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PSO Approach to H∞ Preview Control Design

Nidhika Birla and Akhilesh Swarup

Research Scholar, Department of Electrical Engineering,
National Institute of Technology, Kurukshetra, Haryana, India
Dean, Galgotia College of Engg. and Technology, Greater Noida, U.P., India

Abstract

The need and importance of robustness is highlighted by the fact that real engineering systems are to deal with disturbance, noise and uncertainties, to maintain stability and performance level to a certain degree. The robust control system is designed using H∞ control design technique by solving Discrete Algebraic Riccati Equation (DARE). But the solution is not a perfect controller. Perfect controller can be achieved for a plant with a non-causal controller that uses future information. Preview Control can be a solution to perfect control. Preview Control is a field well suited for application to systems that have reference signals and disturbances known a priori. The use of advance knowledge of such signals can improve the tracking quality and disturbance rejection of the concerned control system. The solution to the Preview Control problem is obtained using the Algebraic Riccati Equation (ARE). The solution to H∞ Control and Preview Control problem gives good results but there is a scope of improvement as the results of Algebraic Riccati Equation strongly depend on the critical parameters (Q, R and γ). An automated and systematic methodology for finding the optimal parameter values is the Particle Swarm Optimization (PSO) technique. This paper presents the step by step design of discrete H∞ Controller and H∞ Preview Controller for reference tracking and disturbance rejection using state augmentation with automated parameter selection using PSO. The algorithms are implemented and simulated in MATLAB environment. The results obtained show that the PSO technique can remarkably improve the performance of the system for discrete H∞ control system and H∞ Preview control system for both reference tracking and disturbance rejection.

Introduction

The issue of robustness is of crucial importance in control system design because real engineering systems are vulnerable to external disturbance and measurement noise and there are always differences between mathematical models used for design and the actual system. Typically, a control engineer is required to design a controller that will stabilize a plant, if it is not stable originally, and satisfy certain performance levels in the presence of disturbance signals, noise interference, unmodelled plant dynamics and plant parameter variations. A solution to this problem was given in early 1980s by Zames [1] and Zames and Francis [2]. In the $H\infty$ approach the designer from the outset specifies a model of system uncertainty, such as additive perturbation and/ or output disturbance that is most suited to the problem at hand. A constrained optimization is then performed to maximize the robust stability of the closed-loop system to the type of uncertainty chosen, the constraint being internal stability of feedback system. In most cases, a feasible controller is designed to achieve robust stability for closed-loop system. Performance objectives can also be included in the optimization cost function. Elegant solution formulae have been developed in this theory, which are based on the solutions of ARE.

Theoretically, perfect control can be achieved for a plant with a non-causal controller. A non-causal controller is the one that uses future information. Preview Control can be a solution to perfect control. It can be used to obtain performance beyond typical achievable performance obtained by a feedback only control design, much the way a pure feed-forward controller allows an extra degree-of-freedom in the controller design. However, the closed – loop sensitivity and stability robustness properties of the system are determined purely by the feedback portion of the control. The notion of anticipative control action is a very general one, with a possibility of many different control-theoretic problem formulations. The term Preview Control is usually associated with a particular class of anticipative control problems with a preview horizon that extends for a fixed time into the future. The field of Preview Control is concerned with using the advance knowledge of disturbances or references in order to improve the tracking quality or disturbance rejection. Preview Control is compatible with optimal control which minimizes evaluation function through all period of time. This field has attracted many researchers as its applications include guidance of autonomous vehicles, robotics and process control.

The classical solution of Preview Control problem is given using $H\infty$ control and state augmentation, solved using Algebraic Riccati Equation. The mathematical formulation and solution of the $H\infty$ Preview Control problem is given by A. Kojima, et. al. and G. Tadmor, et. al., for preview compensation, output feedback setting and fixed lag smoothing [3-6]. The discrete version of the preview control problem and its various issues are studied with numerical examples by Polyakov, et. al. [7]. Y. Kuroiwa, et. al. have analysed the $H\infty$ Preview Control problem for the systems with delay [8]. M. M. Negm, et. al. have synthesized Optimal Preview Control for three-phase induction motor [9]. Analysis and Design of $H\infty$ Preview Tracking Control Systems and its various variations using state augmentation have also been studied [10-12]. The solutions to all the problems are given using Algebraic Riccati Equation for the continuous and Discrete Algebraic Riccati Equation for the discrete – time systems.

Despite the mature theory available for both $H\infty$ control design and Preview control design, there is a scope of improvement in the results obtained using DARE. The solution of Discrete Algebraic Ricccati Equation depends very strongly on the critical parameters chosen, namely, Q (error weighting matrix), R (control weighting matrix) and γ (constraint on $H\infty$ norm). The classical method for choice of these parameters is cumbersome and requires complicated calculations. The solution of the classical method has a scope of improvement. The choice of these parameters is more critical in case of H∞ Preview Control problem and the problem complicates even more with the increase in the length of preview. An automated solution to such a problem is Particle Swarm Optimization (PSO) technique. The PSO algorithm allows the solution parameters to fly over the complete solution space to find the optimal result. This paper aims at bridging the gap between the theory and its automated solution using PSO with an insight to practical applications.

Descrete H∞ Control Problem

Problem Definition:

Consider the following discrete-time invariant plant:

$$\begin{aligned} x(t+1) &= Ax(t) + Ew(t) + Bu(t) \\ z(t) &= Cx(t) + Du(t) \end{aligned} \tag{1}$$

where t=o,1,2,… and the stare $x(t) \in R^n$, disturbance $w(t) \in R^q$, control $u(t) \in R^m$ and the generalized error (controlled output) $z(t) \in R^p$.

The following are the assumptions made:

i. (A,B) is stabilizable, (C,A) detectable.

ii. $D^T[D\ C] = [R_1\ O]$ and $R_1 > O$

The latter assumption assumes the cross-weighting between control signal and state is null.

The aim is to find a control strategy $[u(t)]$ which leads to a bounded H∞ norm of the transfer function matrix T_{zw} from $w(t)$ to $z(t)$. the control signal $u(t) = -Kx(t)$, where K represents the controller in negative feedback notation. the z-transfer function to be minimized in an H∞ norm sense is:

$$T_{zw} = (C - DK)(zl - A + BK)^{-1}E \tag{2}$$

The aim of control may be summarized as follows:

1. The constant gain feedback controller K should ensure the system is closed-loop stable and $(A - BK)$ is an asymptotically stable matrix.
2. The gain T_{zw} must satisfy $\|T\|_{zw} \leq \gamma$ where $\gamma > 0$.

The constraint on the H∞ norm will ensure, from the small gain theorem, that the system will be robustly stable in certain sense.

The H∞ state feedback control law can be obtained from the solution $X \in R^{nxn}$ satisfying:

$$X \geq 0$$
$$X = A^T XA + C^T C - A^T XB(R_1 + B^T XB)^{-1} B^T XA + XE(\gamma^2 + E^T XE)^{-1} E^T X \quad (3)$$

Then the state feedback H∞ optimal control

$$u(t) = -Kx(t) \quad (4)$$

and the state feedback gain:

$$K = (R_1 + B^T XB)^{-1} B^T XA \quad (5)$$

stabilizes the closed-loop system and ensures

$$\| T_{zw} \|_\infty \leq \gamma \quad (6)$$

Solution Methodology:
Consider the system defined by the state-variable model in discrete-domain as:

$$\begin{aligned} x(t+1) &= Ax(t) + Bu(t) \\ y(t) &= Cx(t) + Du(t) \end{aligned} \quad (7)$$

The system model is to be converted to a standard format, so that the performance index can be a part to the system model. The performance index chosen is given as below:

$$\begin{aligned} &J = \sum_{t=0}^{\infty} \{e^T(t)Qe(t) + \tilde{u}^T(t)R\tilde{u}(t)\}Q, R\}0 \\ &e(t) = r(t) - u(t), \tilde{u}(t) = u(t+1) - u(t) \end{aligned} \quad (8)$$

where $e(t)$ is the tracking error, $r(t)$ is the reference signal and $u(t)$ is the control signal.

