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Intuitionistic Fuzzy Ideals in Hyper BCI-Algebras

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Abstract

In this paper we introduce distributive hyper BCI-ideals and the concept of intuitionistic fuzzy set is applied to investigate the relations between intuitionistic fuzzy distributive hyper BCI-ideals and the distributive hyper BCI-ideals of a hyper BCI-algebra.

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Introduction

The hyper structure theory was introduced by F. Marty [13] in 1934. Y.B. Jun et al. applied this concept to BCK-algebras [7] and Xiao Xin Long introduced a hyper BCI-algebra [12], as generalization of a BCI-algebra. Different types of hyper BCI-ideals are also defined in [12]. The proper hyper BCI-algebras which coincide with hyper BCK-algebras and p-semisimple BCI-algebras of order 3 are investigated in [5]. In [8-10] the fuzzification of hyper BCK-ideals is discussed and the related results are developed. The notion of Bi-polar-valued fuzzy hyper subalgebra (briefly BFHS) of a hyper BCI-algebra based on Bi-polar-valued fuzzy set and related properties are established in [6]. Further, Bi-polar-valued fuzzy characteristic hyper subalgebra is also stated.

The idea of "intuitionistic fuzzy set" was first published by Atanassov [1, 2] as a generalization of the notion of fuzzy sets. After that many researchers considered the intuitionistic fuzzification of ideals and subalgebras in BCK/BCI-algebras. In this paper we introduce distributive hyper BCI-ideals and the concept of intuitionistic fuzzy set is applied to investigate the relations between intuitionistic fuzzy distributive hyper BCI-ideals of a hyper BCI-algebra.

Preliminaries

By a BCI-algebra we mean an algebra $(X, *, 0)$ type $(2, 0)$ satisfying the axioms:

(i) $((x*y)*(x*z))*(z*y)=0,$

(ii) $(x*(x*y))*y=0,$

(iii) $x*x=0,$

(iv) $x*y=0$ and $y*x=0$ imply $x=y$, for all $x,y,z\in X.$

If a BCI-algebra X satisfies the following identity:

(v) $0*x=0,$ for all $x\in X$,

then X is called a BCK-algebra. Any BCI/BCK-algebra X satisfies the following axioms:

(vi) $x*0=x,$

(vii) $x\le y$ imply $x*z\le y*z$ and $z*y\le z*x,$

(viii) $(x*y)*z=(x*z)*y,$

(ix) $(x*z)*(y*z)\le x*y,$ for all $x,y,z\in X.$

In a BCI-algebra, we can define a partial ordering "$\le$" by $x\le y$ if and only if $x*y=0.$ A subset S of a BCK/BCI-algebra X is called a subalgebra of X if $x*y\in S$ for all $x,y\in S.$ An ideal of a BCK/BCI-algebra X is a subset I of X containing 0 such that if $x*y\in I$ and $y\in I$ then $x\in I.$ Note that every ideal I of a BCK/BCI-algebra X has the following property:

$x\le y$ and $y\in I$ imply $x\in I.$

In what follows, X will denote a BCI-algebra unless otherwise specified.

An intuitionistic fuzzy set A in a non-empty set X is an object having the form

$$A=\{\langle x,\mu_A(x),\upsilon_A(x)\rangle \setminus x\in X\},$$

where the functions $\mu_A : X\to[0,1]$ and $\upsilon_A : X\to[0,1]$ denote the degree of membership (namely $\mu_A(x)$) and the degree of non membership (namely $\upsilon_A(x)$) of each element $x\in X$ to the set A respectively, and $0\le\mu_A(x)+\upsilon_A(x)\le 1$ for all $x\in X$.

Such defined objects are studied by many authors (see for Example two journals:

1. Fuzzy Sets and 2. Notes on Intuitionistic Fuzzy Sets) and have many interesting applications not only in mathematics (See Chapter 5 in the book [2]).

Let X be a non empty set endowed with a hyper operation $"\circ"$, that is $"\circ"$, is a function from $X \times X$ to $P^*(X) = P(X) - \{\phi\}$. For two subsets A and B of X, denote by $A \circ B$ the set

$\cup_{a \in A, b \in B}\, a \circ b$

Definition 2.1 [14]: By a fuzzy set μ in X, we mean a function $\mu : X \to [0,1]$.

Definition 2.2 [12]: By a hyper BCI-algebra, it is meant a nonempty set X endowed with a hyper operation $"\circ"$ and a constant 0 satisfying the following axioms:

(I) $(x \circ z) \circ (y \circ z) << x \circ y,$

(II) $(x \circ y) \circ z = (x \circ z) \circ y,$

(III) $x << x,$

(IV) $x << y$ and $y << x$ imply $x = y,$

(V) $0 \circ (0 \circ x) << x, x \neq 0,$

for all $x, y, z \in X$, where $x << y$ is defined by $0 \in x \circ y$ and for every $A, B \subseteq X$, $A << B$ is defined by for all $a \in A$, there exists $b \in B$ such that $a << b$. In such case $"<<"$ is called the hyper order in X.

Definition 2.3 [12]: Let A be a non-empty subset of a hyper BCI-algebra X. Then A is said to be a hyper BCI-ideal of X if

(1) $0 \in A,$

(2) $x \circ y << A$ and $y \in A$ imply $x \in A$ for all $x, y \in X$.

Definition 2.4: For an intuitionistic fuzzy set A in X and $t, s \in [0,1]$, the set

$A_{\langle t,s \rangle} = \{x \in X \,/\, \mu_A(x) \geq t, \upsilon_A(x) \leq s\}$ is called a level subset of A.

Definition 2.5 [12]: Let $(H, \circ)$ be a hyper BCI-algebra. Then $(H, \circ, 0)$ is a BCI-algebra if and only if $H = S_I = \{x \in H \,/\, x \circ x = \{0\}\}$.

Definition 2.6 [12]: Hypergroup is defined as a hyperstructure $(X, \cdot)$ such that the following axioms hold:

(3) $(x \cdot y) \cdot z = x \cdot (y \cdot z)$ for all $x, y, z \in X$,

(4) $x \cdot X = X \cdot x = X$ for all $x \in X$.

