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How Graph and Even Number Relates with VLSI Design

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Abstract

In this paper, various properties of graphs, obtained from the graphical partition of even numbers have been discussed. The total number of simple Hamiltonian graphs of vertex $n \geq 6$ for the graphical partition of the even number k where $2i + 10 \leq k \leq m(m+1)$ for $i \geq 1$, with simultaneous changes of $m \geq 5$ and $n \geq 6$ has been found as $(n^2-3n+2)/2$ .The existence of bipartite graph of the form $K_{i,m}$ for $i \geq 3$ with simultaneous changes of $m \geq 5$ have also been discussed for the even number of the type 2(i x m). Besides, a theoretical result has been discussed for existence of line of sight graph obtained from various net patterns used for m x m printed circuit board for the graphical partition of numbers $2i+10 \leq k \leq 4n+20$ for $i \geq 1$, $n \geq 1$ with simultaneous changes of the vertex $m \geq 6$ and the number of such planar line of sight graphs are ≥ 16. Finally two propositions for strongly regular graphs and one conjecture for technically applicable graphical partition of even number greater than or equal to 12 have been cited.

AMS Subject Classification: 11A41, 11A51, 11A051, 05C30, 05C45.

Keywords: Graphical partition, Hamiltonian Graph, Line of Sight Graphs, strongly regular graph.

1.1. Introduction: It has been reported that graph theory started from the solution of Konigsberg bridge problem which was solved by Euler in 1736. Again, it has also been found that the number theory started so many years ago before graph theory. Number theory is considered as a branch of pure mathematics, but on the other hand graph theory is considered as an applied branch. Graph theory has many applications in various branches of science and technology. Therefore, it is necessary to discuss the relationship between number theory and graph theory in the present day context. Various properties, relating to graph and number have been found, even though there are so many conjectural results in both the cases. Ryser Herbert [1] has been opined that …….. Combinatory and Graph Theory are sister discipline. Graph Theory, which has been considered as a branch of combinatory and which had been developed from the solution of Konigsberg bridge problem has been occupying the different fields of modern technology with various applications. On the other hand, the study of Number Theory, which was discovered so many years ago than graph theory, has been considering as a branch of pure mathematics since the existence of number theory. But, it is observed that the Pure and Applied Mathematics have been coming to meet at a point since the existence of some branches of engineering subjects such as computer science, electronic & telecommunication etc. The theoretical computer science needs some fields of pure mathematics, which is supposed as combinatorial branch related with numbers. Hence the pure mathematics can also be considered as applied mathematics as many results of pure Mathematics are applicable in various branches of science and technology. Let us consider a number 14. Simply, we can say that 14 is an even number and it can be expressed as a sum of two primes, say, 7+7 or 11+3, according to Gold Bach conjecture, which was stated that every even number n greater than 2 can be expressed as a sum of two primes in 1742. But, when one study the partition of the number 14, then it is found that one of the partition say, 3+3+3+5=14, gives a graph, whose structure (geometric structure) was forwarded by Euler after the solution of the Konigsberg bridge problem. Again, another partition 2+2+2+2+2+2+2 =14 gives a regular graph of degree two of seven vertex which is Hamiltonian graph and three colorable. Similarly the partitions 14=3+3+2+2+2+2, 14=7+1+1+1+1+1+1+1, 14=4+4+3+3 etc give some of the structure of graphs. Hence, the various properties of graphs, relating to degree of a graph and various applications of graphs are nothing, but the properties of number which are actually prevailed in partition of numbers. It is important to note that Number theory again can be considered as a never ending theory, as and when some conjectural problems have been solved, immediately some new problems exist. Let us discuss first some important cases in number theory and secondly the existences of graphs which are obtained from the graphical partition of even numbers. It has been found that there are so many unsolved problems in number theory which are conjectural in nature. There are many important conjectural problems which lie in prime number systems (all prime number's properties have not been considered here). This paper mainly contains prime numbers, as we know that for any prime p, the number 2p is an even number and our main topic of discussion in this article is related with even numbers of various forms.

