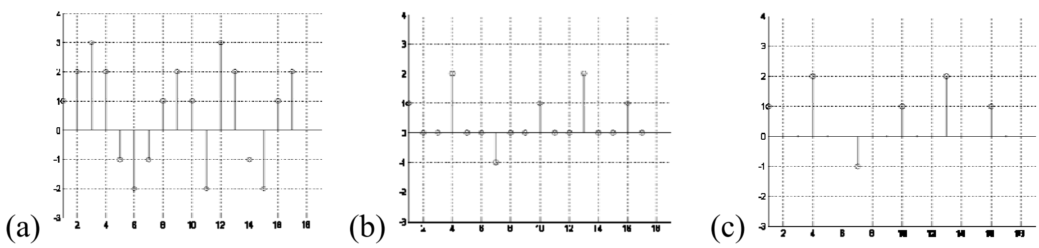


Figure 2: Down-sampling as a two step process (a) Original Signal (b) Replacing the M-1 samples with zero samples at M=3 (c) Removing the zero samples to reduce the sampling rates

This process can be further illustrated in the z-domain. Consider the z-transform of {x(n)}:

$$X(z) = \sum_{n=-\infty}^{\infty} x(n) z^{-n} \tag{2}$$

The z-transform of the sequence {y(m)}, the decimated version of {x(n)}, is a series of shifted images of X (z). However, decimation without filtering can cause degradation in the signal due to aliasing.

The reason is that a sampled signal repeats its spectrum every 2π radians. Decimation without filtering could cause the sampled images to overlap, depending on the signal bandwidth. To prevent aliasing from occurring, an anti-aliasing filter is typically used prior to down-sampling [6]. Sufficient linear filtering before down-sampling, avoids unwanted aliasing in the down-sampled signal. The entire operation in Figure 3 can be thought of as a linear but time-variant filtering operation. Ideally, the low pass filter satisfies the following spectral characteristics,

$$H_{ideal}(e^{i\omega}) = \begin{cases} 1 & -\frac{\pi}{M} \le \omega \le \frac{\pi}{M} \\ 0 & otherwise \end{cases} \tag{3}$$

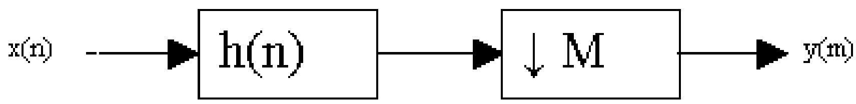


Figure 3: Low pass filter followed by a down-sampling operation

The spectrum of the input signal X(e) is assumed to be nonzero in the frequency interval π ≤ ω ≤ π . Spectrally, the purpose of the filter is to decrease the bandwidth of the signal x(n) by a factor M. The resulting sequence d(n) can be expressed (4) in terms of the convolution of x(n) with the filter transfer function h(n):

$$d(n) = \sum_{l=0}^{\infty} h(l)x(n-l) \tag{4}$$

After down-sampling by M, the resulting sequence y(m) is expressed in (5):

$$y(m) = d(Mm) = \sum_{k=0}^{\infty} h(k)x(Mm-k) \tag{5}$$

Interpolation

Interpolation [1,7] is used to change the sampling rate of a signal without changing its spectral content. In other words, increasing the sampling rate of a given signal (up-sampling) increases the spectral separation between the images of the original spectrum. However, this process doesn’t add any new information to the existing signal even though the sampling rate has increased, yielding more sample points for processing. After zero insertion, get the simplest form of signal interpolation. The up-sampling process is shown in Figure 4.

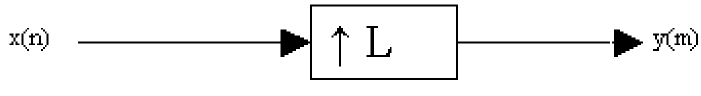


Figure 4: An up-sampling block diagram

Interpolation via zero insertion implies that zeros are inserted at the new rate between the original sample signals according to the relation (6):

$$y(n) = \begin{cases} x(\frac{n}{L}) & for\ n = 0, \pm L, \pm 2L, ... \\ 0 & otherwise \end{cases} \tag{6}$$