Also, let

$$\tilde{x}(t) = x(t+1) - x(t), d(t) = r(t+1) - r(t)$$

From the system model,

$$\begin{aligned} \tilde{x}(t+1) &= A\tilde{x}(t) + B\tilde{u}(t) \\ e(t+1) &= e(t) + d(t) - C\tilde{x}(t) - D\tilde{u}(t) \end{aligned} \quad (9)$$

So, the augmented system becomes,

$$\begin{aligned} \bar{x}(t+1) &= \bar{A}\bar{x}(t) + \bar{B}\tilde{u}(t) + \bar{E}d(t) \\ z(t) &= \bar{C}\bar{x}(t) + \bar{D}\tilde{u}(t) \end{aligned} \tag{10}$$

where

$$\bar{x}(t) = \begin{bmatrix} \tilde{x}(t) \\ e(t) \end{bmatrix}, \quad z(t) = \begin{bmatrix} Q^{1/2}e(t) \\ R^{1/2}u(t) \end{bmatrix}$$

and

$$\bar{A} = \begin{bmatrix} A & O \\ -C & I \end{bmatrix}, \bar{B} = \begin{bmatrix} B \\ -D \end{bmatrix}, \bar{E} = \begin{bmatrix} O \\ I \end{bmatrix}, \bar{C} = \begin{bmatrix} O & Q1/2 \\ O & O \end{bmatrix}, \bar{D} = \begin{bmatrix} O \\ R^{1/2} \end{bmatrix}$$

The augmented system is now ready for solution using Discrete Algebraic Riccati Equation (DARE) by proper choice of parameters, as given in the section II (A).

Parameter Selection using PSO.
The parameters (Q, R and γ) values can be obtained using bisection method or trial and error method. The values of these parameters can be improved to give better results. This variation can be given a random motion but a more systematic methodology to find the best optimal parameter value can be PSO. Each PSO individual flies in the search space, which is multi-dimensional, and presents the optimal solution by dynamically adjusting the velocity of the individuals.

The procedure of automating the tuning technique for Q, R and γ values using PSO consists of evaluating a series of step responses in order to permit the algorithm to converge by minimizing the objective function, Integral of Absolute Error (IAE) given by,

$$IAE = \int |Y_{SP}(t) - Y(t)|\, dt \tag{11}$$

The algorithm for the solution of the automated parameters selection problem is given as below:

i. Initialize a population (array) of particles with random position and velocities on d-dimension in problem space.
ii. For each particle, evaluate the designed optimization fitness function in d variables.
iii. Compare particle's fitness evaluation with particle's pbest. If current value is better than pbest, then set pbest value equal to the current value and the pbest location equal to the current location in d-dimensional space.
iv. Compare fitness evaluation with the population's overall previous gbest. If the current value is better than gbest, then reset gbest to the current particle's array index and value.
v. Change the velocity and position of the particle according to equations given below:

$$v_{id}^{(t+1)} = w.v_{id}^{(t)} + c_1.rand().(pbest - x_{id}^{(t)} 0 + c_2.rand(). \\ (gbest - x_{id}^{(t)}) x_{id}^{(t+1)} = x_{id}^{(t)} + v_{id}^{(t+1)} \quad (12)$$

vi. Loop to step (ii) until a criterion is met, usually a sufficiently good fitness or a max number of iterations (generations).

Application to System Design:
The verification of performance of the procedure was carried out on MATLAB platform for three systems and the results are summed up as follows:

Servomechanism (4th Order):
The state-variable model of the 4th-order Servomechanism is given by:

$$A = \begin{bmatrix} 0.9752 & 0.0248 & 0.1983 & 0.0017 \\ 0.0248 & 0.9752 & 0.0017 & 0.1983 \\ -0.2459 & 0.2459 & 0.9752 & 0.0248 \\ 0.2459 & -0.2459 & 0.0248 & 0.9752 \\ 0.0248 & 0.0017 & 0.0017 & 0.2459 \end{bmatrix}; \; B = \begin{bmatrix} -0.0199 \\ -0.0001 \\ -0.1983 \\ -0.0017 \end{bmatrix}; \; C = [0 \; 1 \; 0 \; 0]; D = [O]$$

The initial values of the parameters chosen are:

$$Q = 1, R = 2, \gamma = 22$$

The system is converted to a standard format using augmentation technique given in section II(B). The parameter values obtained from the PSO procedure are:

$$Q = 7.6085, R = 0.0047, \gamma = 24.8208$$

A comparison of the step response of the system with two solutions is shown in figure 1. Figure 2 shows the placement of closed-loop eigen values of the system with initial and PSO-tuned parameters.

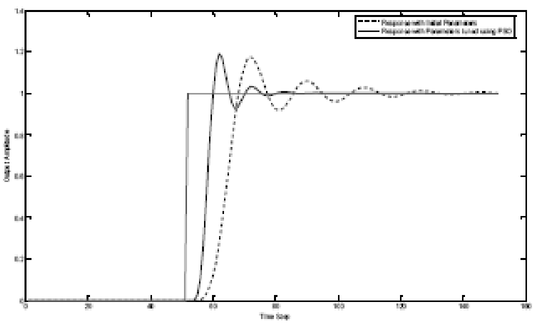

Figure 1: Transient Response of Servomechanism

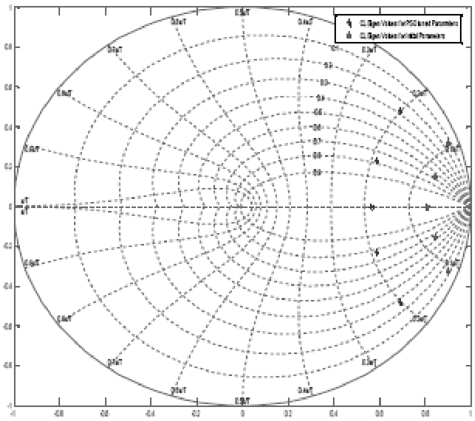

Figure 2: Closed – Loop Eigen Values of Designed System

Missile Model (5th Order):

The state-variable model of the 5th-order Missile model is given by:

$$A=\begin{bmatrix}-1.06 & 0 & 0.81 & 0 & -0.11\\ 0 & 0 & 1 & 0 & 0\\ -33.88 & 0 & -96.8 & 0 & -55.9\\ 272 & -272 & 0 & 0 & 0\\ 251.5 & -0.017 & -0.017 & 0 & 26.9\end{bmatrix};\ B=\begin{bmatrix}0\\0\\0\\0\\0.96\end{bmatrix};\ C=\begin{bmatrix}0 & 0 & 1 & 0 & 0\end{bmatrix}; D=[0]$$

The initial values of the parameters chosen are:

$$Q=50, R=10, \gamma=50$$

The system is converted to a standard format using augmentation technique given in section II (B). The parameter values obtained from the PSO procedure are:

$$Q=50.4687, R=0.0036, \gamma=35.2873$$

A comparison of the step response of the system with two solutions is shown in figure 3. Figure 4 shows the placement of closed-loop eigen values of the system with initial and PSO-tuned parameters.

