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Coefficient Inequalities for a New Subclass of K-uniformly Convex Functions

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Abstract

The object of this paper is to introduce a new subclass of k-uniformly convex functions and to obtain the coefficient inequalities, Fekete-Szego inequality for the functions in this class.

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Introduction

Let $\mathcal{A}$ denote the class of all analytic functions *f* of the form

$$f(z) = z + \sum_{n=2}^{\infty} a_n z^n \tag{1.1}$$

in the open unit disc $U = \{z / z \in C \text{ and } |z| < 1\}$

Let S be the subclass of $\mathcal{A}$ consisting of univalent functions.

Let $S^*(\beta)$ and $C(\beta)$ be the subclasses of S consisting of the functions *f* of the form (1.1) and satisfying the condition

$$\mathrm{Re}\left(\frac{zf'(z)}{f(z)}\right) > \beta$$

and

$$\mathrm{Re}\left(1 + \frac{zf''(z)}{f'(z)}\right) > \beta$$

respectively
for some $\beta(0 \le \beta < 1)$ and $z \in U$.

These classes are known as starlike and convex functions of order β respectively.

S. Shams, S.R. Kulkarni and J.M.Jahangiri [4] have introduced the classes $SD(k,\beta)$ and $KD(k,\beta)$ as follows.

A function $f \in A$ and satisfies the condition

$$\operatorname{Re}\left(\frac{zf'(z)}{f(z)}\right) > k\left|\frac{zf'(z)}{f(z)}-1\right|+\beta, \qquad z \in U$$

is said to be in the class $SD(k,\beta)$ for some $k \ge 0$ and $0 \le \beta < 1$.

Similarly $KD(k,\beta)$ be the subclass of $\mathcal{A}$ consisting of functions $f \in A$ and satisfying the condition.

$$\operatorname{Re}\left(1+\frac{zf''(z)}{f'(z)}\right) > k\left|\frac{zf''(z)}{f'(z)}\right|+\beta, \qquad z \in U \quad \text{for some } k \ge 0 \text{ and } 0 \le \beta < 1.$$

For $0 \le k \le \beta$, these classes have been studied by S.Owa, Y.Polatoglu and E.Yavuz [2]. They have obtained the coefficient inequalities and distortion properties for the functions in these classes.

Several authors have obtained the Fekete-Szego inequality for the normalized analytical functions *f* of the form (1.1) in various subclasses of *S*.

In the present paper we introduce a new subclass of analytic functions and obtain the coefficient inequalities, Fekete-Szego inequalities for the functions in this class. The results of this paper will unify and generalize the earlier results of several authors in this direction.

Defination 1.1: Let $U(\lambda,\alpha,\beta,k)$ be the class of functions $f \in \mathcal{A}$ satisfying the condition

$$\operatorname{Re}\left(\frac{zF'(z)}{F(z)}\right) > k\left|\frac{zF'(z)}{F(z)}-1\right|+\beta \tag{1.2}$$

where $0 \le \alpha \le \lambda \le 1$ and $0 \le k \le \beta < 1$

with $F(z) = \lambda\alpha z^2 f''(z) + (\lambda-\alpha)zf'(z) + (1-\lambda+\alpha)f(z)$

It is noted that

(i) $U(0,\ 0,\ \beta,\ 0) = S^*(\beta)$ and $U(1,\ 0,\beta,\ 0) = C(\beta)$ are usual starlike functions of order β and convex functions of order β

(ii) $U(0,\ 0,\ \beta,\ k) = SD(k,\ \beta)$ and $U(1,\ 0,\beta,\ k) = KD(k,\beta)$ Studied by S.Owa, Y, Polatoglu, E. Yavuz [2]

To prove our results, we require the following Lemmas.

