Development of Model Reduction using Stability Equation and Cauer Continued Fraction Method

Harendra Singh¹ and V.R. Singh²

¹Research Scholar, Department of Electrical Engineering, Mewar University, Gangrar, Chittorgarh (Rajasthan)
²Director, PDM College of Engineering, Bahadurgarh (Haryana)

Abstract

A mixed method is presented for order reduction of linear time invariant system for single input single output system. Numerator of polynomial is derived from Cauer second form of continued fraction and reduced denominator is obtained from stability equation method. Reduced order model retains the steady state value and stability of original system.

Keywords: - Model reduction, Cauer second form (continued fraction expansion), Stability equation method.

I. INTRODUCTION

The design and analysis of high order system becomes complex and tedious. Numerous methods have been developed for approximation of original system of linear time invariant system for single input and single output (SISO). Routh approximation method is good for stability and simplicity (1) but it is found that this method is unable to produce good reduced model. Several methods have been proposed based on pade approximation, (2) continued fraction expansion (3–5) and error minimization technique (6–7) and time moment matching etc. Many papers have been reviewed. These methods contain advantage and disadvantages. Continued fraction method is used as powerful tools for reducing the original system. Several methods have been made in model reduction as pade approximation, Routh approximation (9). This paper deals with both cauer second form of continued fraction and stability equation method. pade approximant method is one of reduced technique based on matching of time moments of reduced order model and original system. this technique is very simple in computation and fitting of initial time moments.
Harendra Singh and V.R. Singh

and steady state value of output of reduced order model and original system in
form of \( \sum a_i t^i \). A known drawback of this method is unstable reduced order
model arises from original stable system, was `. improved by using this method.
Pal and Parthsarthy and Jayasimha (14) combine this method with continued
fraction method to retail stability in reduced order model.

II. Methods Used

2.1 Statement of Approximation Method.

Suppose nth order SISO system is given by transfer function

\[
H(s) = \frac{N(s)}{D(s)} = \frac{B_{21} + B_{22}S + B_{23}S^2 + \ldots + B_{2n}S^{n-1}}{B_{11} + B_{12}S + B_{13}S^2 + \ldots + B_{1n}S^n}
\]

\[
H(s) = \frac{\sum_{j=1}^{n} B_{2j}S^{j-1}}{\sum_{j=1}^{n} B_{1j}S^{j-1}}
\]

(1)

Where \( B_{21}, B_{22}, \ldots, B_{2n} \) and \( B_{11}, B_{12}, \ldots, B_{1n+1} \) are constant.

Reduced order model is given by

\[
R(s) = \frac{N_r(s)}{D_r(s)} = \frac{D_{21} + D_{22}S + D_{23}S^2 + \ldots}{D_{11} + D_{12}S + D_{13}S^2 + \ldots}
\]

\[
R(s) = \frac{\sum_{j=1}^{r} D_{2j}S^{j-1}}{\sum_{j=1}^{r} D_{1j}S^{j-1}}
\]

(2)

2.2 Order Reduction Method.

This method consists of two types

Step 1

First numerator of reduced model is formulated in form of continued fraction

Expansion (11)

\[
H(s) = \frac{1}{h_1 + \frac{1}{h_2 / s + \frac{1}{h_3 / s + \frac{1}{h_4 / s + \frac{1}{\ldots}}}}}
\]

(3)
The quotients $h_i$ ($i = 1, 2, 3... r$) are determined by using Routh algorithm (11).

$$B_{i,j} = B_{i-2j+1} - h_{i-2} B_{i-1,j+1}$$

\[ i = 3, 4 ............ \]
\[ j = 1, 2 ............ \]

$$h_i = \frac{B_{i,j}}{B_{i+1,j}}, i=1, 2...$$

$$B_{i+1,j} \neq 0$$

Eqn. (2) gives the second cauer second form of continued fraction expansion of original system of H(s) and the first 2r quotient are retained and truncated. The Continued fraction is inverted to obtain the $r_{th}$ order of reduced order model R(s).