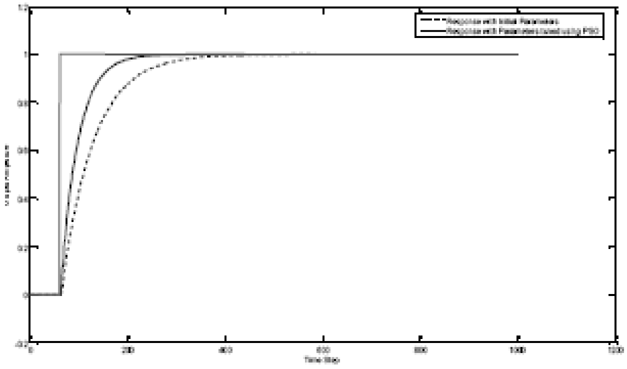

Figure 3: Transient Response of Missile Model

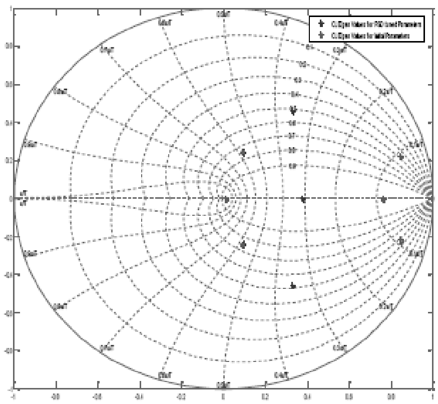

Figure 4: Closed – Loop Eigen Values of Designed System

Temperature Control System with Delay:
The model of the temperature control system with delay in transfer function format is given by.

$$G(s) = e^{-1.5s}\frac{1}{(s+1)} \tag{13}$$

The initial values of the parameters chosen are:

$$Q = 5, R = 0.5, \gamma = 0.7$$

The system is converted to a standard format using augmentation technique given in section II (B). The parameter values obtained from the PSO procedure are :

$$Q = 2.4614, R = 0.5, \gamma = 0.7$$

A comparison of the step response of the sytem with two solutions is shiwn in figure 5. Figure 6 shows the placement of closed-loop eigen values of the system with initial and PSO-tuned parameters.

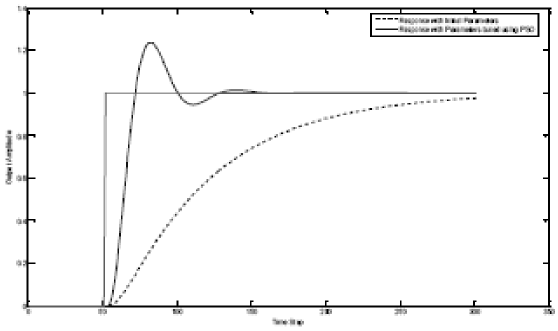

Figure 5: Transient Response of Temperature Control System

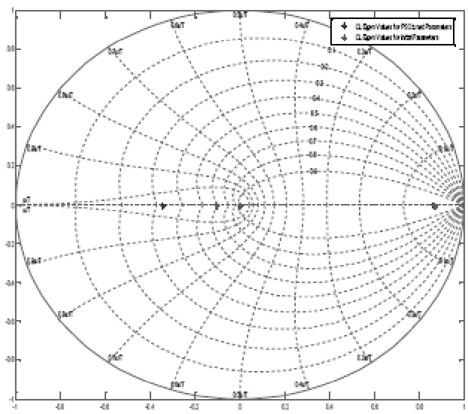

Figure 6: Closed – Loop Eigen Values of Designed System

Comparisons of the step responses show that the parameters tuned using PSO technique produce better results in terms of transient response characteristics. The closed-loop eigen values' plot, also, show that the system becomes more stable with these parameter values.

Discrete H∞ Preview Control Problem with Previewable Reference Signal

Problem Definition:

The general discrete-time system is described by

$$\begin{aligned} x(t+1) &= Ax(t) + Bu(t) + Ed(t) \\ z(t) &= Cx(t) + Du(t) \end{aligned} \tag{14}$$

where $x(t) \in R^n$ and $u(t) \in R^m$ are the state vector and control input, respectively. The signal $z(t) \in R^p$ denotes the controlled putout or the tracking error. Moreover, $d(t) \in R^l$ denotes the exogenous signal which can be considered as the reference signal or the disturbance. Figure 7 shows the block diagram of a general preview control system explaining the relation between various signals of the system.

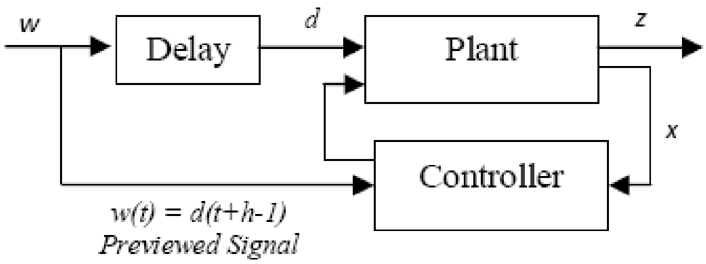

Figure 7: Preview Control Problem.

The following assumptions are made for the system:

i. (A,B) is stabilizable.

ii. $\begin{bmatrix} A - e^{j\theta l} & B \\ C & D \end{bmatrix}$ has full column rank for any $\theta \in [0, 2\pi)$.

iii. The values of $d(t), d(t+1), ..., d(t+h)$ are available for control where h is a nonnegative constant that is called preview length.

The purpose of preview control is to design a controller in the form of

$$u(t) = K_x x(t) + \sum_{i=0}^{h} K_{di} d(t+1) \tag{15}$$

so that the following quadratic performance index is made satisfactorily small even in the presence of the exogenous input d.

$$J = \| z \|_2^2 = \sum_{i=0}^{\infty} \| z(t) \|^2 \tag{16}$$

The first and second terms on the right-hand-side of the controller equation (15) represent the state feedback and preview compensation, respectively.

Solution Methodology:

Consider the system defined by the state-variable model in discrete-domain, as:

$$\begin{aligned} x(t+1) &= Ax(t) + Bu(t) \\ y(t) &= Cx(t) + Du(t) \end{aligned} \tag{17}$$

The system model is to be converted to a standard format, so that the performance index can be a part to the system model. The previewed informatin of the reference signal is made a part of system model using state augmentation, explained as below:

Let $x_d(t)$ be the vector which represents the previewed information that is available for control, namely

$$x_d(t) = \begin{bmatrix} d(t) \\ d(t+1) \\ \vdots \\ d(t+h) \end{bmatrix} \in R^{l(h+1)} \tag{18}$$

This information can be represented as

$$x_d(t+1) = A_d x_d(t) + B_d d(t+h-1) \tag{19}$$

where

$$A_d = \begin{bmatrix} 0 & l & \ddots & 0 \\ & 0 & \ddots & l \\ 0 & & & 0 \end{bmatrix}, B_d = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ l \end{bmatrix}$$

Now, the augmented state vector is defined as,

$$\xi(t) = [x^T(t) \;\; x_d^T(t)]^T$$

The augmented system, now, becomes,

$$\begin{aligned} \xi(t+1) &= F\xi(t) + Gu(t) + Ld(t), \\ z(t) &= H\xi(t) + Du(t) \end{aligned} \tag{20}$$

where

$$F = \begin{bmatrix} A & E & O \\ O & Ad & \end{bmatrix}, G = \begin{bmatrix} B \\ O \end{bmatrix}, L = \begin{bmatrix} O \\ B_d \end{bmatrix}, H = \begin{bmatrix} C & 0 & 0 \end{bmatrix}$$

Thus, the preview controller is a state feedback law for the augmented sytem.