Where $x.y = y \circ (0 \circ x)$ for all $x, y \in X$.

Theorem 2.7 [12]: Let $(X, \circ)$ be a hyper BCI-algebra and satisfy the following conditions:

(5) $x \in a \circ (a \circ x)$,

(6) $x \circ (0 \circ y) = y \circ (0 \circ x)$.

Then $(X, \cdot)$ is a hypergroup.

Definition 2.8: An intuitionistic fuzzy relation on any set X is an intuitionistic fuzzy set

$B = \langle \mu_B, \upsilon_B \rangle$ where $\mu_B : X \times X \to [0,1]$ and $\upsilon_B : X \times X \to [0,1]$.

Definition 2.9: If B is an intuitionistic fuzzy relation on a set X and A is an intuitionistic fuzzy set in X, then B is an intuitionistic fuzzy relation on A if

$\mu_B(x, y) \le \min\{\mu_A(x), \mu_A(y)\}$

and $\upsilon_B(x, y) \ge \max\{\upsilon_A(x), \upsilon_A(y)\}, \forall x, y \in X$.

Definition 2.10: If A is an intuitionistic fuzzy set in a set X, the strongest intuitionistic fuzzy relation on X is an intuitionistic fuzzy relation of X is $B_A = \langle (\mu_B)_{\mu_A}, (\upsilon_B)_{\upsilon_A} \rangle$, given by

$(\mu_B)_{\mu_A}(x, y) = \min\{\mu_A(x), \mu_A(y)\}$

and

$(\upsilon_B)_{\upsilon_A}(x, y) = \max\{\upsilon_A(x), \upsilon_A(y)\}, \forall x, y \in X$.

Definition 2.11: Let A & B be intuitionistic fuzzy sets in a set X. The Cartesian product of A and B is defined by

$(\mu_A \times \mu_B)(x, y) = \min\{\mu_A(x), \mu_B(y)\}$

$(\upsilon_A \times \upsilon_B)(x, y) = \max\{\upsilon_A(x), \upsilon_B(y)\}$ for all $x, y \in X$.

Definition 2.12 [6]: Let X and Y be hyper BCI-algebras. A mapping $f : X \to Y$ is called a hyper homomorphism, if

(7) $f(0) = 0$,

(8) $f(x \circ y) = f(x) \circ f(y)$.

3. Distributive Hyper BCI-Ideals

Definition 3.1: A non-empty set A of a hyper BCI-algebra X is called a distributive hyper BCI-ideal if it satisfies (1) and

(9) $((x \circ z) \circ z) \circ (y \circ z) << A$ and $y \in A \Rightarrow x \in A$.

Example 3.2: Consider a hyper BCI-algebra $X = \{0, a, b, c\}$ with the following Cayley table.

$\circ$	0	a	b	c
0	{0,a}	{0,a}	{0,a}	{0,a}
a	{a}	{0,a}	{0,a}	{0,a}
b	{b}	{b}	{0,a,b}	{0,a,b}
c	{c}	{c}	{c}	{0,a,c}

$\{0,a\},\{0,a,b\}$ are the only hyper BCI-ideals in X which are also distributive hyper BCI-ideals of X.

Example 3.3: Consider a hyper BCI-algebra $X=\{0,a,b,c\}$ with the following Cayley table.

$\circ$	0	a	b	c
0	{0}	{0}	{b}	{b}
a	{a}	{0,a}	{b}	{b}
b	{b}	{b}	{0}	{0}
c	{c}	{b,c}	{a}	{0,a}

It can be easily checked that $A=\{0,a\}$ is a hyper BCI-ideal but A is not a distributive hyper BCI-ideal of X because $((c\circ b)\circ b)\circ(0\circ b) << A$ and $0\in A$ implies $c\in A$ which is a contradiction.

4. Intuitionistic Fuzzy Hyper BCI-Ideals

Definition 4.1: An intuitionistic fuzzy set A in a hyper BCI-algebra X is an intuitionistic fuzzy hyper BCI-ideal if

(10) $x << y$ implies $\mu_A(y) \le \mu_A(x)$ and $\upsilon_A(y) \ge \upsilon_A(x)$,

(11) $\mu_A(x) \ge \min\{\inf_{u\in(x\circ y)} \mu_A(u), \mu_A(y)\}$,

(12) $\upsilon_A(x) \le \max\{\sup_{u\in(x\circ y)} \upsilon_A(u), \upsilon_A(y)\}, \forall x, y \in X$.

Definition 4.2: An intuitionistic fuzzy set A in X is called an intuitionistic fuzzy distributive hyper BCI-ideal if satisfies (10) and

(13) $\mu_A(x) \ge \min\{\inf_{v\in(((x\circ z)\circ z)\circ(y\circ z))} \mu_A(v), \mu_A(y)\}$,

(14) $\upsilon_A(x) \le \max\{\sup_{v\in(((x\circ z)\circ z)\circ(y\circ z))} \upsilon_A(v), \upsilon_A(y)\}, \forall x, y, z \in X$.

Example 4.3: Consider a hyper BCI-algebra $X=\{0,a,b,c\}$ with the following Cayley table.

$\circ$	0	a	b	c
0	{0}	{0}	{b}	{b}
a	{a}	{0}	{b}	{b}
b	{b}	{b}	{0}	{0}
c	{c}	{b}	{a}	{0,a}

Define an intuitionistic fuzzy set A in X by $\mu_A(c) = \mu_A(b) = 0.2, \mu_A(a) = 0.4, \mu_A(0) = 0.6$ and $\upsilon_A(c) = \upsilon_A(b) = 0.6, \upsilon_A(a) = 0.2$ & $\mu_A(0) = 0.1$. It is routine to verify that A is an intuitionistic fuzzy hyper BCI-ideal of X but not an intuitionistic fuzzy distributive hyper BCI-ideal of X. Because

$$\mu_A(x) \geq \min\{\inf_{u \in (((x \circ z) \circ z) \circ (y \circ z))} \mu_A(u), \mu_A(y)\}$$

and $\upsilon_A(x) \leq \max\{\sup_{u \in (((x \circ z) \circ z) \circ (y \circ z))} \upsilon_A(u), \upsilon_A(y)\}$ are not satisfied for $x = b, y = a, z = c.$

Example 4.4: Consider a hyper BCI-algebra $X = \{0, a, b, c\}$ with the following Cayley table.