**Step 2**

**Determination of the denominator coefficients of reduced order model:**

For stable original system $H(s)$, denominator of $H(s)$ is separated in even parts and odd parts, as stability equation

$$D_e(s) = c_{00}\prod_{i=1}^{n_1} \left(1 + \frac{s^2}{z_i^2}\right)$$

$$D_o(s) = c_{01}\prod_{i=1}^{n_2} \left(1 + \frac{s^2}{p_i^2}\right)$$

where $n_1$ and $n_2$ are integer parts of $k/2$ and $k-1/2$ respectively and

$z_{12} < p_{12} < z_{22} < p_{22}.............$

by neglecting factors with large magnitude of $z_i$ and $p_i$ in eqn(5). Stability equation for $r^{th}$ order of reduced order model is obtained as

$$D_e(s) = c_{00}\prod_{i=1}^{n_3} \left(1 + \frac{s^2}{z_i^2}\right)$$

$$D_o(s) = c_{01}s\prod_{i=1}^{n_4} \left(1 + \frac{s^2}{p_i^2}\right)$$

Where $n_3$ and $n_4$ are integer parts of $r/2$ and $r-1/2$
\[
D_r(s) = c_0 \prod_{i=4}^{2} (1 + \frac{s^2}{\zeta_i^2}) + c_{01} \prod_{i=1}^{r+2} (1 + \frac{s^2}{\rho_i}) \tag{7}
\]

Combing these even and odd parts of reduced order denominator and reciprocating, we get

\[D_r(s) = D_c(s) + D_o(s)\]

\[D_r(s)\] is obtained and is given by as

\[D_r(s) = \sum_{j=1}^{r+1} D_{1j} s^{j-1} \tag{8}\]

Coefficients \(D_{1j}\) in eqn. (8) are matched with \(h_i\) in eqn. (4) to determine reduced order numerator \(N_r(s)\), by applying the following reverse Routh algorithm

\[B_{r+1,j} = \frac{B_{r,j}}{h_i}\]

\[B_{r+1,j+1} = (B_{r+j+1} - B_{r+2,j})\]

\[i = 1, 2, \ldots (r-1)\]

\[j = 1, 2, \ldots (r-1)\]

\& \(B_{1,r+1} = 1\)

The reduced order model (ROM) is obtained as

\[R(s) = \frac{N_r(s)}{D_r(s)}\]

**Step 4:**
There is steady state error between the outputs of original and reduced order system. To avoid steady state error we match the steady state responses by the following relationship to obtain correction factor ‘k’ a constant.

\[\frac{B_{21}}{B_{11}} = k \frac{D_{21}}{D_{11}} \tag{9}\]

**III. Results: - Numerical Analysis.**
Let transfer function of 5th order system (14) is given as

\[H(s) = \frac{10s^4 + 82s^3 + 264s^2 + 369s + 156}{2s^3 + 21s^4 + 84s^3 + 173s^2 + 148s + 40} \tag{10}\]
Development of Model Reduction using Stability Equation

from above original transfer function, second order reduced model is to be obtained

**Step 1:-**
The quotient $h_i$ for $i = 1, 2, 3 \ldots \ldots r$ are determined using Routh algorithm as

$h_1 = 0.256, \quad h_2 = 2.92$

$h_3 = 0.872, \quad h_4 = 1.72$

**Step 2:-**
Separating the denominator of the above high order system (HOS) in even and odd parts, we get stability equation as

$$D_e(s) = 40 + 173s^2 + 21s^4$$

$$= 40(1 + \frac{s^2}{0.2381})(1 + \frac{s^2}{8.000})$$

$$D_o(s) = 148s + 84s^3 + 2s^5$$

$$= 148s(1 + \frac{s^2}{1.76})(1 + \frac{s^2}{74})$$

by discarding the factors with large magnitude of $z_i^2$ and $p_i^2$ in $D_e(s)$ and $D_o(s)$ respectively. The stability equation for reduced model of second order is given by

$$D_r(s) = 40(1 + \frac{s^2}{0.2381}) \& D_o(s) = 148s$$

Combing these reduced stability equation, we get denominator of reduced 2nd order model as

$$D_r(s) = 166.67s^2 + 148s + 40$$

**Step 3:-**
By matching the quotients of $h_i$ of cauer second form with the coefficients of reduced denominator and using reverse Routh algorithm to get

$$N_r = 369s + 156$$

The transfer function for reduced order model (ROM) with 2nd order can be expressed as

$$R_2(s) = \frac{369s + 156}{166.67s^2 + 148s + 40}$$
Step 4:-
In this numerical section, there is no steady state error between step response of original order and reduced model, hence $k = 1$ and the final model reduction remains unchanged. The original and reduced order model (ROM) is compared by step responses plot shown in fig.1

![Step Response Plot](image)

**Figure 1**

IV. Discussion:
In proposed model reduction method algorithm used which combines to take both advantages of stability equation method to derive the denominator of polynomial and the cauer second form to determine the numerator of polynomial. It is observed that proposed method preserves the steady state and stability in reduced order model. This algorithm has been implemented in Matlab 7.0. Steady state and stability avoids any error in initial and final value of responses of original and reduced order system. This method is extended for multivariable system. These methods are easily contribution to model reduction method due to flexibility of algorithm. This method is curiosity method to develop reduced order model for approximation of original system.

References