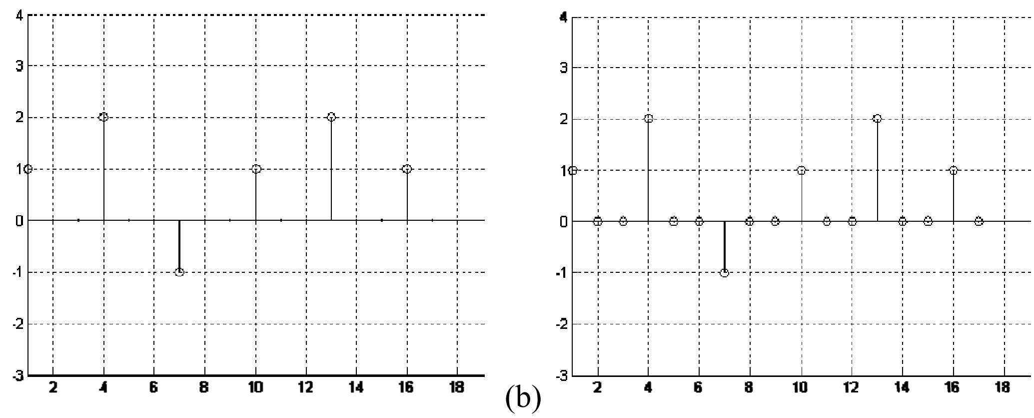
This up-sampling process is illustrated for an up-sampling factor of 4 as shown in Figure 5. The relationship between Y(z) and X(z) is easily derived. Let

$$Y(z) = \sum_{n=-\infty}^{\infty} y(n) z^{-n} = \sum_{\substack{n=-\infty \\ k=nL}}^{\infty} y(k) z^{-k}$$

$$= \sum_{\substack{n=-\infty \\ k=nL}}^{\infty} x(\frac{k}{L}) z^{-k} \tag{7}$$

Define l = k/L, the relation in (7) becomes:

$$Y(z) = \sum_{\substack{n=-\infty \\ k=nL}}^{\infty} x(l) z^{-lL} = X(z^{L}) \tag{8}$$



(a) (b)

Figure 5: Up-sampling process (a) Original Signal (b) Increasing the sampling rate by inserting zero between the original samples

In the frequency domain, this implies that the DTFT of Y(z) is

$$Y(e^{j\omega}) = X(e^{j\omega L}) \tag{9}$$

The relationship in (9) can be depicted for the case when L=4. The spectrum of the up-sampled signal is made up of multiple compressed copies of the original signal spectrum. These compressed copies, known as images, are placed at 2π/L intervals from DC. The original signal to the frequency range π ≤ ω ≤ π implies that the up-sampled signal is now limited to the range π/L ≤ ω ≤ π/L. So, the up-sampling process compresses the signal in the frequency domain by a factor of L with respect to the new sampling rate which is L times higher than the original sampling rate.

In order to suppress the images, the up-sampler of Figure 4 is typically followed by a low pass filter, as shown in Figure 6. The cutoff frequency of this filter is typically set to π/L. The resulting signal is an interpolated version of the original signal. The zero valued samples inserted between the original samples are now replaced with nonzero samples due to the convolution operation between the up-sampled signal and the impulse response of the low pass filter h(n).

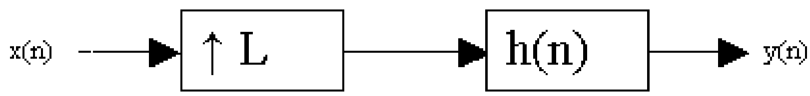


Figure 6: An up-sampling operation followed by a low pass filter

Identities

There are six identities for decimation and interpolation in multirate signal processing [8,9]. Identities first to third are decimation identities and fourth to sixth are interpolation identities. All the six identities are described as follows:

First Identitity

The first identity is shows in Figure 7. The scaling of the signals in the branches, their addition at the node, and down sampling is equivalent to down-sample the signals prior to scaling and addition. Transforming the structure from Figure 7(a) to that of Figure 7(b) leads to the arithmetic operations (multiplications and addition) being evaluated at the M times lower sampling rate.

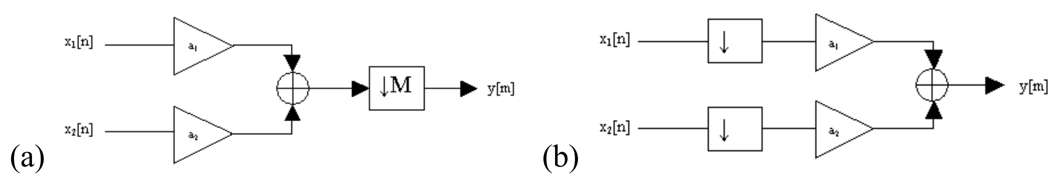


Figure 7(a) & (b): Block diagram of 1st Identity

Second Identitity

The second identity states that the delay-by-M followed by a down-sampler-by-M is equivalent to the down-sampler-by-M followed by a delay-by-one. Figure 8 shows the proper interchange in the cascade connection.