Recently, Kalita, B, etal [10] has been discussed some structures of graphs obtained from some graphical partitions. It has been found [10] that the graphical partition of the numbers 2n+4 for $n \geq 1$ always give at least two regular graphs of degree 1 and 2. In addition to this, it has been again found that the graphical partition of the numbers 4(n+1) for $n \geq 1$ produce a binary tree. Further, it has been discussed the existence of some complete graphs for the graphical partition of numbers (n+1) (n+2) for n

≥ 1.Some properties of partition of numbers and graphical partition of number have been forwarded by Harary [8]. Interestingly, the graphical partitions of number play an important role with graph theory. Various properties of graphs and graphical partitions have been discussed by Kalita [10-15] and [2-5]. It is interesting to note that when one studies number theory, he may not feel/think to study graph theory simultaneously with number theory. But, it should be discussed simultaneously. Graph theory has many applications in different branches of science and technology, whereas number theory is considered as a branch of pure mathematics.

But it has been found that the proof of Goldbach conjecture related with prime numbers has been forwarded by Kalita [5], where he applied graph theory results to prove the conjecture with many new definitions of graph theory. This proof can be considered as first application of graph theory used in proving number theory conjecture. It has been found that the various kinds of planar and non-planar graphs are used for printed circuited board. The most important class of non-planar graphs H= $\{H_k$ (2m+3,6m+6) for m ≥ 2, k=1 to 4m-2$\}$ have been constructed and various properties relating to maximum and minimum number of crossing of the class have been discussed [24]. Recently [25] Kalita, B etal found the Even number finding graph related to the proof of Gold Bach Conjecture and forwarded an algorithm to prove that all even number can be expressed as a sum of two primes.

In this paper, the relationship of graph theory and the graphical partition of numbers have been discussed. Some theoretical results have also been considered and proved. Finally the existence of various kinds of line of sight graphs have been cited with theoretical results for printed circuit board for different types of net patterns.

The paper is organized as follows.

The section 1.1of the paper focuses some works on number theory with some sophisticated structure of graphs. The definition of partition and graphical partitions are included in section 2.1 Section 3.1includes the theoretical explanations on graphical partitions of some even numbers related with graphs. In section 4.1 the application of some graphical partitions of numbers relating to line of sight graph constructed for VLSI design technology of printed circuit board m x m pattern, has been discussed with theoretical explanation. Two propositions and one conjecture have also been included in section 5.1. The conclusion is included in section 6.1

2.1. Definition: We now remind the following definitions:

2.1.1 Partition of number: The partition of a non-negative number/ integer n has been defined as a representation of n as a sum of positive integers [8]. For example, there are five partitions of number 4 and they are 4, 3+1, 2+2, 2+1+1, 1+1+1+1.

2.1.2 Graphical partition: The graphical partition of an even number n is a partition, whose summands/parts of the partition give the degrees of a graph. That is there exist a graph for the partition of even number n into p parts, where n= $\sum p_i$ = 2.q, q are the numbers of edges of the graph. Hence, it is observed that the relation of even numbers and graphical partition plays an important role both in number theory and graph theory. The even numbers of the forms $\{4n+10/\ n \geq 4\}$ and $\{4n+12/\ n \geq 4\}$ [2] have been considered here and study different types of graphs obtained from the graphical partition of them.

2.1.3 Line of sight graph:

A line of sight graph G (V, E) is a graph where the set of vertices V represents nets and the set of edges E represents edges connecting two vertices if and only if the corresponding nets have a short circuit between them when placed the nets in a printed circuit board. It is known that the Hamiltonian graph has been occupied a field of applied graph theory, which is found in many real life problems. Hence it is necessary to study the various properties of Hamiltonian graph and the following theorem gives a direction to know the number of simple Hamiltonian graphs.