$\circ$	0	a	b	c
0	{0,a}	{0,a}	{b}	{b}
a	{a}	{0,a}	{b}	{b}
b	{b}	{b}	{0,a}	{0,a}
c	{c}	{b}	{a}	{0,a}

Define an intuitionistic fuzzy set A in X by $\mu_A(a) = \mu_A(c) = \mu_A(b) = 0.2, \mu_A(0) = 0.4$ and $\upsilon_A(a) = \upsilon_A(c) = \upsilon_A(b) = 0.6$ & $\mu_A(0) = 0.2$. It is routine to verify that A is an intuitionistic fuzzy distributive hyper BCI-ideal as well as an intuitionistic fuzzy hyper BCI-ideal of X.

Example 4.5: Consider a hyper BCI-algebra $X = \{0, a, b, c\}$ with the following Cayley table.

$\circ$	0	a	b	c
0	{0,a}	{0,a}	{b}	{b}
a	{a}	{0,a}	{b}	{b}
b	{b}	{b}	{0,a}	{0,a}
c	{c}	{b,c}	{a}	{0,a}

Define an intuitionistic fuzzy set A in X by $\mu_A(c) = 0.1, \mu_A(b) = 0.2, \mu_A(a) = 0.3, \mu_A(0) = 0.5$ and $\upsilon_A(c) = 0.7, \upsilon_A(b) = 0.5, \upsilon_A(a) = 0.4$ & $\mu_A(0) = 0.3.$ It can be easily evaluated that A is an intuitionistic fuzzy hyper BCI-ideal of X but A is not an intuitionistic fuzzy distributive hyper BCI-ideal of X. Since

$$\mu_A(c) \geq \min\{\inf_{u \in ((c \circ b) \circ b) \circ (a \circ b)} \mu_A(u), \mu_A(a)\}$$

and $\upsilon_A(c) \leq \max\{\sup_{u \in ((c \circ b) \circ b) \circ (a \circ b)} \upsilon_A(u), \upsilon_A(a)\}$ are not satisfied.

Proposition 4.6: An intuitionistic fuzzy set A is an intuitionistic fuzzy hyper BCI-ideal of a hyper BCI-algebra X iff $A_{\langle t,s \rangle}$ is a hyper BCI-ideal of X whenever

$A_{\langle t,s \rangle} \neq \phi$ and $t, s \in [0,1]$.

Proof. Let A be an intuitionistic fuzzy hyper BCI-ideal of X and $A_{\langle t,s \rangle} \neq \phi$ for $t, s \in [0,1]$.

Since $\mu_A(0) \geq \mu_A(x) \geq t$ and $\upsilon_A(0) \leq \upsilon_A(x) \leq s$ for some $x \in A_{\langle t,s \rangle}$ therefore $0 \in A_{\langle t,s \rangle}$. Let $x, y \in X$ such that $x \circ y << A_{\langle t,s \rangle}$ and $y \in A_{\langle t,s \rangle}$ implies $\mu_A(y) \geq t$ and $\upsilon_A(y) \leq s$.

For any $v \in x \circ y$ there exists $w \in A_{\langle t,s \rangle}$ such that $v << w$ which implies that

$$t \leq \mu_A(w) \leq \mu_A(v)$$

and $s \geq \upsilon_A(w) \geq \upsilon_A(v)$ then

$$\mu_A(x) \geq \{\inf_{v \in x \circ y} \mu_A(v), \mu_A(y)\} \geq \{t, t\} \geq t$$

and $\upsilon_A(x) \leq \{\sup_{v \in x \circ y} \upsilon_A(v), \upsilon_A(y)\} \leq \{s, s\} \leq s$ implies that $x \in A_{\langle t,s \rangle}$.

So $A_{\langle t,s \rangle}$ is a hyper BCI-ideal.

Conversely, suppose $A_{\langle t,s \rangle}$ is a hyper BCI-ideal of X. For any $x \in X$ setting $\mu_A(x) = t$ and $\upsilon_A(x) = s$ then $x \in A_{\langle t,s \rangle}$.

Since $0 \in A_{\langle t,s \rangle}$ which implies that $\mu_A(0) \geq t, \upsilon_A(0) \leq s$ so

$$\mu_A(0) \geq \mu_A(x)$$

and $\upsilon_A(0) \leq \upsilon_A(x), \forall x \in X$.

For any $x, y \in X$, let

$$t = \{\inf_{w \in x \circ y} \mu_A(w), \mu_A(y)\}$$

and $s = \{\sup_{w \in x \circ y} \upsilon_A(w), \upsilon_A(y)\}$.

Then for $y \in A_{\langle t,s \rangle}$ and $u \in x \circ y$ we have

$$\mu_A(u) \geq \{\inf_{w \in x \circ y} \mu_A(w)\} \geq \{\inf_{w \in x \circ y} \mu_A(w), \mu_A(y)\} = t$$

and $\upsilon_A(u) \leq \{\sup_{w \in x \circ y} \upsilon_A(w)\} \leq \{\sup_{w \in x \circ y} \upsilon_A(w), \upsilon_A(y)\} = s$

which implies that $u \in A_{\langle t,s \rangle}$ so $x \circ y << A_{\langle t,s \rangle}$, $y \in A_{\langle t,s \rangle}$ implies that $x \in A_{\langle t,s \rangle}$. Therefore

$$\mu_A(x) \geq t = \{\inf_{w \in x \circ y} \mu_A(w), \mu_A(y)\}$$

and $\upsilon_A(x) \leq s = \{\sup_{w \in x \circ y} \upsilon_A(w), \upsilon_A(y)\}$.

Hence A is an intuitionistic fuzzy hyper BCI-ideal of X.