By applying the standard H∞ control theory to the augmented system, the solution to the Preview Control problem is obtained as follows:

Let a positive constant γ be given. There exists a stabilizing state feedback control with preview action satisfying the H∞ performance level

$$\sup \frac{\| z \|_2}{\| d \|_2} < \gamma \tag{21}$$

if and only if there exists a positive semi-definite stabilizing solution P to the following Algebraic Riccati Equation such that $W := \gamma^2 - L^T PL > 0$.

$$\begin{aligned} P &= F^T PF - \begin{bmatrix} G^T PF + D^T H \\ L^T PF \end{bmatrix}^T V^{-1} \begin{bmatrix} G^T PF + D^T H \\ L^T PF \end{bmatrix} + H^T H \\ V &= \begin{bmatrix} D^T D & 0 \\ 0 & -\gamma^2 l \end{bmatrix} + \begin{bmatrix} G^T \\ L^T \end{bmatrix} P[G \;\; L] \end{aligned} \tag{22}$$

In this case, one of the desired H∞ preview controller is given by

$$\begin{aligned} &[K_x \quad K_{d0} \quad \dots \quad K_{dh}] = -(D^T D + G^T \hat{P} G)^{-1} (G^T \hat{P} F + D^T H) \\ &\hat{P} = P + PLW^{-1} L^T P \end{aligned} \tag{23}$$

Parameter Selection using PSO.

The parameters (Q,R and γ) values obtained using bisection method or trial and error method can be improved to give better results using PSO technique. Each PSO individual flies in the search space which is multi-dimensional and presents the optimal solution by dynamically adjusting the velocity of the individuals.

The procedure of automating the tuning technique for Q, R and γ values using PSO consists of evaluating a series of step responses in order to permit the algorithm to converge by minimizing the objectiv function, Integral of Absolute Error (IAE) given by,

$$IAE = \int | Y_{SP}(t) - Y(t) | \, dt \tag{24}$$

The algorithm for the solution of the automated parameters selection problem is explained in section II (C).

Application to System Design:

The verification of performance of the procedure was carried out on MATLAB platform for three systems and the results are summed up as follows:

Servomechanism (4^{th} Order):

The state-variable model of the 4^{th}-order Servomechanism is given by:

$$A = \begin{bmatrix} 0.9752 & 0.0248 & 0.1983 & 0.0017 \\ 0.0248 & 0.9752 & 0.0017 & 0.1983 \\ -0.2459 & 0.2459 & 0.9752 & 0.0248 \\ 0.2459 & -0.2459 & 0.0248 & 0.9752 \\ 0.0248 & 0.0017 & 0.0017 & 0.2459 \end{bmatrix}; \quad B = \begin{bmatrix} -0.0199 \\ -0.0001 \\ -0.1983 \\ -0.0017 \end{bmatrix}; \quad C = [0\ 1\ 0\ 0]; D = [0]$$

The initial values of the parameters chosen are:

$$Q = 20, R = 1, \gamma = 22$$

The system is converted to a standard format using augmentation technique given in section II(B). The previewed information is included in the system model as explained in section III (B). The parameter values obtained from the PSO procedure are:

$$Q = 7.6085, R = 0.0047, \gamma = 24.8208$$

A comparison of the step response of the system with two solution is shown in figure 8, 10 and 12 for preview lengths h = 1, 5, 10. The figures 9, 11 and 13 show the placement of closed-loop eigen values of the system with initial and PSO-tuned parameters.

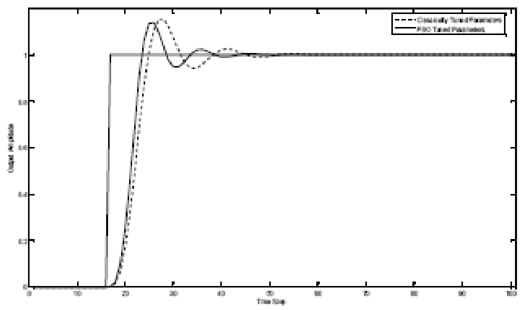

Figure 8: Transient Response of Servomechanism for Preview Length =1

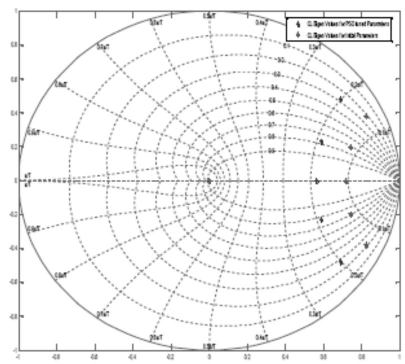

Figure 9: Closed – Loop Eigen Values of Designed System

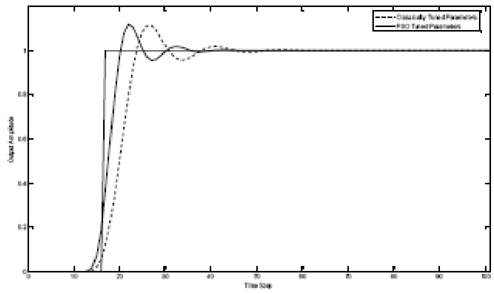

Figure 10: Transient Response of Servomechanism for Preview Length = 5

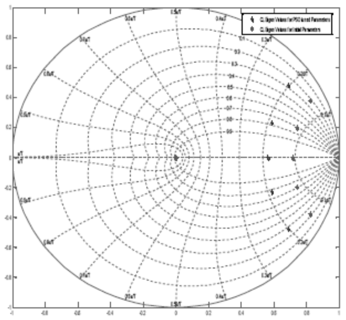

Figure 11: Closed – Loop Eigen Values of Designed System

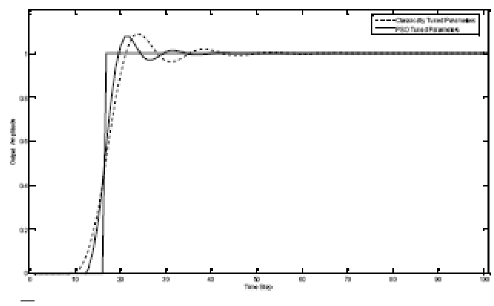

Figure 12: Transient Response of Servomechanism for Preview Length = 10

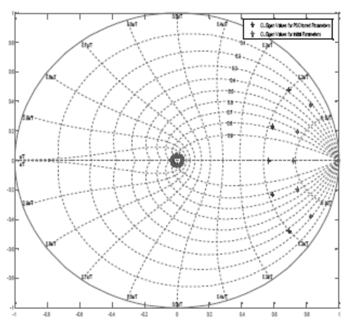

Figure 13: Closed – Loop Eigen Values of Designed System

Missile Model (5th Order):

The state-variable model of the 5th-order Missile model is given by:

$$A=\begin{bmatrix}-1.06 & 0 & 0.81 & 0 & -0.11\\ 0 & 0 & 1 & 0 & 0\\ -33.88 & 0 & -96.8 & 0 & -55.9\\ 272 & -272 & 0 & 0 & 0\\ 251.5 & -0.017 & 71.35 & 0 & 26.9\end{bmatrix};\ B=\begin{bmatrix}0\\0\\0\\0\\0.96\end{bmatrix};\ C=[0\ 0\ 1\ 0\ 0]; D|=[0]$$

The initial values of the parametes were chosen as:

$$Q=50, R=10, \gamma=50$$

The system is converted to a standard format using augmentation technique given in section II (B). The previewed information is included in the system model as explained in section III (B). The parameter values obtained from the PSO procedure are:

$$Q=50.4687, R=0.0036, \gamma=35.2873$$

A comparison of the step response of the system with two solution is shown in figure 14, 16 and 18 for preview lengths h = 1, 2, 3. The figures 15, 17 and 19 shows the placement of closed-loop eigen values of the system with initial and PSO-tuned parameters.

