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Development of Model Reduction using Stability Equation and Cauer Continued Fraction Method

Harendra Singh[1] and V.R. Singh[2]

[1]Research Scholar, Department of Electrical Engineering, Mewar University, Gangrar, Chittorgarh (Rajasthan)
[2]Director, PDM College of Engineering, Bahadurgarh (Haryana)

Abstract

A mixed method is presented for order reduction of linear time invariant system for single input single output system. Numerator of polynomial is derived from Cauer second form of continued fraction and reduced denominator is obtained from stability equation method. Reduced order model retains the steady state value and stability of original system.

Keywords: - Model reduction, Cauer second form (continued fraction expansion), Stability equation method.

I.INTRODUCTION

The design and analysis of high order system becomes complex and tedious. Numerous methods have been developed for approximation of original system of linear time invariant system for single input and single output (SISO).Routh approximation method is good for stability and simplicity (1) but it is found that this method is unable to produce good reduced model. .Several methods have been proposed based on pade approximation, (2) continued fraction expansion (3-5) and error minimization technique (6-7) and time moment matching etc.Many papers have been reviewed. These methods contain advantage and disadvantages. Continued fraction method is used as powerful tools for reducing the original system. Several methods have been made in model reduction as pade approximation, Routh approximation (9) .This paper deals with both cauer second form of continued fraction and stability equation method. .pade approximant method is one of reduced technique based on matching of time moments of reduced order model and original system. this technique is very simple in computation and fitting of initial time moments and steady state value of output of reduced order model and original system in form of $\sum \alpha_i t^i$ .A known drawback of this method is unstable reduced order model arises from original stable system, was ˙, improved by using this method . Pal and Parthsarthy and Jayasimha (14) combine this method with continued fraction method to retail stability in reduced order model.

II. Methods Used

2.1 Statement of Approximation Method.

Suppose nth order SISO system is given by transfer function

$$\mathrm{H(s)} = \frac{N(s)}{D(s)} = \frac{B_{21} + B_{22}S + B_{23}S^2 + \dots + B_{2n}S^{n-1}}{B_{11} + B_{12}S + B_{13}S^2 + \dots + B_{1n}S^n}$$

$$\mathrm{H(s)} = \frac{\sum_{j=1}^{n-1} B_{2j} s^{j-1}}{\sum_{j=1}^{n} B_{1j} s^{j-1}} \tag{1}$$

Where B_{21} ,B_{22} …………B_{2n} and B_{11} ,B_{12} ………..B_{1n+1} are constant.

Reduced order model is given by

$$R(s) = \frac{N_r(s)}{D_r(s)} = \frac{D_{21} + D_{22}s + D_{23}s^2 + \dots\dots}{D_{11} + D_{12}s + D_{13}s^2 + \dots\dots}$$

$$R(s) = \frac{\sum_{j=1}^{r} D_{2j} s^{j-1}}{\sum_{j=1}^{r+1} D_{1j} s^{j-1}} \tag{2}$$

2.2 Order Reduction Method.

This method consists of two types

Step 1

First numerator of reduced model is formulated in form of continued fraction

Expansion (11)

$$H(s) = \cfrac{1}{h_1 + \cfrac{1}{h_2/s + \cfrac{1}{h_3 + \cfrac{1}{h_4/s + \cfrac{1}{\ddots}}}}} \tag{3}$$

The quotients h_i (i = 1, 2, 3… r) are determined by using Routh algorithm (11).

$B_{i,j} = B_{i-2,j+1} - h_{i-2}\, B_{i-1,j+1}$

i = 3, 4………..

j = 1,2 ………

$$h_i = \frac{B_{i,j}}{B_{i+1,j}}, \text{ i=1, 2…}$$

$$B_{i+1,j} \neq 0 \tag{4}$$

Eqn. (2) gives the second cauer second form of continued fraction expansion of original system of H(s) and the first 2r quotient are retained and truncated. The Continued fraction is inverted to obtain the r_{th} order of reduced order model R(s).