Proposition 4.7: Let A be an intuitionistic fuzzy hyper BCI-ideal of X then $x \circ y << z$ implies that $\mu_A(x) \geq \min\{\mu_A(z), \mu_A(y)\}$ and $\upsilon_A(x) \leq \max\{\upsilon_A(z), \upsilon_A(y)\}$.

Proof: Since A is an intuitionistic fuzzy hyper BCI-ideal then

$$\mu_A(x) \geq \min\{\inf_{u \in x \circ y} \mu_A(u), \mu_A(y)\} \geq \min\{\mu_A(z), \mu_A(y)\}$$

and $\upsilon_A(x) \leq \max\{\sup_{u \in x \circ y} \upsilon_A(u), \upsilon_A(y)\} \leq \max\{\upsilon_A(z), \upsilon_A(y)\}$ because $x \circ y << z$ implies that

$$\mu_A(z) \leq \mu_A(x \circ y)$$

and $\upsilon_A(z) \geq \upsilon_A(x \circ y)$.

Theorem 4.8: Every intuitionistic fuzzy distributive hyper BCI-ideal is an intuitionistic fuzzy hyper BCI-ideal.

Proof: Let A be an intuitionistic fuzzy distributive hyper BCI-ideal of X. Then

$$\begin{aligned}\mu_A(x) &\geq \min\{\inf_{u \in ((x \circ z) \circ z) \circ (y \circ z)} \mu_A(u), \mu_A(y)\} \\ &= \min\{\inf_{u \in ((x \circ 0) \circ 0) \circ (y \circ 0)} \mu_A(u), \mu_A(y)\} \\ &= \min\{\inf_{u \in x \circ y} \mu_A(u), \mu_A(y)\} \text{ for all } x, y \in X,\end{aligned}$$

and

$$\begin{aligned}\upsilon_A(x) &\leq \max\{\sup_{u \in ((x \circ z) \circ z) \circ (y \circ z)} \upsilon_A(u), \upsilon_A(y)\} \\ &= \max\{\sup_{u \in ((x \circ 0) \circ 0) \circ (y \circ 0)} \upsilon_A(u), \upsilon_A(y)\} \\ &= \max\{\sup_{u \in x \circ y} \upsilon_A(u), \upsilon_A(y)\} \text{ for all } x, y \in X.\end{aligned}$$

Hence A is an intuitionistic fuzzy hyper BCI-ideal of X. The converse of the Theorem 4.8 may not be true as seen in examples 4.3, 4.5.

Theorem 4.9: An intuitionistic fuzzy set A of a hyper BCI-algebra X is an intuitionistic fuzzy distributive hyper BCI-ideal of X iff for all $t,s\in[0,1]$, $A_{\langle t,s\rangle}$ is a distributive hyper BCI-ideal of X, whenever $A_{\langle t,s\rangle}\neq\phi$.

Proof: Suppose A is an intuitionistic fuzzy distributive hyper BCI-ideal of X and $A_{\langle t,s\rangle}\neq\phi$ for $t,s\in[0,1]$ since $\mu_A(0)\geq\mu_A(x)\geq t$ and $\upsilon_A(0)\leq\upsilon_A(x)\leq s$ for some $x\in A_{\langle t,s\rangle}$, we get $0\in A_{\langle t,s\rangle}$. If $((x\circ z)\circ z)\circ(y\circ z)<<A_{\langle t,s\rangle}$ and $y\in A_{\langle t,s\rangle}$ then for any $u\in((x\circ z)\circ z)\circ(y\circ z)$ there exists $v\in A_{\langle t,s\rangle}$ with $\mu_A(v)\geq t$ and $\upsilon_A(v)\leq s$ such that $u<<v$ which implies that $t\leq\mu_A(v)\leq\mu_A(u)$ and $s\geq\upsilon_A(v)\geq\upsilon_A(u)$. Since A is an intuitionistic fuzzy distributive hyper BCI-ideal therefore for all $x,y,z\in X$, we have

$$\mu_A(x)\geq\min\left\{\inf_{u\in((x\circ z)\circ z)\circ(y\circ z)}\mu_A(u),\mu_A(y)\right\}\geq\min\{t,t\}=t$$

and $\upsilon_A(x)\leq\max\left\{\sup_{u\in((x\circ z)\circ z)\circ(y\circ z)}\upsilon_A(u),\upsilon_A(y)\right\}\leq\max\{s,s\}=s$ because $y\in A_{\langle t,s\rangle}$ with $\mu_A(y)\geq t$ and $\upsilon_A(y)\leq s$.

So $x\in A_{\langle t,s\rangle}$. Hence $A_{\langle t,s\rangle}$ is a distributive hyper BCI-ideal of X. Conversely suppose that for all $t,s\in[0,1]$, $A_{\langle t,s\rangle}(\neq\phi)$ is a distributive hyper BCI-ideal which implies that $A_{\langle t,s\rangle}$ is a hyper BCI-ideal of X and hence A is a intuitionistic fuzzy hyper BCI-ideal of X. We have to prove that A is a intuitionistic fuzzy distributive hyper BCI-ideal of X. i.e. A has to satisfy

$$\mu_A(x)\geq\min\left\{\inf_{u\in((x\circ z)\circ z)\circ(y\circ z)}\mu_A(u),\mu_A(y)\right\}$$

and $\upsilon_A(x)\leq\max\left\{\sup_{u\in((x\circ z)\circ z)\circ(y\circ z)}\upsilon_A(u),\upsilon_A(y)\right\}$.

If not then

$$\mu_A(x)<\min\left\{\inf_{u\in((x\circ z)\circ z)\circ(y\circ z)}\mu_A(u),\mu_A(y)\right\}$$

and $\upsilon_A(x)>\max\left\{\sup_{u\in((x\circ z)\circ z)\circ(y\circ z)}\upsilon_A(u),\upsilon_A(y)\right\}$.