Lemma (1.1) [3]: If $p(z) = 1 + c_1 z + c_2 z^2 + ...$ is a function with positive real part then for any complex number *v*, we have

$\left| c_2 - v\,c_1^2 \right| \le 2\ \max\ \{1, |\ 2v-1|\ \}$

This result is sharp for the functions

$p(z) = \dfrac{1+z^2}{1-z^2}$ and $p(z) = \dfrac{1+z}{1-z}$

Lemma (1.2) [1]: If $p_1(z) = 1 + c_1 z + c_2 z^2 + ...$ is an analytic function with positive real part in *U* then for any real number *v*, we have

$\left| c_2 - vc_1^2 \right| \quad \le -4v+2$ if $v \le 0$ ≤ 2 if $0 \le v \le 1$ $\le 4v-2$ if $v \ge 1$

when $v < 0$ or $v > 1$ the equality holds if and only if $p_1(z) = \dfrac{1+z}{1-z}$ or one of its rotations. If $0 < v < 1$ then the equality holds if and only if $p_1(z) = \dfrac{1+z^2}{1-z^2}$ or one of its rotations. If $v = 0$ then the equality holds if and only if

$p_1(z) = \left(\dfrac{1+\lambda}{2}\right)\left(\dfrac{1+z}{1-z}\right) + \left(\dfrac{1-\lambda}{2}\right)\left(\dfrac{1-z}{1+z}\right),\ (0 \le \lambda \le 1)$ or one of its rotations.

If $v = 1$ the equality holds only for the reciprocal of $p_1(z)$ for the case $v = 0$. Also the above upper bound is sharp and it can be further improved as follows when $0 < v < 1$.

$\left| c_2 - v\,c_1^2 \right| + v \left| c_1^2 \right| \le 2, \qquad (0 \le v \le 1/2)$

$\left| c_2 - v\,c_1^2 \right| + (1-v) \left| c_1^2 \right| \le 2, \quad (1/2 < v \le 1)$

In the next sections, we obtain the coefficient inequalities and Fekete-Szego in equality for the function *f* in the class $U(\lambda, \alpha, \beta, k)$.

Coefficient inequalities

Theorem 2.1: If $f(z) \in U(\lambda, \alpha, \beta, k)$ with $0 \le k \le \beta$ then $f(z) \in U\left(\lambda, \alpha, \dfrac{\beta-k}{1-k}, 0\right)$

Proof: Since $f(z) \in U(\lambda, \alpha, \beta, k)$ from definition (1.1) we have

$$\operatorname{Re}\left(\frac{zF'(z)}{F(z)}\right) > k\left|\frac{zF'(z)}{F(z)} - 1\right| + \beta$$

$$\Rightarrow \operatorname{Re}\left(\frac{zF'(z)}{F(z)}\right) > k\operatorname{Re}\left(\frac{zF'(z)}{F(z)} - 1\right) + \beta$$

$$\Rightarrow \operatorname{Re}\left(\frac{zF'(z)}{F(z)}\right) > \frac{\beta-k}{1-k} \qquad (z \in U) \tag{2.1}$$

$$\Rightarrow f(z) \in U\left(\lambda, \alpha, \frac{\beta-k}{1-k}, 0\right)$$

This completes the proof of the theorem

Theorem 2.2: If $f(z)\in U(\lambda,\alpha,\beta,k)$ then

$$|a_2|\le\frac{2(1-\beta)}{(1-k)(2\lambda\alpha+\lambda-\alpha+1)} \tag{2.2}$$

$$|a_n|\le\frac{2(1-\beta)}{(1-k)[(n-1)(n\lambda\alpha+\lambda-\alpha)+1]}\prod_{j=1}^{n-1}\left[1+\frac{2(1-\beta)}{j(1-k)}\right] \text{ for } n\ge 3,\quad z\in U \tag{2.3}$$

Proof: Since $f(z)\in U(\lambda,\alpha,\beta,k)$, from equation (2.1) we have

$$\Rightarrow \operatorname{Re}\left(\frac{zF'(z)}{F(z)}\right)>\frac{\beta-k}{1-k}$$

Define a function $p(z)$ such that

$$p(z)=\frac{(1-k)\left[\frac{zF'(z)}{F(z)}\right]-(\beta-k)}{1-\beta}=1+\sum_{n=1}^{\infty}c_n z^n,\quad z\in U \tag{2.4}$$

Here $p(z)$ is analytic in U with $p(0)=1$ and $\operatorname{Re}(p(z))>0$.