2.1.4. Strongly Regular Graph: It is found that the strongly regular graphs are the special types of graph with two parameters λ, μ with vertex v and this is k-regular. It is not possible to have a strongly regular graphs SRG (v, k λ, μ) or there is no rule how to construct it from the graphical partition of all even numbers due to the parameter λ and μ.

3.1. Theorems:

3.1.1.Theorem : The total number of simple Hamiltonian graph of vertex $n \geq 6$ is $(n^2 -3n +2)/2$ for the graphical partition of the even number k where $2i+10 \leq k \leq m\,(m+1)$ for $i \geq 1$ with simultaneous changes of $m \geq 5$ and $n \geq 6$.

Proof: The theorem is true for n=6, i=1 and m=5. That is for the graphical partition of the even number k where $12 \leq k \leq 30$, and there are 10 number of Hamiltonian simple graphs, which are obtained as follows:

Let us consider a graph of six vertices [n=6]. It is clear that 12 has the partition 2+2+2+2+2+2 which gives a graph of six vertex of degree two each and this is simple and Hamiltonian. Now join any two vertices by one edge which gives two vertices of degree three and other four vertices are of degree two. Then we have 3+3+2+2+2+2=14 , which is the partition of the number 14 and this gives a graph of six vertices and this is also simple and Hamiltonian [Figure-1]

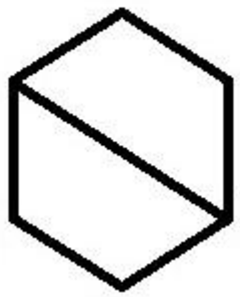

Figure - 1.

Continuing the process of introduction of new edges, joining the vertices, till the completion of construction of complete graph, we have 10 simple Hamiltonian graphs for the number $12 \leq k \leq 30$. Again we know that the complete graph of 6 vertex have 15 edges. Hence we must consider 15-6= 9 times of addition of edges to construct simple Hamiltonian graph and this 9 times of construction of various graphs and the first graph of six vertices altogether give the 10 number of simple Hamiltonian graphs. Hence for the graphical partition of other even numbers, we have different types of simple Hamiltonian graphs till the completion of complete graphs K_n for $n \geq 6$, using the formula $\{n(n-1)/2 - n\} + 1$ and this gives the total number of simple Hamiltonian graphs as $(n^2 -3n+2)/2$, which completes the proof.

Remarks1: It is true that there may exist other structure of graphs except Hamiltonian graph for other graphical partitions of the same even number. But, our theorem gives only the total number of Hamiltonian simple graphs.

3.1.2.**Theorem**: There does not exist any complete graphs for the graphical partitions of all even numbers except the even numbers of the form (n+1) (n+2) for $n \geq 4$.

Proof: If possible, we suppose that there exist a complete graph K_n for $n \geq 5$ for the graphical partition of even number, say L, and this L is the even number other than the even number of the form (n+1) (n+2) for $n \geq 4$. Then it is clear that the even number L must be a factor of the form (n+1) (n+2), as this factor gives an even number and (n+1) (n+2)/2 gives the number of edges of K_n for $n \geq 5$ [9]. But it is simple to prove that L can not be factored as stated above.[if L=28 say , them 28 can not be factored as (n+1) (n+2).Hence there does not exist complete graph for the graphical partition of L, which completes the proof.

Remarks2: From the above theorem it is easy to show from what types of even numbers one can find the existence and non-existence of complete graphs.

Hence if one takes the even numbers say, 32,34, 36, 38, 40 or 44, 46, 48, 50, 52, 54 ……… , and study the graphical partition, then immediately it will help to know that there does not exist graphical partition from which complete graph exist.