Taking t_0,s_0 satisfying

$$\mu_A(x_0)<t_0<\min\left\{\inf_{u\in((x_0\circ z_0)\circ z_0)\circ(y_0\circ z_0)}\mu_A(u_0),\mu_A(y_0)\right\}$$

and $\upsilon_A(x_0)>s_0>\max\left\{\sup_{u\in((x_0\circ z_0)\circ z_0)\circ(y_0\circ z_0)}\upsilon_A(u_0),\upsilon_A(y_0)\right\}$, then $((x_0\circ z_0)\circ z_0)\circ(y_0\circ z_0)<<A_{\langle t_0,s_0\rangle}$ and $y\in A_{\langle t_0,s_0\rangle}$ implies that $x_0\notin A_{\langle t_0,s_0\rangle}$ which is a contradiction. So A is an intuitionistic fuzzy distributive hyper BCI-ideal of X.

Theorem 4.10: For any subset A of X, let A be an intuitionistic fuzzy set in X defined by

$$\mu_A(x)=\begin{cases} t & if \quad x\in A \\ 0 & if \quad x\notin A \end{cases} \quad \text{and} \quad \upsilon_A(x)=\begin{cases} s & if \quad x\in A \\ 1 & if \quad x\notin A \end{cases}$$

for all $x\in X$ where $t,s\in(0,1]$. Then A is a distributive hyper BCI-ideal if and only if A is an intuitionistic fuzzy distributive hyper BCI-ideal of X.

Proof: Since A is an distributive hyper BCI-ideal of X therefore if $((x\circ z)\circ z)\circ(y\circ z) << A$ and $y\in A$ then $x\in A$. Hence

$$\mu_A(((x\circ z)\circ z)\circ(y\circ z))=t=\mu_A(y)=\mu_A(x)$$

and $\upsilon_A(((x\circ z)\circ z)\circ(y\circ z))=s=\upsilon_A(y)=\upsilon_A(x)$

which implies that

$$\mu_A(x)=\min\{\inf_{u\in((x\circ z)\circ z)\circ(y\circ z)}\mu_A(u),\mu_A(y)\}$$

and $\upsilon_A(x)=\max\{\sup_{u\in((x\circ z)\circ z)\circ(y\circ z)}\upsilon_A(u),\upsilon_A(y)\}$.

If atleast one of $((x\circ z)\circ z)\circ(y\circ z)$ does not a hyper order in A and $y\notin A$ then

$$\mu_A(x)\geq\min\{\inf_{u\in((x\circ z)\circ z)\circ(y\circ z)}\mu_A(u),\mu_A(y)\}$$

and $\upsilon_A(x)\leq\max\{\sup_{u\in((x\circ z)\circ z)\circ(y\circ z)}\upsilon_A(u),\upsilon_A(y)\}$.

Since $0\in A$ implies that $\mu_A(0)=t\geq\mu_A(x)$ and $\upsilon_A(0)=s\leq\upsilon_A(x)$ for all $x\in X$. So A is an intuitionistic fuzzy distributive hyper BCI-ideal. Converse follows from Theorem (4.9).

Theorem 4.11: Let A_1,A_2 be intuitionistic fuzzy distributive hyper BCI-ideals of X. Then $A_1\times A_2$ is an intuitionistic fuzzy distributive hyper BCI-ideal of $X\times X$.

Proof:
Since

$$(\mu_{A_1}\times\mu_{A_2})(x,y)=\min\{\mu_{A_1}(x),\mu_{A_2}(y)\},$$
$$(\upsilon_{A_1}\times\upsilon_{A_2})(x,y)=\max\{\upsilon_{A_1}(x),\upsilon_{A_2}(y)\},$$

then

$$(\mu_{A_1}\times\mu_{A_2})(0,0)=\min\{\mu_{A_1}(0),\mu_{A_2}(0)\}\geq\min\{\mu_{A_1}(x),\mu_{A_2}(y)\}=(\mu_{A_1}\times\mu_{A_2})(x,y)$$

and

$$(\upsilon_{A_1}\times\upsilon_{A_2})(0,0)=\max\{\upsilon_{A_1}(0),\upsilon_{A_2}(0)\}\leq\max\{\upsilon_{A_1}(x),\upsilon_{A_2}(y)\}=(\upsilon_{A_1}\times\upsilon_{A_2})(x,y)$$

so

$$(\mu_{A_1} \times \mu_{A_2})(0,0) \geq (\mu_{A_1} \times \mu_{A_2})(x, y)$$

and $(\upsilon_{A_1} \times \upsilon_{A_2})(0,0) \leq (\upsilon_{A_1} \times \upsilon_{A_2})(x, y)$, for all $(x, y) \in X$.

Since

$$\begin{aligned}(\mu_{A_1} \times \mu_{A_2})((x_1, y_1) \circ (x_2, y_2)) &= (\mu_{A_1} \times \mu_{A_2})(x_1 \circ x_2, y_1 \circ y_2) \\ &= \min\{\mu_{A_1}(x_1 \circ x_2), \mu_{A_2}(y_1 \circ y_2)\}\end{aligned}$$

and

$$\begin{aligned}(\upsilon_{A_1} \times \upsilon_{A_2})((x_1, y_1) \circ (x_2, y_2)) &= (\upsilon_{A_1} \times \upsilon_{A_2})(x_1 \circ x_2, y_1 \circ y_2) \\ &= \max\{\upsilon_{A_1}(x_1 \circ x_2), \upsilon_{A_2}(y_1 \circ y_2)\}.\end{aligned}$$

Consider

$$\begin{aligned}&(\mu_{A_1} \times \mu_{A_2})(x_1, y_1) \\ &= \min\{(\mu_{A_1}(x_1), \mu_{A_2}(y_1)\} \\ &\geq \min\left[\min\{\inf_{t_1 \in ((x_1 \circ x_3) \circ x_3) \circ (x_2 \circ x_3)} \mu_{A_1}(t_1), \mu_{A_1}(x_2)\}, \min\{\inf_{t_2 \in ((y_1 \circ y_3) \circ y_3) \circ (y_2 \circ y_3)} \mu_{A_2}(t_2), \mu_{A_2}(y_2)\}\right] \\ &= \min\left[\min\{\inf_{t_1 \in ((x_1 \circ x_3) \circ x_3) \circ (x_2 \circ x_3)} \mu_{A_1}(t_1), \inf_{t_2 \in ((y_1 \circ y_3) \circ y_3) \circ (y_2 \circ y_3)} \mu_{A_2}(t_2)\}, \min\{\mu_{A_1}(x_2), \mu_{A_2}(y_2)\}\right] \\ &= \min\left[\min\{\inf(\mu_{A_1}(t_1), \mu_{A_2}(t_2))\}, \{(\mu_{A_1} \times \mu_{A_2})(x_2, y_2)\}\right], \text{ where}\end{aligned}$$

