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# Simplex Method and Non-Linear Programming

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**Abstract**

In this paper, we have introduced a technique that is used to solve a set of nonlinear programming problems by simplex method. This technique also helps to provide the solution of linear programming problems. It also provides better alternate solutions to mathematical programming problems. Numerical illustrations are also given to support the technique.

**Keywords:** Mathematical Programming, Lingo software, Nonlinear Programming, Simplex Method.

## Introduction:

Mathematical programming problems having nonlinear/linear objective functions and linear/nonlinear constraints are called nonlinear programming problems. Large number of methods are available to solve different types of non-linear programming problems which depend on the type of objective function and the type of constraints [2,6]. It becomes very difficult to solve nonlinear programming problems with large number of variables and constraints, large number of iterations can be used to reach the solution, no doubt there are software that are used to solve the nonlinear programming problems but still there is the scarcity of methods that can be used to solve most of non-linear programming problems as in case of linear programming, most of the problems can be solved by well-known methods like Simplex Method, etc. [3,5].

There are relationships between mathematical programming and regression analysis, experimental design and testing of hypothesis etc [1,4] in which we can solve the problems by alternate methods. Here we present the technique which is totally linked internal to mathematical programming. We select a set of mathematical programming problems and solve any one of them and the solution of rest of the problems comes automatically, “n” number of mathematical programming problems can be produced and solution of one can be found by any methods like simplex method for linear programming and the solutions of the rest "(n-1)" problems are obtained directly from these solutions. Some special cases of non-linear programming can be solved by this technique.

One more advantage of this technique is that it provides a alternate solution to the mathematical programming problem which is much better than other alternate solutions provided by other methods. The Simplex method solves linear programming problems but fails to supply the solution in "n" variables in a system with 'n' decision variables and 'm' constraints. It always provides the "m" variable solution ($m < n$) as rest of the variables (treated as non basic variables) are set equal to zero. By this method we can get the "n" variable solution. In a mathematical programming problem, having "n" controlling variables, it is always preferable to have a solution having all the controlling variables non zero in the solution to the one with only "m" variable solution.

**Method**

We consider a Mathematical Programming Problem of type:

**(A) Optimize** $F(x) = \sum_{j=1}^{n} Xj^{k}$

Subject to

$$a_{ij}\, x_j^{p} \leq b_i$$
$$x_j \geq 0$$

1. $k = p = 1$, the problem becomes linear programming problem with optimal solutions $x_j^*$
2. $k = 1$ and $p > 1$, a set of nonlinear programming problems formed for different values of "p", the solution of these problems can be found by using the solutions of (i) which is $(x_j^*)^{1/p}$. It means that we can get the solutions of all nonlinear programming problem formed by the (A) by simplex method solution.

**(B) Optimize** $F(x) = \sum_{j=1}^{n} Xj^{Kj}$

Subject to

$$a_{ij} x_j^{Kj} \leq b_i$$
$$x_i \geq 0$$

Similarly here also we can find the solution of all nonlinear programming problems formed by (B) by using simplex method.

Put $x_j^{kj} = y_j$ the problem (B) becomes linear programming problem and can be solved by any linear programming technique, the solution of the new formed linear programming problem say $(y_j^*)$, form these solutions we can find the solution of all nonlinear programming problems formed by (B) which is $(y_j^*)^{1/kj}$.

**Example 1**

Max F(x) = $X_1^{\,k} + X_2^{\,k} + X_3^{\,k}$

**Subject to**

$$2X_1^p + 3X_2^p \le 8$$

$$2X_1^p + 5X_3^p \le 10$$

$$3X_1^p + 2X_2^p + 4X_3^p \le 15$$

$$X_i \ge 0$$

When k = p =1, the mathematical programming problem becomes a linear programming problem and it can be easily solved by Simplex Method which provides the solution to the above problem

$X_1 = 2.1707$ $X_2 = 1.2195$ $X_3 = 1.5121$

When p > 1 the mathematical programming is a nonlinear programming problem and can be solved by many methods/software, but we find the solution of nonlinear programming problems by the solution of linear programming problem which is equal to the $p^{th}$ root of solution of linear programming problem. Thus,