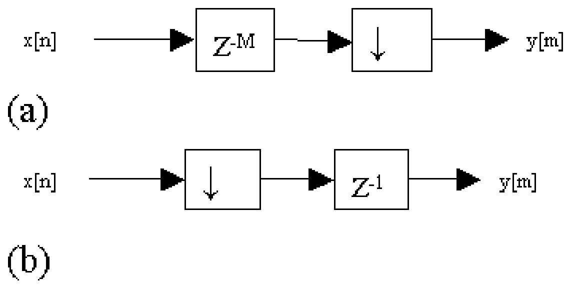


Figure 8(a) & (b): Block diagram of 2nd Identity

Third Identitity

Figure 9 shows the third identity. This identity is related to the cascade connection of a linear time-invariant system H(z) and a down-sampler. Filtering with $H(z^M)$ and down-sampling by M is equal to the down-sampling by M and filtering with H(z). The third identity may be considered as a more general version of the second identity.

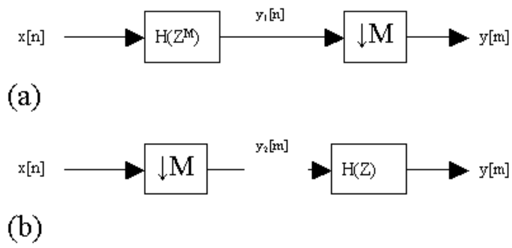


Figure 9(a) & (b): Block diagram of 3rd Identity

Fourth Identitity

The fourth identity is shows in Figure 10. The up-sampling prior to the branching and scaling is equivalent to branching and scaling prior to up-sampling. Transforming the structure from Figure 10(a) to that of Figure 10(b) leads to the arithmetic operations (multiplications and addition) being evaluated at the L times lower sampling rate.

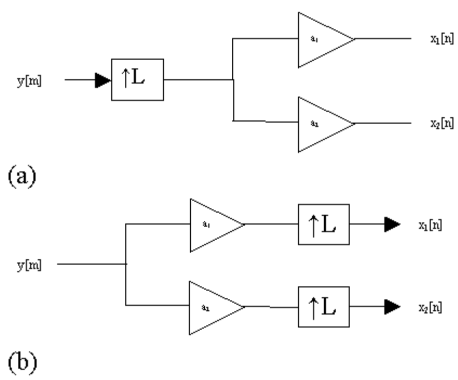


Figure 10(a) & (b): Block diagram of 4th Identity

Fifth Identitity

The fifth identity states that delayed-by-1 signal and up-sampled-by-L is equivalent to the signal up-sampled-by-L and delayed-by-L. Figure 11 shows the proper interchange in the cascade of the delay and the up-sampler.

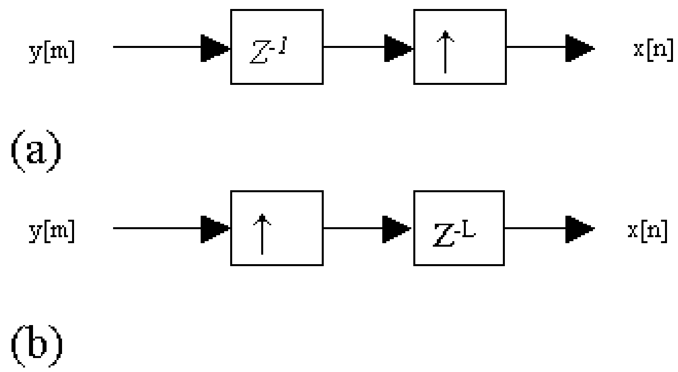


Figure 11(a) & (b): Block diagram of 5th Identity

Sixth Identitity

The sixth identity, Figure 12, is cascade connection of a linear time-invariant system and an up-sampler. Filtering with H(z) and up-sampling by L is equal to the up-sampling by L and filtering with H(z^L). The sixth identity may be considered as a more general version of the fifth identity.