$t_1 \in ((x_1 \circ x_3) \circ x_3) \circ (x_2 \circ x_3)$,

$t_2 \in ((y_1 \circ y_3) \circ y_3) \circ (y_2 \circ y_3)$,

$$\begin{aligned}&= \min\left[\inf\{(\mu_{A_1} \times \mu_{A_2})(t_1, t_2)\}, \{(\mu_{A_1} \times \mu_{A_2})(x_2, y_2)\}\right] \\ &= \min\left[\inf\{(\mu_{A_1} \times \mu_{A_2})(((x_1 \circ x_3) \circ x_3) \circ (x_2 \circ x_3), ((y_1 \circ y_3) \circ y_3) \circ (y_2 \circ y_3))\}, \{(\mu_{A_1} \times \mu_{A_2})(x_2, y_2)\}\right] \\ &= \min\left[\{\inf_{t \in [(((x_1, y_1) \circ (x_3, y_3)) \circ (x_3, y_3)) \circ ((x_2, y_2) \circ (x_3, y_3))]} (\mu_{A_1} \times \mu_{A_2})(t)\}, \{(\mu_{A_1} \times \mu_{A_2})(x_2, y_2)\}\right]\end{aligned}$$

and

$$\begin{aligned}&(\upsilon_{A_1} \times \upsilon_{A_2})(x_1, y_1) \\ &= \max\{(\upsilon_{A_1}(x_1), \upsilon_{A_2}(y_1)\} \\ &\leq \max\left[\max\{\sup_{t_1 \in ((x_1 \circ x_3) \circ x_3) \circ (x_2 \circ x_3)} \upsilon_{A_1}(t_1), \upsilon_{A_2}(x_2)\}, \max\{\sup_{t_2 \in ((y_1 \circ y_3) \circ y_3) \circ (y_2 \circ y_3)} \upsilon_{A_2}(t_2), \upsilon_{A_2}(y_2)\}\right] \\ &= \max\left[\max\{\sup_{t_1 \in ((x_1 \circ x_3) \circ x_3) \circ (x_2 \circ x_3)} \upsilon_{A_1}(t_1), \sup_{t_2 \in ((y_1 \circ y_3) \circ y_3) \circ (y_2 \circ y_3)} \upsilon_{A_2}(t_2)\}, \max\{\upsilon_{A_1}(x_2), \upsilon_{A_2}(y_2)\}\right] \\ &= \max\left[\max\{\sup(\upsilon_{A_1}(t_1), \upsilon_{A_2}(t_2))\}, \{(\upsilon_{A_1} \times \upsilon_{A_2})(x_2, y_2)\}\right], \text{ where}\end{aligned}$$

$t_1 \in ((x_1 \circ x_3) \circ x_3) \circ (x_2 \circ x_3)$,

$t_2 \in ((y_1 \circ y_3) \circ y_3) \circ (y_2 \circ y_3)$,

$$\begin{aligned}&= \max\left[\sup\{(\upsilon_{A_1} \times \upsilon_{A_2})(t_1, t_2)\}, \{(\upsilon_{A_1} \times \upsilon_{A_2})(x_2, y_2)\}\right] \\ &= \max\left[\sup\{(\upsilon_{A_1} \times \upsilon_{A_2})(((x_1 \circ x_3) \circ x_3) \circ (x_2 \circ x_3), ((y_1 \circ y_3) \circ y_3) \circ (y_2 \circ y_3))\}, \{(\upsilon_{A_1} \times \upsilon_{A_2})(x_2, y_2)\}\right] \\ &= \max\left[\{\sup_{t \in [(((x_1, y_1) \circ (x_3, y_3)) \circ (x_3, y_3)) \circ ((x_2, y_2) \circ (x_3, y_3))]} (\upsilon_{A_1} \times \upsilon_{A_2})(t)\}, \{(\upsilon_{A_1} \times \upsilon_{A_2})(x_2, y_2)\}\right]\end{aligned}$$

which implies that $A_1 \times A_2$ is an intuitionistic fuzzy distributive hyper BCI-ideal of X.

Proposition 4.12: Let A be an intuitionistic fuzzy distributive hyper BCI-ideal of X. Then

$$\mu_A(x) \geq \inf_{u\in((x\circ z)\circ z)\circ(0\circ z)} \mu_A(u) \text{ and } \upsilon_A(x) \leq \sup_{u\in((x\circ z)\circ z)\circ(0\circ z)} \upsilon_A(u).$$

Proof: Since A is an intuitionistic fuzzy distributive hyper BCI-ideal of X. Then

$$\begin{aligned}\mu_A(x) &\geq \min\{\inf_{u\in((x\circ z)\circ z)\circ(y\circ z)} \mu_A(u), \mu_A(y)\}\\ &= \min\{\inf_{u\in((x\circ z)\circ z)\circ(0\circ z)} \mu_A(u), \mu_A(0)\}\\ &= \inf_{u\in((x\circ z)\circ z)\circ(0\circ z)} \mu_A(u) \text{ because } \mu_A(0) = \mu_A(t) \text{ for all } t \in X,\end{aligned}$$

Also,

$$\begin{aligned}\upsilon_A(x) &\leq \max\{\sup_{u\in((x\circ z)\circ z)\circ(y\circ z)} \upsilon_A(u), \upsilon_A(y)\}\\ &= \max\{\sup_{u\in((x\circ z)\circ z)\circ(0\circ z)} \upsilon_A(u), \upsilon_A(0)\}\\ &= \sup_{u\in((x\circ z)\circ z)\circ(0\circ z)} \upsilon_A(u) \text{ because } \upsilon_A(0) = \upsilon_A(t) \text{ for all } t \in X.\end{aligned}$$

Theorem 4.13: Let A be an intuitionistic fuzzy set in a hyper BCI-algebra X and let λ_A be the strongest intuitionistic fuzzy relation on X. Then A is an intuitionistic fuzzy distributive hyper BCI-ideal of X iff λ_A is an intuitionistic fuzzy distributive hyper BCI-ideal of $X \times X$.