Step 2

Determination of the denominator coefficients of reduced order model:-

For stable original system H(s), denominator of H(s) is separated in even parts and odd parts, as stability equation

$$D_e(s) = c_{00}\prod_{i=1}^{n_1}\left(1+\frac{s^2}{z_i^2}\right) \tag{5}$$

$$Do(s) = c_{01}\prod_{i=1}^{n_2}\left(1+\frac{s^2}{p_i^2}\right)$$

where n_1 and n_2 are integer parts of k/2 and k−1/2 respectively and

$z_{12} < p_{12} < z_{22} < p_{22}$……………

by neglecting factors with large magnitude of z_i and p_i in eqn(5).Stability equation for r^{th} order of reduced order model is obtained. as

$$D_e(s) = c_{00}\prod_{i=1}^{n_3}\left(1+\frac{s^2}{z_i^2}\right)$$

$$D_o(s) = c_{01}s\prod_{i=1}^{n_4}\left(1+\frac{s^2}{p_i^2}\right) \tag{6}$$

Where n_3 and n_4 are integer parts of r/2 and $\frac{r-1}{2}$

$$D_r(s) = c_{00}\prod_{i=1}^{r/2}(1+\frac{s^2}{z_i^2}) + c_{01}\prod_{i=1}^{r-1/2}(1+\frac{s^2}{p_i^2}) \tag{7}$$

Combing these even and odd parts of reduced order denominator and reciprocating, we get

$D_r(s) = D_e(s) + D_o(s)$

$D_r(s)$ is obtained and is given by as

$$D_r(s) = \sum_{j=1}^{r+1} D_{1,j} s^{j-1} \tag{8}$$

Coefficients $D_{1,j}$ in eqn. (8) are matched with h_i in eqn. (4) to determine reduced order numerator $N_r(s)$, by applying the following reverse Routh algorithm

$$B_{i+1,j} = \frac{B_{i,j}}{h_i}$$

$$B_{i+1}, {}_{j+1} = (B_i, {}_{j+1} - B_{i+2,j})$$

i, = 1,2 ....(r −1)

j = 1, 2............... (r−1)

&$B_{1,r+1} = 1$

The reduced order model (ROM) is obtained as

$$R(s) = \frac{N_r(s)}{D_r(s)}$$

Step 4:-

There is steady state error between the outputs of original and reduced order system. To avoid steady state error we match the steady state responses by the following relationship to obtain correction factor ‘k’ a constant.

$$\frac{B_{21}}{B_{11}} = k\frac{D_{21}}{D_{11}} \tag{9}$$

III. Results: - Numerical Analysis.

Let transfer function of 5th order system (14) is given as

$$H(s) = \frac{10s^4 + 82s^3 + 264s^2 + 369s + 156}{2s^5 + 21s^4 + 84s^3 + 173s^2 + 148s + 40} \tag{10}$$

from above original transfer function, second order reduced model is to be obtained

Step 1:-

The quotient h_i for i =1, 2, 3 …………r are determined using Routh algorithm as

$h_1 = 0.256$, $h_2 = 2.92$

$h_3 = 0.872$ $h_4 = 1.72$

Step 2:-

Separating the denominator of the above high order system (HOS) in even and odd parts, we get stability equation as

$D_e(s) = 40 + 173s^2 + 21s^4$

$$40(1+\frac{s^2}{0.2381})(1+\frac{s^2}{8.000})$$

$D_o(s) = 148s + 84s^3 + 2s^5$

$$148s(1+\frac{s^2}{1.76})(1+\frac{s^2}{74})$$

by discarding the factors with large magnitude of z_i^2 and p_i^2 in $D_e(s)$ and $D_o(s)$ respectively. The stability equation for reduced model of second order is given by

$$D_e(s) = 40(1+\frac{s^2}{0.2381})\ \&\ D_o(s) = 148s$$

Combing these reduced stability equation, we get denominator of reduced 2^{nd} order model as

$D_r(s) = 166.67\ s^2 + 148s + 40$

Step 3:-

By matching the quotients of h_i of cauer second form with the coefficients of reduced denominator and using reverse Routh algorithm to get

$N_r = 369s + 156$

The transfer function for reduced order model (ROM) with 2^{nd} order can be expressed as

$$R_2(s) = \frac{369s+156}{166.67s2+148s+40}$$

Step 4:-
In this numerical section, there is no steady state error between step response of original order and reduced model, hence k =1 and the final model reduction remains unchanged. The original and reduced order model (ROM) is compared by step responses plot shown in fig.1