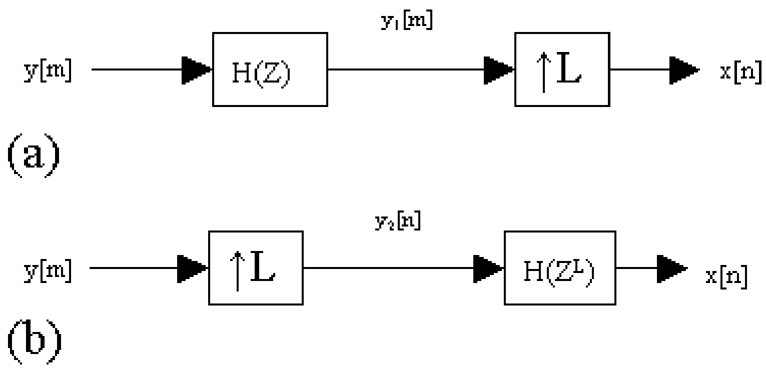


Figure 12(a) & (b): Block diagram of 6th Identity

Simulation Results

Simulation results for all six identities are as follows:

First Identitity

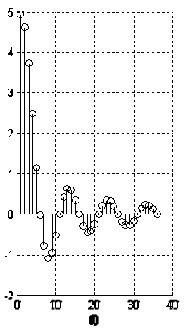
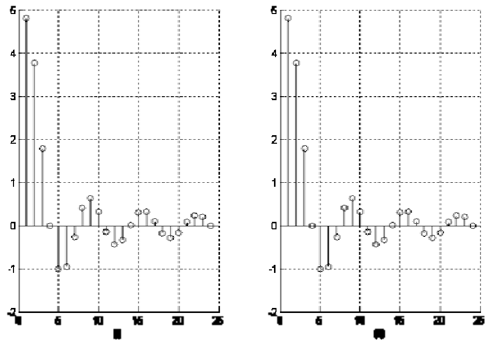
(a)  (b) 

Figure 13: 1st Identity results at (a) M=2 & (b) M=3

Second Identitity

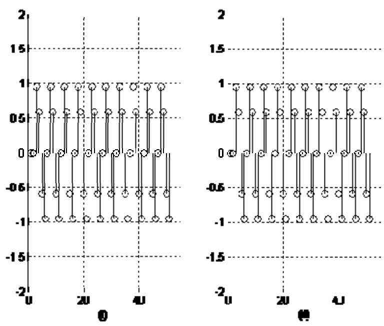
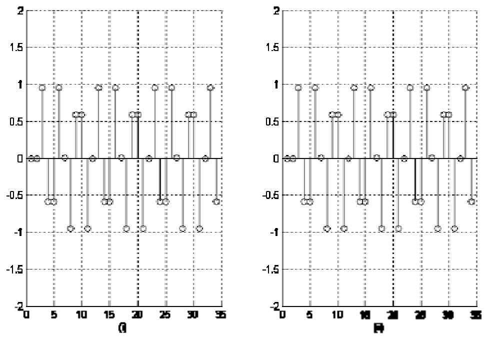
(a)  (b) 

Figure 14: 2nd Identity results at (a) M=2 & (b) M=3

Third Identitity

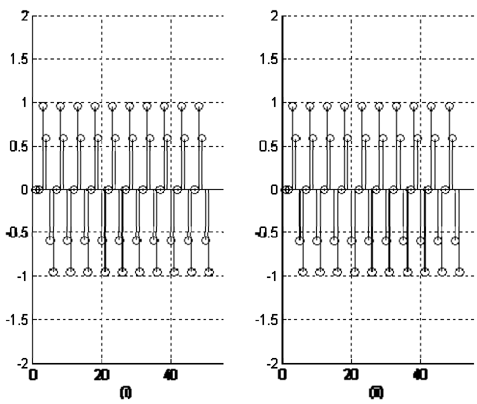
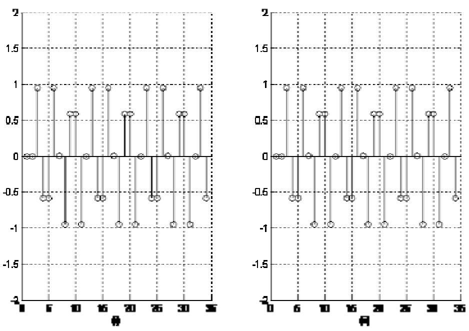
(a)  (b) 