Proof: Let A be an intuitionistic fuzzy distributive hyper BCI-ideal of X then

$$\begin{aligned}\lambda_{\mu_A}(0,0) &= \min\{\mu_A(0), \mu_A(0)\}\\ &\geq \min\{\mu_A(x_1), \mu_A(x_2)\}\\ &= \lambda_{\mu_A}(x_1, x_2) \text{ for all } (x_1, x_2) \in X \times X,\end{aligned}$$

and

$$\begin{aligned}\lambda_{\upsilon_A}(0,0) &= \max\{\upsilon_A(0), \upsilon_A(0)\}\\ &\leq \max\{\upsilon_A(x_1), \upsilon_A(x_2)\}\\ &= \lambda_{\upsilon_A}(x_1, x_2) \text{ for all } (x_1, x_2) \in X \times X.\end{aligned}$$

Consider

$$\begin{aligned}&\lambda_{\mu_A}(x_1, x_2) = \min\{\mu_A(x_1), \mu_A(x_2)\}\\ &\geq \min[\min\{\inf_{u\in((x_1\circ z_1)\circ z_1)\circ(y_1\circ z_1)} \mu_A(u), \mu_A(y_1)\}, \min\{\inf_{v\in((x_2\circ z_2)\circ z_2)\circ(y_2\circ z_2)} \mu_A(v), \mu_A(y_2)\}]\\ &= \min[\min\{\{\inf_{u\in((x_1\circ z_1)\circ z_1)\circ(y_1\circ z_1)} \mu_A(u)\}, \{\inf_{v\in((x_2\circ z_2)\circ z_2)\circ(y_2\circ z_2)} \mu_A(v)\}\}, \min\{\mu_A(y_1), \mu_A(y_2)\}]\\ &= \min[\min\{\{\inf_{u\in((x_1\circ z_1)\circ z_1)\circ(y_1\circ z_1)} \mu_A(u)\}, \{\inf_{v\in((x_2\circ z_2)\circ z_2)\circ(y_2\circ z_2)} \mu_A(v)\}\}, \lambda_{\mu_A}(y_1, y_2)]\\ &= \min[\min\{\inf(\mu_A(u), \mu_A(v))\}, \lambda_{\mu_A}(y_1, y_2)],\end{aligned}$$

where

$$\begin{aligned}&u \in ((x_1 \circ z_1) \circ z_1) \circ (y_1 \circ z_1), v \in ((x_2 \circ z_2) \circ z_2) \circ (y_2 \circ z_2)\\ &\quad = \min[\inf\{\min(\mu_A(u), \mu_A(v))\}, \lambda_{\mu_A}(y_1, y_2)]\end{aligned}$$

where
$u \in ((x_1 \circ z_1) \circ z_1) \circ (y_1 \circ z_1), v \in ((x_2 \circ z_2) \circ z_2) \circ (y_2 \circ z_2)$
$= \min\lfloor \inf \{\lambda_{\mu_A}(u,v), \lambda_{\mu_A}(y_1, y_2)\}\rfloor$
where $(u,v) \in (((x_1, x_2) \circ (z_1, z_2)) \circ (z_1, z_2)) \circ ((y_1, y_2) \circ (z_1, z_2))$

Also,
$\lambda_{\upsilon_A}(x_1, x_2) = \max\{\upsilon_A(x_1), \upsilon_A(x_2)\}$
$\leq \max\lfloor \max\{\sup_{u\in((x_1\circ z_1)\circ z_1)\circ(y_1\circ z_1)} \upsilon_A(u), \upsilon_A(y_1)\}, \max\{\sup_{v\in((x_2\circ z_2)\circ z_2)\circ(y_2\circ z_2)} \upsilon_A(v), \upsilon_A(y_2)\}\rfloor$
$= \max\lfloor \max\{\{\sup_{u\in((x_1\circ z_1)\circ z_1)\circ(y_1\circ z_1)} \upsilon_A(u)\}, \{\sup_{v\in((x_2\circ z_2)\circ z_2)\circ(y_2\circ z_2)} \upsilon_A(v)\}\}, \max\{\upsilon_A(y_1), \upsilon_A(y_2)\}\rfloor$
$= \max\lfloor \max\{\{\sup_{u\in((x_1\circ z_1)\circ z_1)\circ(y_1\circ z_1)} \upsilon_A(u)\}, \{\sup_{v\in((x_2\circ z_2)\circ z_2)\circ(y_2\circ z_2)} \upsilon_A(v)\}\}, \lambda_{\upsilon_A}(y_1, y_2)\rfloor$
$= \max\lfloor \max\{\sup(\upsilon_A(u), \upsilon_A(v))\}, \lambda_{\upsilon_A}(y_1, y_2)\rfloor,$
where
$u \in ((x_1 \circ z_1) \circ z_1) \circ (y_1 \circ z_1), v \in ((x_2 \circ z_2) \circ z_2) \circ (y_2 \circ z_2)$
$= \max\lfloor \sup\{\max(\upsilon_A(u), \upsilon_A(v))\}, \lambda_{\upsilon_A}(y_1, y_2)\rfloor$
where
$u \in ((x_1 \circ z_1) \circ z_1) \circ (y_1 \circ z_1), v \in ((x_2 \circ z_2) \circ z_2) \circ (y_2 \circ z_2)$
$= \max\lfloor \sup\{\lambda_{\upsilon_A}(u,v), \lambda_{\upsilon_A}(y_1, y_2)\}\rfloor$
where $(u,v) \in (((x_1, x_2) \circ (z_1, z_2)) \circ (z_1, z_2)) \circ ((y_1, y_2) \circ (z_1, z_2))$
which implies that λ_{μ_A} and λ_{υ_A}, they are intuitionistic fuzzy distributive hyper BCI-ideals of $X \times X$.