3.1.3.Theorem: There exist different type of colorings for the simple Hamiltonian graph existed from the graphical partition of the even number 2n+10 for $n \geq 1$

Proof: We prove the theorem for the graphical partition of even numbers 2n+10 for $n \geq 1$.Let n=1.Then the graphical partition of 12=2+2+2+2+2+2(when n=1), gives a regular graph of degree 2 with six vertices and this graph is a two colorable graph. We have another graphical partition of 12 which is 3+3+3+3=12. We see that one can draw a complete graph of four vertices with degree 3 which is four colorable. Further, we see that the graphical partition of 12 as 3+3+2+2+2=12, which gives a graph of five vertices with degrees 3,3,2,2 and 2 and when constructed it immediately shows that it requires three colors. Hence, continuing the process of obtaining different types of simple Hamiltonian graphs from graphical partitions of the number 2n+10 for n=1, we see that there exist different types graph colorings from different types of graphical partition of numbers and this is true.

Now, we suppose that the result is true for n=k. Hence there exist different colorings for the graph obtained from the graphical partitions of number 2k+10. It is found that the number 2k+12 is an even number for $k \geq 0$.Now we see that the even number 2(k+1)+10= 2k+2 +10= 2k+12 gives different types of graphs and different types of colorings which is true and this completes the proof.

3.1.4.Theorem: For the even numbers 4n+10 for $n \geq 5$, there exist different coloring of the complete graph $K_{4L+2} - \{e_i\}$ i=1,2,3,4,.. for some fixed values of $n \geq 5$ with simultaneous changes of $L \geq 1$

Proof: We know that the complete graph K_{4L+2} for $L \geq 1$ is 4L+2 colorable. It is clear that when we remove one edge from this complete graph then the graph thus obtained is also 4L+2-1=4L+1 colorable. Continuing the same process of deletion of edges from the complete graph we have different coloring for the graph K_{4L+1}-$\{e_i\}$,i=1,2,3,4…..[for example, the fixed values of n=5,20 and 43 we have K_6,K_{10}, K_{14} with simultaneous changes of L=1,2 and 3.]. This completes the proof.

3.1.5.Theorem: The graphical partition of the form i + i + i…+ m + m + m…of the number 2(i x m) give the bipartite graph of the form $K_{i,m}$ for $i \geq 3$ with simultaneous changes of $m \geq 5$.

Proof: We know that a bipartite graph is a graph which partitions the set of vertices into two sets such that one end of edges are in one set and the other end lies in other sets. Here we consider the value of $m \geq 5$ and $i \geq 3$. If m=5 and i=3, then we have the even numbers 2(3x5)=30 , and there exist a bipartite graph of the form $K_{3,5}$. Hence for any values of $m \geq 5$ and $i \geq 3$, there always exist bipartite graphs.

Corollary: The graphical partition of the form i + i + i +…gives the bipartite graph $K_{i,i}$ for $i \geq 3$ where 2i are the number of vertices and i^2 are the number of edges.

Proof : From definition (2.1.1) It is clear that there are two graphs.

4.1. Theoretical Result for formation of Line of Sight Graphs obtained from Some graphical partitions of even numbers with special reference to printed circuit board of the form m x m for m≥6:

It is interesting to note that the line of sight graph plays an important role in calculating the short circuit in a printed circuit board. In this case, the coloring properties mainly chromatic number of the line of sight graph helps. It has also been known that any planar graph is a line of sight graph and therefore certain types of circuit board exist. The lay out of line of sight graph in a floor plan is an important area of VLSI design technology.

The following theorem has been considered for our purposes.

4.1.1.Theorem:

There always exist planar simple graph (not acyclic but Hamiltonian) for the graphical partitions of the number k where $2i+10 \leq k \leq 4n+20$ for $i \geq 1$, $n \geq 1$ with simultaneous changes of the vertex $m \geq 6$,for the printed circuit board of the form m x m and they are the line of sight graph and the number of such graph are ≥16**.**

Proof: Let there be a printed circuit board m x m where m ≥6. We first prove it taking the different types of even numbers and their graphical partitions between 12 and 24 which is found putting i=1 and n=1. Let us consider the printed circuit board for m=6 which is shown in figure-2, with six nets N1, N2 ,N3,N4,N5 and N6.