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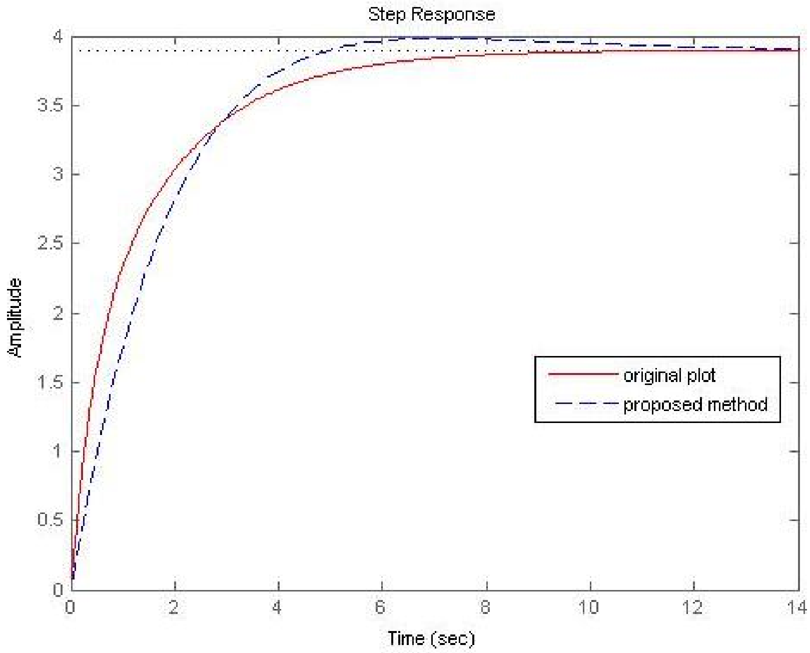

Figure 1

IV. Discussion:

In proposed model reduction method algorithm used which combines to take both advantages of stability equation method to derive the denominator of polynomial and the cauer second form to determine the numerator of polynomial .it is observed that proposed method preserves the steady state and stability in reduced order model. This algorithm has been implemented in metlab 7.0. Steady state and stability avoids any error in initial and final value of responses of original and reduced order system. This method is extended for multivariable system. These methods are easily contribution to model reduction method due to flexibility of algorithm. This method is curiosity method to develop reduced order model for approximation of original system.

References

[1] Hutton M F and Friedland "Routh approximation for reduced order of linear time invariant system. IEEE 1975, AC-20,pp-329-332.

[2] Y Shamash “stable reduced order model using pade type approximation”. IEEE TRANS.on automatic control. Vol 19, 1974, pp-615-616.
[3] Chuang SC “application of continued fraction method for modeling transfer function to give more accurate initial transient response”. Electronic, Lett. , 1970, 6, pp-861-863.
[4] T, N LUCAS “Continued fraction algorithm for biased model reduction.” Electronic Letters, 9th June 1983, Vol 19, No12
[5] R Prasad and J Pal “A stable reduction of linear system by continued fraction.” J Inst.Engrs.India, IE (I) Journal-EL, Vol 72, pp113-116 October 1991.
[6] S.Mukherjee and R.N Mishra “order reduction of linear system using an error minimization technique”. Journal of Franklin Inst, Vol.323, No 1,pp. 23-32,1987.
[7] K. Ramesh, A. Nirmal Kumar and G. gurusamy “order reduction by error minimization technique”. Proceeding of the 2008 international conference on computing, communication and network(ICCCN 2008), 978-1-4244-3595-1/08,2008IEEE
[8] M F Hutton, B. friendland “ROUTH approximation for reducing order of linear time varying system” IEEE trans.Automatic control ,vol-44,No 9,pp-1782-1787.
[9] T C Chen, CY Chang and KW Han “Stable reduced order Pade approximants using stability equation method”. Electronic Letter .Vol 16,No. 9,pp.345-346.1980.
[10] Jayanta pal “system reduction by a mixed method”.Transaction on Automatic control, vol .AC-25, No 5, October 1980.
[11] J. Pal “Improved pade approximants using stability equation method “Electronic Letters, vol 19, No 11, pp-426-427.may, 1983.
[12] Chen C F and Sheih LS “Novel approach to linear model simplification”. int.j.control, 1968, No 8, pp-561-570.
[13] R. Parthasarthy and K.N Jayasimha “system reduction using stability equation and modified cauer continued fraction method”. Proceeding IEEE, vol 70, No10,pp-1234-1236.October1982.
[14] Lucas. T N “ Biased model reduction by factor division method”Electronic Letter, 1984, 20 pp582-583.
[15] Bai-Wu Wan “Linear model reduction using Mihailov criterion and Pade approximation technique”Int.J.Control,Vol. 33,pp 1073-1089,1981.