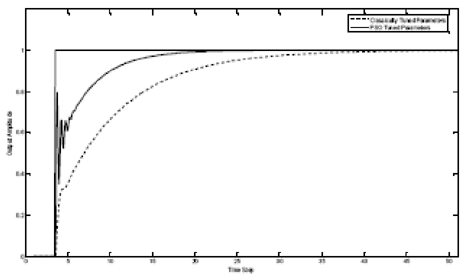

Figure 14: Transient Response of Missile Model for Preview Length = 1

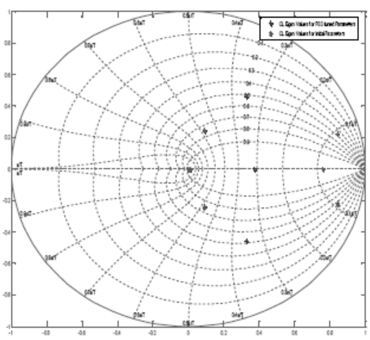

Figure 15: Closed – Loop Eigen Values of Designed System

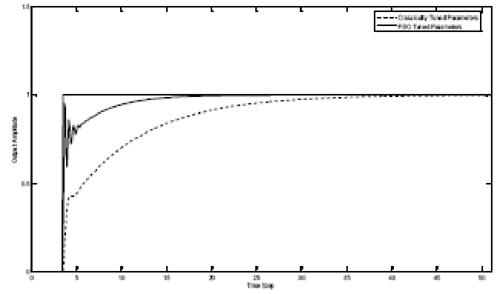

Figure 16: Transient Response of Missile Model for Preview Length = 2

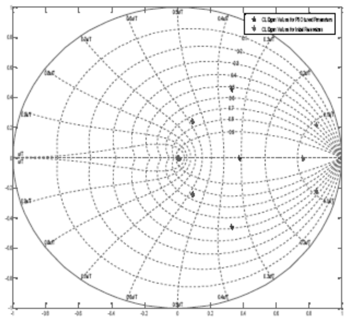

Figure 17: Closed – Loop Eigen Values of Designed System

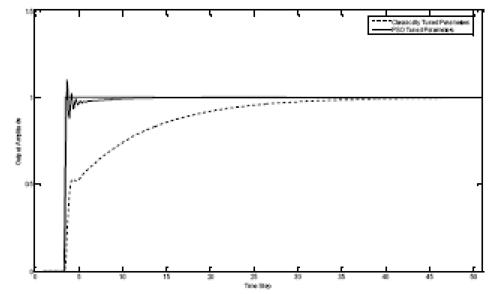

Figure 18: Transient Response of Missile Model for Preview Length = 3

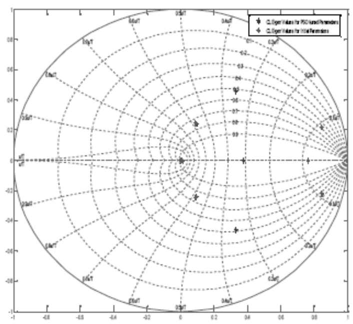

Figure 19: Closed – Loop Eigen Values of Designed System

Temperature Control System with Delay:
The model of the system in transfer function format is given by:

$$G(s) = e^{-1.5s} \frac{1}{(s+1)} \tag{25}$$

The initial values of the parameters were chosen as:

$$Q = 2.7, R = 0.1, \gamma = 3$$

The system is converted to a standard format using augmentation technique given in section II (B). The previewed information is included in the system model as explained in Section III (B). The parameter values obtained from the PSO procedrue are :

$$Q = 3.3093, R = 0.0039, \gamma = 4.2572$$

A comparison of the step response of the system with two solutions is shown in figure 20, 22 and 24 for preview lengths h = 1, 5, 10. The figures 21, 23 and 25 show the placement of closed-loop eigen values of the system with initial and POS-tuned parameters.

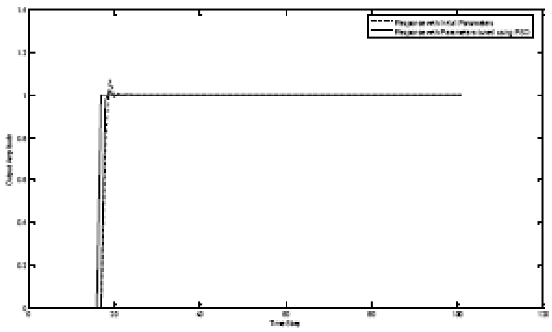

Figure 20: Transient Response of Temperature Control System for Preview Length = 1

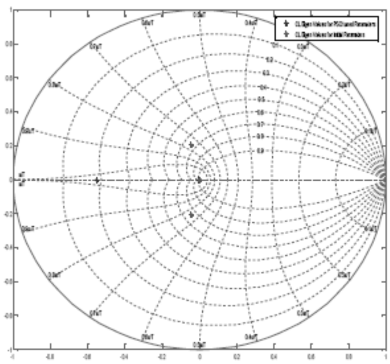

Figure 21: Closed – Loop Eigen Values of Designed System

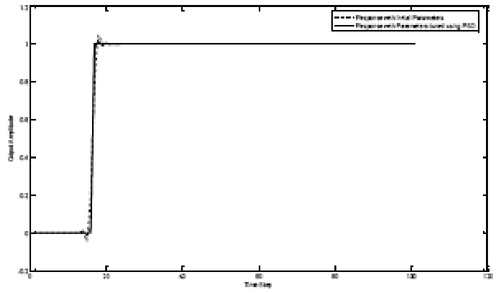

Figure 22: Transient Response of Temperature Control System for Preview Length = 5

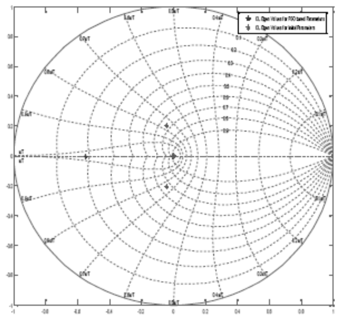

Figure 23: Closed – Loop Eigen Values of Designed System

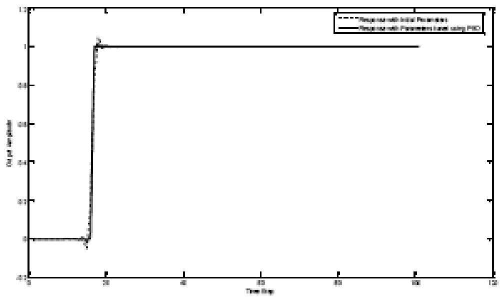

Figure 24: Transient Response of Temperature Control System for Preview Length = 10

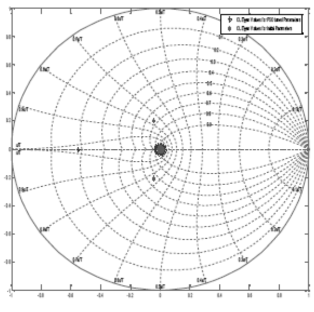

Figure 25: Closed – Loop Eigen Values of Designed System

Comparisons of the step responses show that the parameters tuned using PSO technique produce better results in terms of transient response characteristics. The closed-loop eigen values' plot, also, show that the system becomes more stable with these parameter values.