Conversely suppose that λ_{μ_A} and λ_{υ_A}, they are intuitionistic fuzzy distributive hyper BCI-ideals of $X \times X$ then for all $(x, y) \in X \times X$.

$\mu_A(0) = \min\{\mu_A(0), \mu_A(0)\} = \lambda_u(0,0) \geq \lambda_{\mu_A}(x,x) = \min\{\mu_A(x), \mu_A(x)\} = \mu_A(x)$

and $\upsilon_A(0) = \max\{\upsilon_A(0), \upsilon_A(0)\} = \lambda_u(0,0) \leq \lambda_{\upsilon_A}(x,x) = \max\{\upsilon_A(x), \upsilon_A(x)\} = \upsilon_A(x)$

which implies that $\mu_A(0) \geq \mu_A(x)$ and $\upsilon_A(0) \leq \upsilon_A(x)\, \forall x \in X$.
$\min\{\mu_A(x_1), \mu_A(y_1)\} = \lambda_{\mu_A}(x_1, y_1)$
$= \min\{\inf_{w\in(((x_1,y_1)\circ(x_3,y_3))\circ(x_3,y_3))\circ((x_2,y_2)\circ(x_3,y_3))} \lambda_{\mu_A}(w), \lambda_{\mu_A}(x_2, y_2)\}$
$= \min\{\inf_{w\in(((x_1\circ x_3)\circ x_3)\circ(x_2\circ x_3),((y_1\circ y_3)\circ y_3)\circ(y_2\circ y_3))} \lambda_{\mu_A}(w), \lambda_{\mu_A}(x_2, y_2)\}$

Also,
$\max\{\upsilon_A(x_1), \upsilon_A(y_1)\} = \lambda_{\upsilon_A}(x_1, y_1)$
$= \max\{\sup_{w\in(((x_1,y_1)\circ(x_3,y_3))\circ(x_3,y_3))\circ((x_2,y_2)\circ(x_3,y_3))} \lambda_{\upsilon_A}(w), \lambda_{\upsilon_A}(x_2, y_2)\}$
$= \max\{\sup_{w\in(((x_1\circ x_3)\circ x_3)\circ(x_2\circ x_3),((y_1\circ y_3)\circ y_3)\circ(y_2\circ y_3))} \lambda_{\upsilon_A}(w), \lambda_{\upsilon_A}(x_2, y_2)\}$
so if we put $y_1 = y_2 = y_3 = 0$ or $(x_1 = x_2 = x_3 = 0)$ then we get
$\mu_A(x_1) \geq \min\{\inf_{u\in((x_1\circ x_3)\circ x_3)\circ(x_2\circ x_3)} \mu_A(u), \mu_A(x_2)\}$
and $\upsilon_A(x_1) \leq \max\{\sup_{u\in((x_1\circ x_3)\circ x_3)\circ(x_2\circ x_3)} \upsilon_A(u), \upsilon_A(x_2)\}$

or $\mu_A(y_1) \geq \min\{\inf_{v\in((y_1\circ y_3)\circ y_3)\circ(y_2\circ y_3)} \mu_A(v), \mu_A(y_2)\}$
$\upsilon_A(y_1) \leq \max\{\sup_{v\in((y_1\circ y_3)\circ y_3)\circ(y_2\circ y_3)} \upsilon_A(v), \upsilon_A(y_2)\}$
which implies that A is an intuitionistic fuzzy distributive hyper BCI-ideal of X.

Theorem 4.14: Let $f : X \to Y$ be an onto hyper homomorphism from a hyper BCI-algebra X to a hyper BCI-algebra Y. If A_2 is a intuitionistic fuzzy distributive hyper BCI-ideal of Y then BCI hyper homomorphic pre image A_1 of A_2 under f is also a intuitionistic fuzzy distributive hyper BCI-ideal of X.

Proof: Define $A_2(f(x)) = A_1(x), \forall x \in X$ and since A_2 is an intuitionistic fuzzy distributive hyper BCI-ideal of Y with $f(x) \in Y$.

Therefore, $\mu_{A_1}(0) = \mu_{A_2}(f(0)) \geq \mu_{A_2}(f(x)) = \mu_{A_1}(x)$

and $\upsilon_{A_1}(0) = \upsilon_{A_2}(f(0)) \leq \upsilon_{A_2}(f(x)) = \upsilon_{A_1}(x) \forall x \in X$.

$$\begin{aligned}
\mu_{A_1}(x) &= \mu_{A_2}(f(x)) \\
&\geq \min\{\inf_{u\in((f(x)\circ z')\circ z')\circ(y'\circ z')} \mu_{A_2}(u), \mu_{A_2}(y')\} \\
&= \min\{\inf_{u\in((f(x)\circ f(z))\circ f(z))\circ(f(y)\circ f(z))} \mu_{A_2}(u), \mu_{A_2}(f(y))\} \\
&= \min\{\inf_{v\in(((x\circ z)\circ z)\circ(y\circ z))} \mu_{A_1}(v), \mu_{A_1}(y)\} \text{because } f \text{ is onto,}
\end{aligned}$$

Also,

$$\begin{aligned}
\upsilon_{A_1}(x) &= \upsilon_{A_2}(f(x)) \\
&\leq \max\{\sup_{u\in((f(x)\circ z')\circ z')\circ(y'\circ z')} \upsilon_{A_2}(u), \upsilon_{A_2}(y')\} \\
&= \max\{\sup_{u\in((f(x)\circ f(z))\circ f(z))\circ(f(y)\circ f(z))} \upsilon_{A_2}(u), \upsilon_{A_2}(f(y))\} \\
&= \max\{\sup_{v\in(((x\circ z)\circ z)\circ(y\circ z))} \upsilon_{A_1}(v), \upsilon_{A_1}(y)\} \text{because } f \text{ is onto.}
\end{aligned}$$

Therefore A_1 is an intuitionistic fuzzy distributive hyper BCI-ideal of X.

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