Figure 15: 3rd Identity results at (a) M=2 & (b) M=3

Fourth Identitity

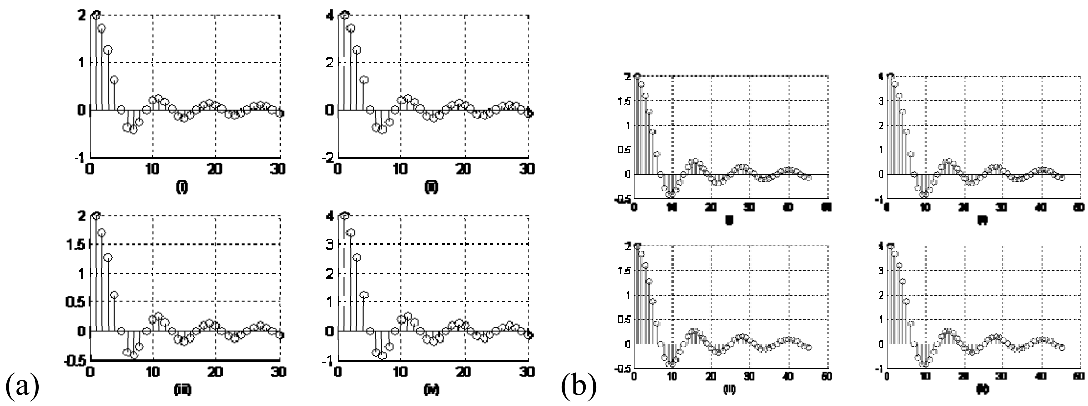


Figure 16: 4^{th} Identity results at (a) L=2 & (b) L=3

Fifth Identitity

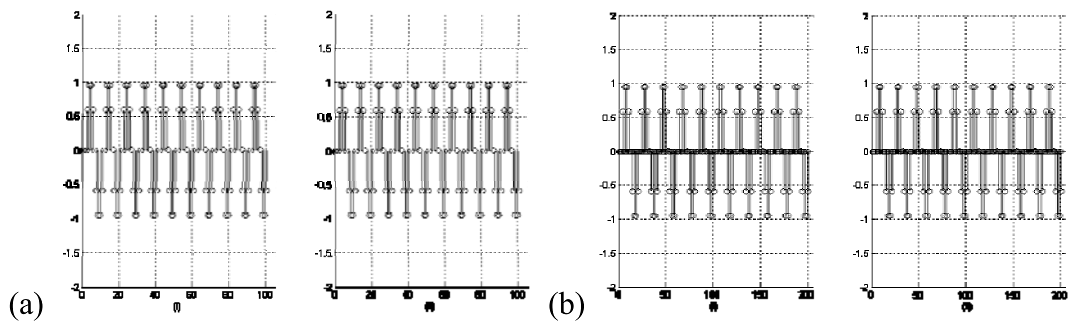


Figure 17: 5^{th} Identity results at (a) L=2 & (b) L=3

Sixth Identitity

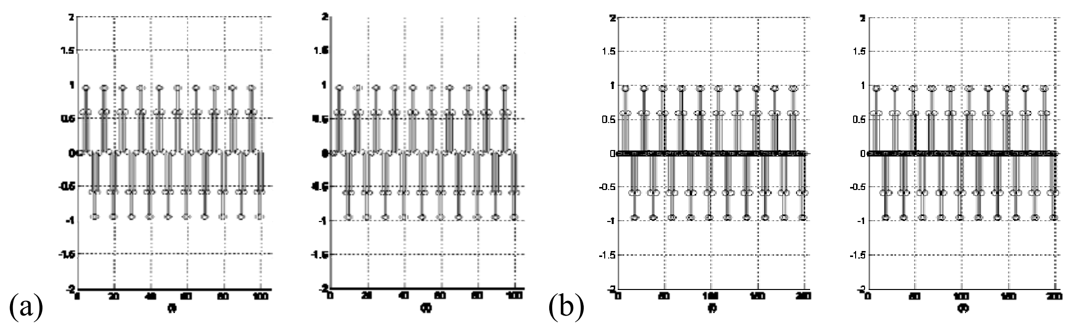


Figure 18: 6^{th} Identity results at (a) L=2 & (b) L=3

Figures 13-18 are graphical representation of first to sixth identities respectively. Here, two rates are consider for the simulation. For decimation identities M=2 & 3 and for interpolation identities L=2 & 3. Sinc/Sin function is used as input signal. For first identity only assume constant $a_1=a_2$ & input signal $x_1(n)=x_2(n)$ and also for fourth identity assume constant $a_1=a_2$. Figure 16(a) part (i)-(iv) represents the outputs for forth identity at L=2 and similarly Figure 16(b) represents the outputs at L=3. All other identities represent the outputs in Figures 13-18 except 16 in part (i) & (ii) only.

Conclusion

This work presented decimation and interpolation, which are rate conversion techniques. These techniques are demonstrated by six identities. The first three identities demonstrated the decimation identities and next three identities are presented the interpolation identities. These identities are used in multirate signal processing for different applications as signal compression systems, digital recorders, mobile phones, & other digital devices. The responses of decimation and interpolation are presented graphically using MATLAB/SIMULINK.

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