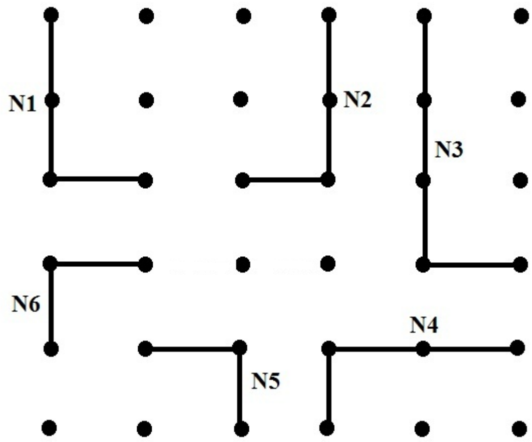

Figure-2

Then a line of sight graph of six vertices and six edges can be constructed which is Hamiltonian and regular of degree two, as shown in figure-3.

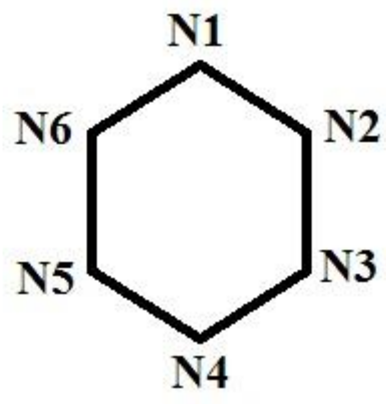

Figure-3

It is interesting to note that the line of sight graph is obtained from the graphical partition of even number 12=2+2+2+2+2+2. It has been found that there are $^6C_2$ =15 tests needed for testing short circuit in the board. But, from the application of graph coloring properties one requires only two tests, where only two partitions { N1,N3, N5} and {N2,N4,N6} contains two colors .

Let us join the vertices N6 and N3 by an edge, and thereafter we have the structure of new graph as shown in figure-4.

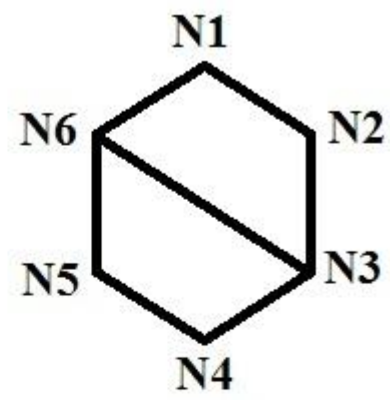

Figure-4

This graph is also a Hamiltonian graph, which is the graphical partition of the even number 14=2+2+2+2+3+3.After joining the edge, there arises a question? Is this graph a line of sight graph? The answer is yes and for yes answer we only require to change the net pattern. Hence the new pattern of net N6 should be imposed with the extended dotted net connecting two another grid points as shown in figure-5.

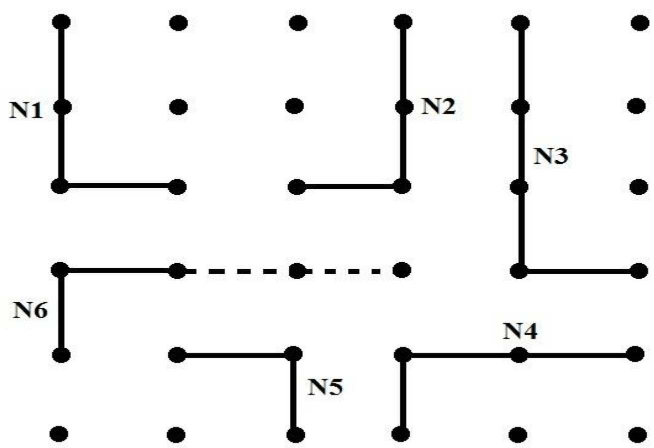

Figure-5

It is found that the graph of figure-4 is the line of sight graph of the net pattern of figure-5 and it is two colorable and hence two test need for short circuit testing. Continuing the process of introduction of various patterns of nets to the printed circuit board 6 x 6,it can be shown that the following graphical partitions