From (2.4) we get

$$(1-\beta)\left[1+\sum_{n=1}^{\infty}c_n z^n\right][F(z)]=(1-k)zF'(z)-(\beta-k)F(z) \tag{2.5}$$

Replacing $F(z), zF'(z)$ with their equivalent expressions in series on bothsides of the above equation (2.5) we get

$$(1-\beta)\left[z+\sum_{n=1}^{\infty}c_n z^{n+1}+\sum_{n=2}^{\infty}[(n-1)(n\lambda\alpha+\lambda-\alpha)+1]a_n z^n+\right.$$
$$\left.\left[\sum_{n=2}^{\infty}[(n-1)(n\lambda\alpha+\lambda-\alpha)+1]a_n z^n\right]\left[\sum_{n=1}^{\infty}c_n z^n\right]\right]$$
$$=(1-k)\left[z+\sum_{n=2}^{\infty}n[(n-1)(n\lambda\alpha+\lambda-\alpha)+1]a_n z^n\right]$$
$$-(\beta-k)\left[z+\sum_{n=2}^{\infty}(n-1)(n\lambda\alpha+\lambda-\alpha)+1\right]a_n z^n$$

Comparing the coefficient of z^n, on both sides of the above equation, we get

$$a_n=\frac{(1-\beta)}{(1-k)(n-1)[(n-1)(n\lambda\alpha+\lambda-\alpha)+1]}\left[c_{n-1}+(2\lambda\alpha+\lambda-\alpha+1)a_2c_{n-2}+\right.$$
$$[2(3\lambda\alpha+\lambda-\alpha)+1]a_3c_{n-3}+....+[(n-3)[(n-2)\lambda\alpha+\lambda-\alpha]+1]a_{n-2}c_2$$
$$\left.+[(n-2)[(n-1)\lambda\alpha+\lambda-\alpha]+1]a_{n-1}c_1\right] \qquad ..... (2.6)$$

Taking Modulus on both sides of equation (2.6) and applying Caratheodary inequality $|c_n|\le 2\ \ \forall n\ge 1$, we get,

$$|a_n|\le\frac{2(1-\beta)}{(1-k)(n-1)[(n-1)(n\lambda\alpha+\lambda-\alpha)+1]}\left[1+(2\lambda\alpha+\lambda-\alpha+1)|a_2|+\right.$$

$$[2(3\lambda\alpha+\lambda-\alpha)+1]|a_3|+......[(n-3)[(n-2)\lambda\alpha+\lambda-\alpha]+1]|a_{n-2}|$$
$$+[(n-2)[(n-1)\lambda\alpha+\lambda-\alpha]+1]|a_{n-1}|] \qquad (2.7)$$

When $n = 2$ $|a_2|\leq\dfrac{2(1-\beta)}{(1-k)(2\lambda\alpha+\lambda-\alpha+1)}$

which proves the result in (2.2)

For n = 3 $|a_3|\leq\dfrac{2(1-\beta)}{2(1-k)[2(3\lambda\alpha+\lambda-\alpha)+1]}\left[1+\dfrac{2(1-\beta)}{(1-k)}\right].$

Thus the result in (2.3) is true for $n = 3$. Suppose that the result in (2.3) is true for $4 \leq n \leq m$.