Discrete H∞ Preview Control Problem With Previewable Reference And Disturbance Signal

Problem Definition:

The basic problem definition for discrete H∞ control problem with previewable reference and disturbance signal is similar to that explained in section III (A), as both the previewed signals, that is reference and disturbance signals, are considered as a part of one vector. Thus, this preview controller with similar assumptions:

i. (A,B) is stabilizable.

ii. $\begin{bmatrix} A-e^{j\theta} & B \\ C & D \end{bmatrix}$ has full column rank for any $\theta \in [0, 2\pi)$.

iii. The values of $d(t), d(t+1), ..., d(t+h)$ are available for control, where h is a nonnegative constant that is called preview length.

Designs the controller of the form:

$$u(t) = K_x x(t) + \sum_{i=0}^{h} K_{di} d(t+i) \tag{26}$$

so that the following quadratic performance index is made satisfactorily small even in the presence of the exogenous input d.

$$J = \| z \|_2^2 = \sum_{i=0}^{\infty} \| z(t) \|^2 \tag{27}$$

The first and second terms on the right-hand-side of the controller equation (26) represent the state feebback and preview compensation, respectively.

Solution Methodology:

Consider the system defined by the state-variable model in discrete-domain, as:

$$\begin{aligned} x(t+1) &= Ax(t) + Bu(t) \\ y(t) &= Cx(t) + Du(t) \end{aligned} \tag{28}$$

The system model is to be converted to a standard format, so that the performance index can be a part to the system model. The previewed information of the reference and disturbance signals is made a part of system model using state augmentation, explained as below:

Let $x_{dw}(t)$ be the vector which represents the previewed information of disturbance signal that is available for control, namely

$$x_{dw}(t)=\begin{bmatrix} \vartriangle w(t) \\ \vartriangle w(t+1) \\ \vartriangle w(t+h) \end{bmatrix} \in R^{l(h+1)} \tag{29}$$

This information can be represented as

$$x_{dw}(t+1)=A_{dw}x_{dw}(t)+B_{dw}w(t+h-1) \tag{30}$$

where

$$A_{dw}=\begin{bmatrix} 0 & l & & 0 \\ & 0 & \ddots & \\ & & \ddots & l \\ 0 & & & 0 \end{bmatrix}, B_{dw}=\begin{bmatrix} 0 \\ \vdots \\ 0 \\ l \end{bmatrix}$$

Let $x_{dr}(t)$ be the vector which represents the previewed information of reference signal that is available for control, namely

$$x_{dr}(t)=\begin{bmatrix} \vartriangle(t) \\ \vartriangle r(t+1) \\ \vdots \\ \vartriangle r(t+h) \end{bmatrix} \in R^{l(h+1)} \tag{31}$$

This information can be represented as

$$x_{dr}(t+1)=A_{dr}x_{dr}(t)+B_{dr}r(t+h-1) \tag{32}$$

where

$$A_{dr}=\begin{bmatrix} 0 & l & & 0 \\ & 0 & \ddots & \\ & & \ddots & l \\ 0 & & & 0 \end{bmatrix}, B_{dr}=\begin{bmatrix} 0 \\ \vdots \\ 0 \\ l \end{bmatrix}$$

Now, the augmented stated vector is defined as,

$$\xi(t)=[x^T(t)\ \ x_{dr}^T(t)\ \ x_{dw}^T(t)]^T$$

The augmented sytem, now, becomes,

$$\xi(t+1)=F\xi(t)+Gu(t)+Ld(t), z(t)=H\xi(t)+Du(t) \tag{33}$$

where

$$F=\begin{bmatrix} A & \vdots & E_1 & 0 & \vdots & E_2 & 0 \\ 0 & \vdots & & A_{dr} & \vdots & & 0 \\ 0 & \vdots & & 0 & \vdots & & A_{dw} \end{bmatrix}, G=\begin{bmatrix} B \\ 0 \\ 0 \end{bmatrix}, L=\begin{bmatrix} 0 \\ B_{dr} \\ B_{dw} \end{bmatrix}, H=\begin{bmatrix} C & \vdots & 0 & 0 \end{bmatrix}$$

Thus, the preview controller is a state feedback law for the augmented system.

By applying the standard H∞ control theory to the augmented sytem, the solution to the Preview Control problem is obtained as follows:

Let a positive constant γ be given. There exists a stabilizing state feedback control with preview action satisfying the H∞ performance level

$$\sup \frac{\| z \|_2}{\| d \|_2} < \gamma \tag{34}$$

if and only if there exists a positive semi-definite stabilizing solution P to the following Algebraic Riccati Equation such that $W := \gamma^2 - L^T PL > 0$.

$$\begin{aligned} P &= F^T PF - \begin{bmatrix} G^T PF + D^T H \\ L^T PF \end{bmatrix}^T V^{-1} \begin{bmatrix} G^T PF + D^T H \\ L^T PF \end{bmatrix} + H^T H \\ V &= \begin{bmatrix} D^T D & 0 \\ 0 & -\gamma^2 l \end{bmatrix} + \begin{bmatrix} G^T \\ L^T \end{bmatrix} P[G \quad L] \end{aligned} \tag{35}$$

In this case, one of the desired H∞ preview controller is given by

$$[K_x \ K_{do} \ \ldots \ K_{dh}] = -(D^T D + G^T \hat{P} F + D^T H)\hat{P} = P + PLW^{-1} L^T P \tag{36}$$

Parameter Selection using PSO:

The parameters (Q, R and γ) values obtained using bisection method or trial and error method can be improved to give better results using PSO technique. Each PSO individual flies in the search space which is multi-dimensional and presents the optimal solution by dynamically adjusting the velocity of the individuals.

The procedure of automating the tuning techniques for Q, R and γ values using PSO consists of evaluating a series of step responses in order to permit the algorithm to converge by minimizing the objective functin, Integral of Absolute Error (IAE) given by,

$$IAE = \int | Y_{SP}(t) - Y(t) | \, dt \tag{37}$$

The algorithm for the solution of the automated parameters selection problem is explained in section II (C).

Application to System Design:

The verification of performance of the procedure was carried out on MATLAB platform for three systems and the results are summed up as follows:

Servomechanism (4^{th} Order):

The state-variable model of the 4^{th}-order Servomechanism is given by:

$$A=\begin{bmatrix} 0.9752 & 0.0248 & 0.1983 & 0.0017 \\ 0.0248 & 0.9752 & 0.0017 & 0.1983 \\ -0.2459 & 0.2459 & 0.9752 & 0.0248 \\ 0.2459 & -0.2459 & 0.0248 & 0.9752 \\ 0.0248 & 0.0017 & 0.9752 & 0.2459 \end{bmatrix};\; B=\begin{bmatrix} -0.0199 \\ -0.0001 \\ -0.1983 \\ -0.0017 \end{bmatrix}; C=\begin{bmatrix} 0 & 1 & 0 & 0 \end{bmatrix}; D=[0]$$

The initial values of the parameters chosen are:

$$Q=1, R=2, \gamma=22$$

The system is converted to a standard format using augmentation technique given in section II (B). The previewed information is included in the system model as explained in section III (B). The parameter values obtained from the PSO procedure are:

$$Q=0.1380, R=1.3784, \gamma=23.4591$$

A comparison of the step response of the system with two solutions is shiwn in figure 26, 28 and 30 for preview lengths h = 2, 5, 10. The figures 27, 29 and 31 show the placement of closed-loop eigen values of the system with initial and PSO-tuned parameters.