12=2+2+2+2+2+2, 14=2+2+2+2+3+3,

16=2+2+3+3+3+3, 18=3+3+3+3+3+3,

18=2+3+3+3+3+4, 18=2+2+3+3+4+4,

20=4+4+3+3+3+3, 20=4+4+4+2+3+3,

22=4+4+4+4+4+2, 22=4+4+4+4+3+3,

24=3+3+4+4+5+5, 24=4+4+4+4+4+4,

24=3+4+4+4+4+5, 24=2+5+5+z4+4+4,

24=3+3+3+5+5+5, 24=3+3+4+4+5+5

Which give different structures of graphs, those are nothing but the line of sight graphs of 6×6 printed circuit board.

It can be shown that among different number of partitions of the even numbers between 12 and 24 there are only sixteen graphical partitions and they are line of sight graphs. All the line of sight graphs are found to be planar**. p**roceeding in the same way for the values of n=2 , i=2, we have the even numbers between 14 and 28,and from various graphical partitions, various types of line of sight graphs obtained from the various patterns of nets for the printed circuit board of the form 7 x 7 (m=7). All the line of sight graphs are planar and the short circuit test can be done applying the coloring properties of graph. Hence increasing the values for n=3, i=3 ………………we have different types of even numbers and different types of line of sight graphs which are actually obtained from various nets patterns for the printed circuit board 8 x 8 ………………., which completes the proof.

5.1. Proposition

5.1.1.Proposition: The strongly regular graphs exist only for some particular values of n greater than 4, for the graphical partition of the even numbers 4n+10/4n+12

Proof: We know that a strongly regular graph G (v, k, λ, μ) is a graph of v vertices and k regular with the properties that for any two adjacent vertices x, y there is λ vertices adjacent to x and y and for any two non-adjacent vertices z, t there is μ vertices adjacent to z and t.

Suppose n=4, Then we have two numbers 26 & 28 and from these two numbers we cannot have any strongly regular graph. But, when we put n=5, then we have another two numbers 30 & 32. From the graphical partition of 30=3+3+3+3+3+3+3+3+3+3, we have Petersen graph , which is strongly regular graph of the form (10,3,0,1) and similarly we have another strongly regular graph (8,4,0,4) from the graphical partition of 32. Hence for some selected values of n ≥4, we can have some strongly regular graph**.**

5.1.2.Proposition: The graphical partitions of every even numbers larger than 12 is technically applicable (TA) if there exist at least one graphical partition which gives only Hamiltonian graph and this graph is the line of sight graph obtained from net patterns placed in a printed circuit board.

Example**:** The graphical partition of 12= 2+2+2+2+2+2 is TA since it gives a circuit which is Hamiltonian but the graphical

5.1.3.Conjecture: From the two different graphical partition of an even number ≥80, there exist two different types of strongly regular graphs of different vertices.

Proof: It is found that for the number 80, we have two different types of strongly regular graphs (10,8, 6, 8) which is known as 5- cocktail party graph and (16,5,0,2) which is known as clebsch graph. For other even numbers greater than 80, we can search the other pair of strongly regular graphs

6.1. Conclusion:

We conclude from this article that the graph theory, though we suppose it as an application oriented subject where there are so many NP-complete problems related with many branches of science and technology has important relationship with number theory specially in even numbers.

REFERENCES:

[1] Ryser, Herbert., " Combinatorial Mathematics" (Carus Monograph No .4) MAA, 1963.