For $n = m + 1$, consider

$$|a_{m+1}|\leq\frac{2(1-\beta)}{(1-k)\cdot m.[m[(m+1)\lambda\alpha+\lambda-\alpha]+1]}\left[1+\frac{2(1-\beta)}{(1-k)}\right.$$
$$\left.+\frac{2(1-\beta)}{(1-k)}\left[1+\frac{2(1-\beta)}{1-k}\right]+........+\frac{2(1-\beta)}{(1-k)}\pi_{j=1}^{m-2}\left[1+\frac{2(1-\beta)}{j(1-k)}\right]\right]$$
$$|a_{m+1}|\leq\frac{2(1-\beta)}{(1-k)\cdot m.[m[(m+1)\lambda\alpha+\lambda-\alpha]+1]}\prod_{j=1}^{m=1}\left[1+\frac{2(1-\beta)}{j(1-k)}\right]$$

Hence the result in (2.3) is true for $n = m + 1$. Therefore by Mathematical induction the result in (2.3) is true for all $n \geq 3$.

This completes the proof of the theorem (2.2)

Fekete-Szego inequality

Theorem 3.1: If $f(z)\in U(\lambda,\alpha,\beta,k)$, then for any complex number μ, we have

$$\left|a_3-\mu a_2^2\right|\leq\frac{2(1-\beta)}{2(1-k)[2(3\lambda\alpha+\lambda-\alpha)+1]}\max\{1,\ |2v-1|\} \qquad (3.1)$$

where $v=\left(\dfrac{1-\beta}{1-k}\right)\left[\dfrac{2\mu[2(3\lambda\alpha+\lambda-\alpha)+1]}{(2\lambda\alpha+\lambda-\alpha+1)^2}-1\right]$ and the result is sharp

Proof: Since $f(z)\in U(\lambda,\alpha,\beta,k)$ from equation (2.6) we have

$$a_2=\frac{1-\beta}{(1-k)(2\lambda\alpha+\lambda-\alpha+1)}c_1$$
$$a_3=\frac{1-\beta}{2(1-k)[2(3\lambda\alpha+\lambda-\alpha)+1]}[c_2+(2\lambda\alpha+\lambda-\alpha+1)a_2c_1]$$

For any complex number μ, we have

$$a_3-\mu a_2^2=\frac{1-\beta}{2(1-k)[2(3\lambda\alpha+\lambda-\alpha)+1]}[c_2+(2\lambda\alpha+\lambda-\alpha+1)a_2c_1]$$

$$-\mu\left[\frac{(1-\beta)^2}{(1-k)^2\,[2\lambda\alpha+\lambda-\alpha+1]^2}c_1^2\right]$$

$$a_3-\mu a_2^2=\frac{1-\beta}{2(1-k)[2(3\lambda\alpha+\lambda-\alpha)+1]}\left[c_2-v\,c_1^2\right] \tag{3.2}$$

where $v=\frac{1-\beta}{1-k}\left[\frac{2\mu(2(3\lambda\alpha+\lambda-\alpha)+1)}{(2\lambda\alpha+\lambda-\alpha+1)^2}-1\right]$

Taking modulus on both sides of (3.2) and applying the Lemma [1.1], we get

$$|a_3-\mu a_2^2|\le\frac{2(1-\beta)}{2(1-k)\,[2(3\lambda\alpha+\lambda-\alpha)+1]}\max\left\{1,|2v-1|\right\}$$

$$|a_3-\mu a_2^2|\le\frac{1-\beta}{(1-k)\,[2(3\lambda\alpha+\lambda-\alpha)+1]}\max$$

$$\left\{1,\left|\frac{2(1-\beta)}{(1-k)}\left[\frac{2\mu[2(3\lambda\alpha+\lambda-\alpha)+1]}{(2\lambda\alpha+\lambda-\alpha+1)^2}-1\right]-1\right|\right\}$$

This completes the proof of the theorem. And the result is sharp.

$$\left|a_3-\mu a_2^2\right|=\frac{1-\beta}{(1-k)[2(3\lambda\alpha+\lambda-\alpha)+1]},\qquad \text{if } P(z)=\frac{1+z^2}{1-z^2} \text{ and}$$

$$|a_3-\mu a_2^2|=\frac{1-\beta}{(1-k)[2(3\lambda\alpha+\lambda-\alpha)+1]}\left[\left|\frac{2(1-\beta)}{(1-k)}\left[\frac{2\mu[2(3\lambda\alpha+\lambda-\alpha)+1]}{(2\lambda\alpha+\lambda-\alpha+1)^2}-1\right]-1\right|\right],$$

if $P(z)=\frac{1+z}{1-z}$.