$X_1^* = (2.1707)^{1/p}$ $X_2^* = (1.2195)^{1/p}$ $X_3^* = (1.5121)^{1/p}$

where $X_i^*$ is the solution of any non linear programming problem of any order depending on the value of "p". When p=5, the example 1 changes into example (1.1) given below:

**Example 1.1**

Max F(x) = $X_1 + X_2 + X_3$

**Subject to**

$$2X_1^5 + 3X_2^5 \le 8$$

$$2X_1^5 + 5X_3^5 \le 10$$

$$3X_1^5 + 2X_2^5 + 4X_3^5 \le 15$$

$$X_i \ge 0$$

The solution of the example (1.1) is

$X_1 = (2.1707)^{1/5}$ $X_2 = (1.2195)^{1/5}$ $X_3 = (1.5121)^{1/5}$

$X_1 = 1.16767$ $X_2 = 1.04048$ $X_3 = 1.0862$

**Example 2**

$$\text{Max } Z = 4X_1^2 + 5X_2^3 + 9X_3^4 + 11X_4^5$$

**Subject to**

$$X_1^2 + X_2^3 + X_3^4 + X_4^5 \leq 15$$
$$7X_1^2 + 5X_2^3 + 3X_3^4 + 2X_4^5 \leq 120$$
$$3X_1^2 + 5X_2^3 + 10X_3^4 + 15X_4^5 \leq 100$$
$$X_i \geq 0$$

In order to get the solution of example (2) we solve the linear programming problem related to example (2) which is example (2.2).

**Example 2.2**

$$\text{Max } Z = 4X_1 + 5X_2 + 9X_3 + 11X_4$$

**Subject to**

$$X_1 + X_2 + X_3 + X_4 \leq 15$$
$$7X_1 + 5X_2 + 3X_3 + 2X_4 \leq 120$$
$$3X_1 + 5X_2 + 10X_3 + 15X_4 \leq 100$$
$$X_i \geq 0$$

$X_1 = 7.14$ $X_2 = 0$ $X_3 = 7.85$ $X_4 = 0$

The solution of original example (2) is

$X_1 = (7.14)^{1/2}$ $X_2 = (0)^{1/3}$ $X_3 = (7.85)^{1/4}$ $X_4 = (0)^{1/5}$

$X_1 = 2.6726$ $X_2 = 0$ $X_3 = 1.6742$ $X_4 = 0$

**Example 3**

$$\text{Max } Z = X_1 + X_2 + X_3 + X_4$$

**Subject to**

$$X_1 + X_2 + X_3 + X_4 \leq 15$$
$$7X_1 + 5X_2 + 3X_3 + 2X_4 \leq 120$$
$$3X_1 + 5X_2 + 10X_3 + 15X_4 \leq 100$$
$$X_i \geq 0$$

By using Simplex Method, we have the following solution to the above problem:

$X_1 = 15$ $X_2 = 0$ $X_3 = 0$ $X_4 = 0$

The problem has four (4) controlling variables and the simplex method provides the solution (15,0,0,0) means only one controlling variable plays active part while others are zero. Now we apply same technique in reverse direction i,e, here we convert linear programming problem (3) into non linear programming problem (3.1) and find the solution of original problem (3).

**Example 3.1**

Max Z = $X_1 + X_2 + X_3 + X_4$

**Subject to**

$$X_1^2 + X_1^2 + X_1^2 + X_1^2 \leq 15$$

$$7X_1^2 + 5X_1^2 + 3X_1^2 + 2X_1^2 \leq 120$$

$$3X_1^2 + 5X_1^2 + 10X_1^2 + 15X_1^2 \leq 100$$

$$X_i \geq 0$$

The solution of this problem by using LINGO software is as

$X_1 = 2.33276$ $X_2 = 2.11821$ $X_3 = 1.72223$ $X_4 = 1.45097$

The solution of original linear programming problem (3) which is the squares of the solution of example (3.1)

$X_1 = 5.4$ $X_2 = 4.5$ $X_3 = 3.0$ $X_4 = 2.1$

This way, we can find the solution of any problem in this class either by solving linear programming problem or solving nonlinear programming problem.

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