[2] Kalita, B., "Proof of goldbach conjecture and related conjecture of primes" Bull. Pure & Applied Sciences, Vol.25E (NO-2) ,2006, pp 451-459.

[3] Kalita, B., "Certain forms of even numbers and their properties" Global journal of pure and applied mathematics, Vol. 5 Number 1(2009) ,pp 53-58.

[4] [4]. Kalita, B.,"Some conjectural problems of primes and partial solution of it" Abstracted, International conference on recent advances in mathematical sciences, I.I.T, Kharagpur,2000, dec 20-22.

[5] Kalita, B., "GRAPH AND GOLDBACH CONJECTURE" International journal of pure and Applied Maths, Vol.4.no.1,2013,pp531-546.

[6] Edgar G. Goodaire, Michael M Parmenter.,"Discrete mathematics with graph theory" Pearson education, 2008, pp 136-144**.**

[7] Alexvand. Etal "Goldback-suspect: a step further" on line article, Language: Dutch, source: kennislink, Date : 27/4/2007.

[8] Harary, F., "Graph theory" Narosa publishing house, 1988, pp 57-63.

[9] George, E. Andrews " Number Theory" HPC, pp 149-159.

[10] Kalita, B., etal "Some properties of graphical partitions" International journal of applied Engineering Research, Vol.4, Number10 (2009) pp1851-1856.

[11] Kalita, B.,"Lower bound of crossing and some properties of sub-graphs of complete graph related VLSI Design" Proc. International multi-conference of engineers & computer scientists/2009, Hong Kong, pp547- 555.

[12] Kalita, B.,etal "crossing and thickness of special type of non-planar graph" Jour. Pure Maths, Vol.24, 2007, pp39-47

[13] Kalita, B., etal "Line graphs of some kind of special graphs", Bull. Pure & applied sciences, Vol.27E (No-2), 2008, pp323-330.

[14] Kalita,B., "Sub-graphs of complete graphs", Proc. International conference on foundation of computer science/2006, Las Vegas, U.S.A, pp 71-77.

[15] Kalita, B.," Some investigation on graph theory"(Ph..D) thesis) Finance India, Vol. xix.No 4, Dec/2005, pp1430-1438.

[16] R.C.Bose., " Strongly regular graphs, partial geometries and partially balanced design"Pacific J. Math, 13, 1963, pp 389-419**.**

[17] J.H.Vant Lint & R.M.Wilson " A course in combinatorics" Cambridge University Press., 1996.

[18] X.Hubaut., " Strongly regular graphs" Discrete Maths., 13. 1975, pp357-381.

[19] J.H. Vant & Schrijver., " Construction of strongly regular graphs, two weight codes and partial geometries by finite fields" Combinatorica, 1, 1981, pp63-73.

[20] Kalita, B., "Construction of strongly regular graphs from TEEP graphs". Bull. Pure & applied sciences,Vol.22E (No-1).pp113-117, 2003.

[21] Erdos, P. etal., "Graphs with prescribed degrees of vertices" (Hungarian) Mat.Lapok11 (1960) , pp 264-274.

[22] Hakimi, S., "On the realizability of a set of integers as degrees of vertices of a graph" J. SIAM, Appl. Math. 10 (1962), pp 496-506.

[23] Havel, V., "A remark on the existence of finite graphs" (Hungarian) Casopis pest . Math. 80 (1955), pp 477-480.

[24] Kalita, B, "A new set of Non-planar graphs" Bull. Pure & applied Sc. Vol.24E (No-1) 2005, pp29-38.

[25] Kalita, B. etal " Consecutive Even Number Finding Graph Related to Gold bach Conjecture" International Journal of Pure & Applied Mathematics. Vol.89, No.1 2013, 55-70

[26] MichaelR.Garey, etal " Application of graph coloring to printed circuit testing " IEEE Transaction on circuit and System" vol.CAS-23,1976, NO 10, PP591-599.

[27] http://mathworld.wolfram.com/strongly regular graph.html.