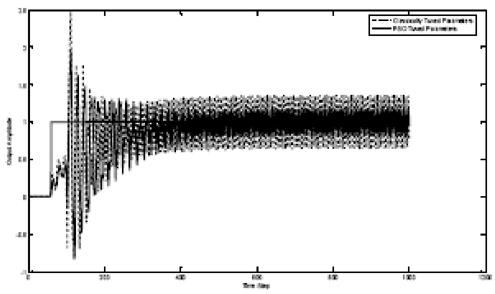

Figure 26: Transient Response of Servomechanism for Preview Length = 2

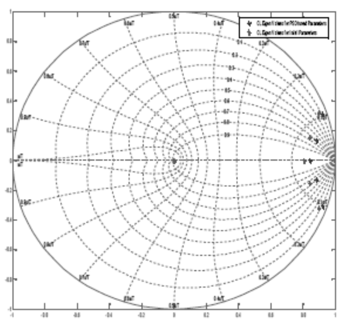

Figure 27: Closed – Loop Eigen Values of Designed System

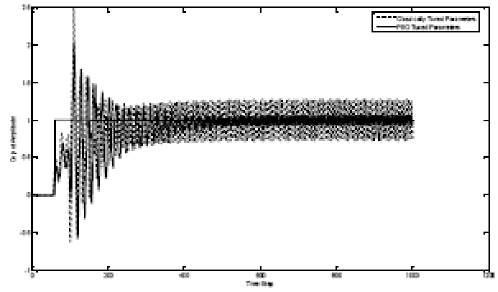

Figure 28: Transient Response of Servomechanism for Preview Length = 5

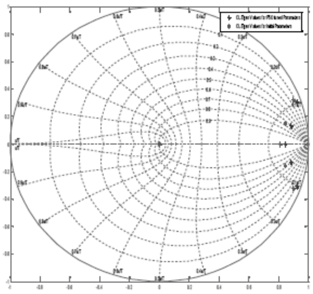

Figure 29: Closed – Loop Eigen Values of Designed System

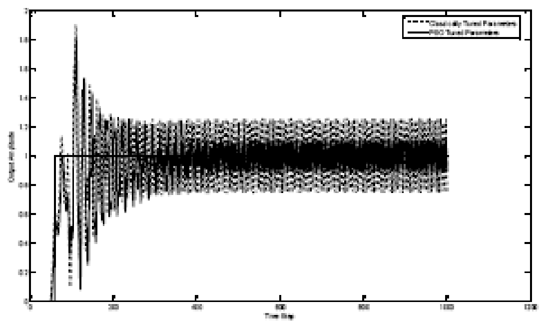

Figure 30: Transient Response of Servomechanism for Preview Length = 10

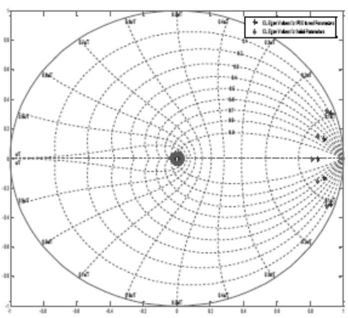

Figure 31: Closed – Loop Eigen Values of Designed System

Missile Model (5th Order):

The state-variable model of the 5th-order Missile model is given by:

$$A=\begin{bmatrix}-1.06 & 0 & 0.81 & 0 & -0.11\\ 0 & 0 & 1 & 0 & 0\\ -33.88 & 0 & -96.8 & 0 & -55.9\\ 272 & -272 & 0 & 0 & 0\\ 251.5 & -0.017 & 71.35 & 0 & 26.9\end{bmatrix};B=\begin{bmatrix}0\\0\\0\\0\\0.96\end{bmatrix};C=\begin{bmatrix}0 & 0 & 1 & 0 & 0\end{bmatrix},D=[0]$$

The initial values of the parameters chosen are:

$$Q=50, R=10, \gamma=50$$

The system is converted to a standard format using augmentation technique given in section II (B). The previewed information is included in the sytem model as explained in section III (B). The parameter values obtained form the PSO procedure are:

$$Q=51.3401, R=0.1102, \gamma=49.7557$$

A comparison of the step response of the system with two solutions is shown in figure 32, 33 and 36 for preview lengths h = 1, 2, 3. The figures 33, 35 and 37 show the placement of closed-loop eigen values of the system with initial and PSO-tuned parameters.

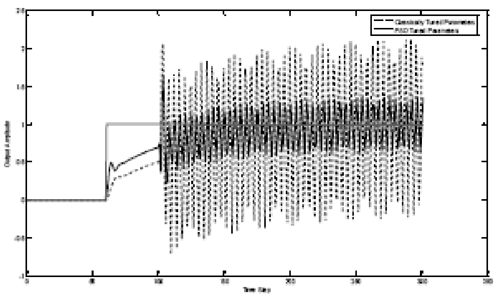

Figure 32: Transient Response of Missile Model for Preview Length = 1

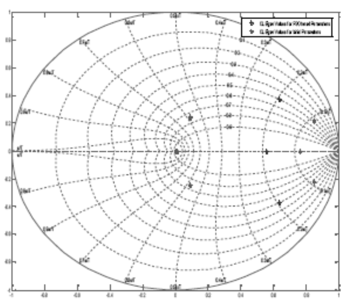

Figure 33: Closed – Loop Eigen Values of Designed System

The initial values of the parameters chosen are:

$$Q = 50, R = 10, \gamma = 50$$

The system is converted to a standard format using augmentation technique given in section II (B). The previewed information is included in the system model as explained from the PSO procedure are:

$$Q = 50.5602, R = 0.1806, \gamma = 48.3357$$

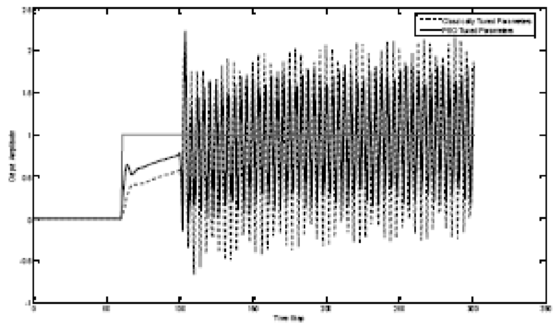

Figure 34: Transient Response of Missile Model for Preview Length = 2

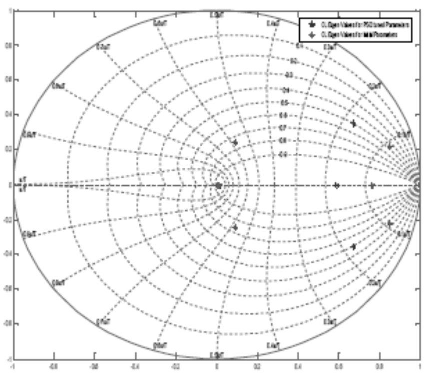

Figure 35: Closed – Loop Eigen Values of Designed System

The initial values of the parameters chosen are:

$$Q = 50, R = 10, \gamma = 50$$

The system is converted to a standard format using augmentation technique given in section II (B). The previewed information is included in the system model as explained in section III (B). The parameter values obtained from the PSO procedure are:

$$Q = 59,1011, R = 0.1108, \gamma = 50.9622$$

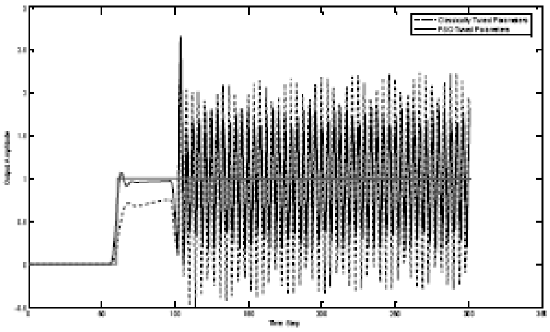

Figure 36: Transient Response of Missile Model for Preview Length = 3

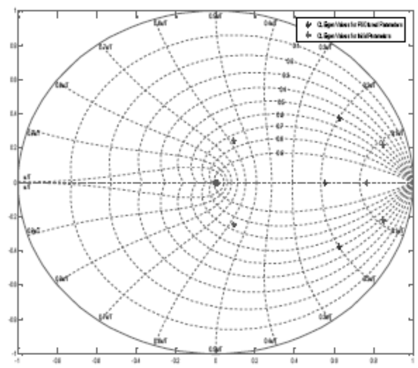

Figure 37: Closed – Loop Eigen Values of Designed System

Temperature Control System with Delay:

The model of the system in transfer function format is given by,

$$G(s) = e^{-1.5s} \frac{1}{(s+1)} \tag{38}$$

The initial values of the parameters chosen are:

$$Q = 2.7, R = 0.1, \gamma = 3$$

The system is converted to a standard format using augmentation technique given in section II (B). The previewed information is included in the sytem model as explained in section III (B). The parameter values obtained from the PSO procedure are:

$$Q = 1.8104, R = 1.3280, \gamma = 11.1947$$

A comparison of the step response of the system with two solutions is shown in figure 38, 40 and 42 for preview lengths h = 1, 5, 10. The figures 39, 41 and 43 show the placement of closed-loop eigen values of the system with initial and PSO-tuned parameters.

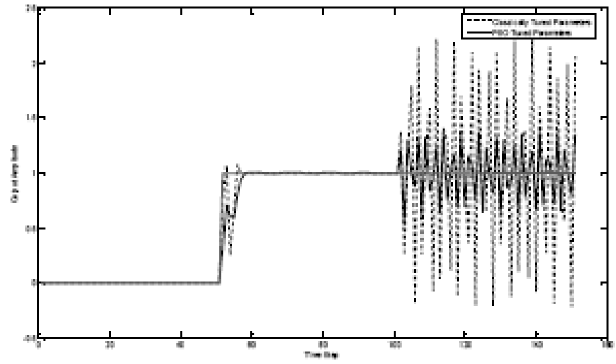

Figure 38: Transient Response of Temperature Control System for Preview Length = 1

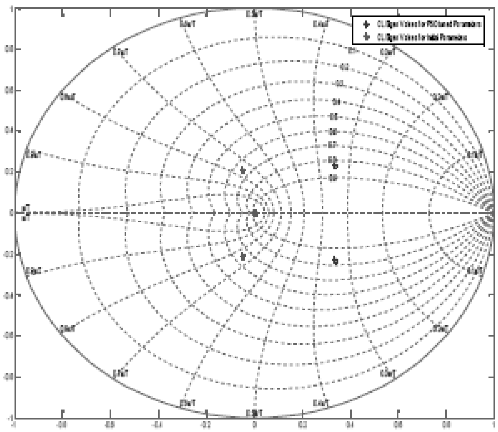

Figure 39: Closed – Loop Eigen Values of Designed System

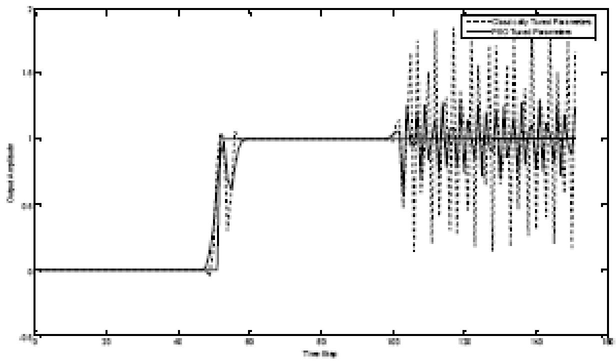

Figure 40: Transient Response of Temperature Control System for Preview Length = 5

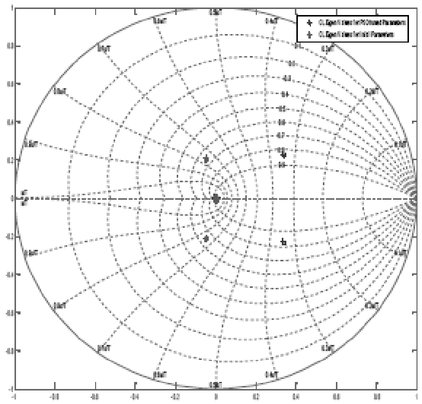

Figure 41: Closed – Loop Eigen Values of Designed System

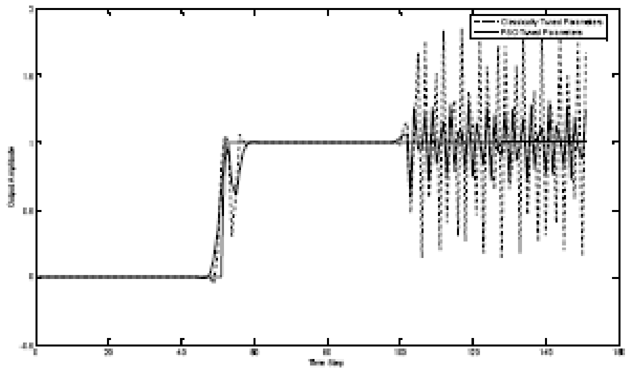

Figure 42: Transient Response of Temperature Control System for Preview Length =10

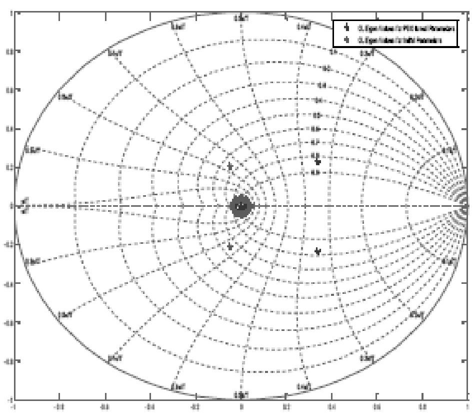

Figure 43: Closed – Loop Eigen Values of Designed System

Comparisons of the step responses show that the parameters tuned using PSO technique produce better results in terms of transient response characteristics. The closed-loop eigen values' plot, also, show that the system becomes more stable with these parameter values.

Conclusion

This paper has discussed and explained various issues in application of Discrete H∞ Control to practical systems. The use of PSO technique has also been verified to automatically tune the critical parameters (Q, R and γ) for optimal values and relieve the designer from the complicated calculations for the choice of these parameters. The next phase of Discrete H∞ Control for systems with previewable reference and disturbance signals have also been discussed, with a focus on automatic tuning of the critical parameters (Q, R and γ) with PSO technique. The application of the system design is verified on MATLAB platform for three practical system models (Servomechanism, Missile Model and Temperature Control System with delay). The results show that the previewable signal improves the transient response of the system remarkably. Also, the automatic tuning of the critical parameters makes the system more robust, improves the transient response and stabilizes the system. The application area of the presented techniques can be extended to the non-linear systems to present a more a generalized autotuning algorithm of the problem.

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