Theorem (3.2):- If $f\in U(\lambda,\alpha,\beta,k)$, then for any real number μ and for

$$\sigma_1=\frac{(2\lambda\alpha+\lambda-\alpha+1)^2}{2\left[2(3\lambda\alpha+\lambda-\alpha)+1\right]}$$

$$\sigma_2=\frac{(2-k-\beta)(2\lambda\alpha+\lambda-\alpha+1)^2}{2(1-\beta)\left[2(3\lambda\alpha+\lambda-\alpha)+1\right]}$$

and

$$\sigma_3=\frac{(3-k-2\beta)(2\lambda\alpha+\lambda-\alpha+1)^2}{4(1-\beta)\left[2(3\lambda\alpha+\lambda-\alpha)+1\right]}$$

then

$$\left|a_3-\mu a_2^2\right|$$

$$\le\frac{1-\beta}{2(1-k)\left[2(3\lambda\alpha+\lambda-\alpha)+1\right]}\left[\frac{(1-\beta)}{(1-k)}\left[4-\frac{8\mu\left[2(3\lambda\alpha+\lambda-\alpha)+1\right]}{(2\lambda\alpha+\lambda-\alpha+1)^2}\right]+2\right]$$

if $\mu\le\sigma_1$

$$\leq \frac{1-\beta}{(1-k)\left[2(3\lambda\alpha+\lambda-\alpha)+1\right]}$$

if $\sigma_1 \leq \mu \leq \sigma_2$

$$\leq \frac{1-\beta}{2(1-k)\left[2(3\lambda\alpha+\lambda-\alpha)+1\right]}\left[\frac{(1-\beta)}{(1-k)}\left[\frac{8\mu\left[2(3\lambda\alpha+\lambda-\alpha)+1\right]}{(2\lambda\alpha+\lambda-\alpha+1)^2}-4\right]-2\right]$$

if $\mu \geq \sigma_2$ (3.3)

Further more if $\sigma_1 \leq \mu \leq \sigma_3$, , then

$$\left|a_3-\mu a_2{}^2\right|+\left[\mu-\frac{(2\lambda\alpha+\lambda-\alpha+1)^2}{2\left[2(3\lambda\alpha+\lambda-\alpha)+1\right]}\right]\left|a_2\right|^2 \leq \frac{1-\beta}{(1-k)\left[2(3\lambda\alpha+\lambda-\alpha)+1\right]}, \quad (3.4)$$

And if $\sigma_3 \leq \mu \leq \sigma_2$, then

$$\left|a_3-\mu a_2{}^2\right|+\left[1-\frac{(1-\beta)}{(1-k)}2\mu\frac{(2(3\lambda\alpha+\lambda-\alpha)+1)-1}{(2\lambda\alpha+\lambda-\alpha+1)^2}\right]\frac{\left|a_2\right|^2}{(1-k)^2(2\lambda\alpha+\lambda-\alpha+1)^2} \leq \frac{(1-\beta)}{(1-k)\left[2(3\lambda\alpha+\lambda-\alpha)+1\right]} \quad (3.5)$$

Proof:- Since $f \in U(\lambda,\alpha,\beta,k)$ from equation (3.2), we have

$$a_3-\mu a_2^2 = \frac{1-\beta}{2(1-k)\left[2(3\lambda\alpha+\lambda-\alpha)+1\right]}\left[c_2-v c_1^2\right]$$

where $v = \frac{1-\beta}{1-k}\left[\frac{2\mu(2(3\lambda\alpha+\lambda-\alpha)+1)}{(2\lambda\alpha+\lambda-\alpha+1)^2}-1\right]$

Taking modulus on both sides of the above equation and applying Lemma (1.2) on the right hand side, we get the following cases :

Case 1) If $\mu \leq \sigma_1$, then

$$\mu \leq \frac{(2\lambda\alpha+\lambda-\alpha+1)^2}{2\left[2(3\lambda\alpha+\lambda-\alpha)+1\right]},$$

which on simplification, we get,

$\Rightarrow v \leq 0$.

$\Rightarrow \left|c_2-vc_1{}^2\right| \leq -4v+2$.

$$\left|c_2-vc_1{}^2\right| \leq \left[\frac{1-\beta}{1-k}\right]\left[4-\frac{8\mu\left[2(3\lambda\alpha+\lambda-\alpha)+1\right]}{(2\lambda\alpha+\lambda-\alpha+1)^2}\right]+2. \quad (3.6)$$

Case 2) If $\sigma_1 \leq \mu \leq \sigma_2$, then,

$$\frac{(2\lambda\alpha+\lambda-\alpha+1)^2}{2\left[2(3\lambda\alpha+\lambda-\alpha)+1\right]}\le\mu\le\frac{(2-k-\beta)(2\lambda\alpha+\lambda-\alpha+1)^2}{2(1-\beta)\left[2(3\lambda\alpha+\lambda-\alpha)+1\right]}$$

After simplification, we get,

$0\le v\le 1.$

$$\left|c_2-vc_1^{\,2}\right|\le 2. \tag{3.7}$$

Case 3) If $\mu\ge\sigma_2$, then,

$$\mu\ge\frac{(2-k-\beta)(2\lambda\alpha+\lambda-\alpha+1)^2}{2(1-\beta)\left[2(3\lambda\alpha+\lambda-\alpha)+1\right]},$$

after simplification, we get,

$\Rightarrow v\ge 1.$

$\Rightarrow \left|c_2-vc_1^{\,2}\right|\le 4v-2.$

$$\left|c_2-vc_1^{\,2}\right|\le\frac{1-\beta}{1-k}\left[\frac{8\mu\left[2(3\lambda\alpha+\lambda-\alpha)+1\right]}{(2\lambda\alpha+\lambda-\alpha+1)^2}-4\right]-2. \tag{3.8}$$

From equations (3.2), (3.6), (3.7) and (3.8), we get the result in (3.3)

Case 4) Further more if $\sigma_1\le\mu\le\sigma_3$, then,

$$\frac{(2\lambda\alpha+\lambda-\alpha+1)^2}{2\left[2(3\lambda\alpha+\lambda-\alpha)+1\right]}\le\mu\le\frac{(3-k-2\beta)(2\lambda\alpha+\lambda-\alpha+1)^2}{4(1-\beta)\left[2(3\lambda\alpha+\lambda-\alpha)+1\right]}$$

After simplification, we get,

$0\le v\le\frac{1}{2}.$

$\left|c_2-vc_1^{\,2}\right|+v\left|c_1^{\,2}\right|\le 2.$

$$\left|a_3-\mu a_2^{\,2}\right|+\left[\mu-\frac{(2\lambda\alpha+\lambda-\alpha+1)^2}{2\left[2(3\lambda\alpha+\lambda-\alpha)+1\right]}\right]\left|a_2\right|^2\le\frac{1-\beta}{(1-k)\left[2(3\lambda\alpha+\lambda-\alpha)+1\right]}$$

Which prove the result in (3.4)

Case 5) Further more if $\sigma_3\le\mu\le\sigma_2$, then,

$$\frac{(3-k-2\beta)(2\lambda\alpha+\lambda-\alpha+1)^2}{4(1-\beta)\left[2(3\lambda\alpha+\lambda-\alpha)+1\right]}\le\mu\le\frac{(2-k-\beta)(2\lambda\alpha+\lambda-\alpha+1)^2}{2(1-\beta)\left[2(3\lambda\alpha+\lambda-\alpha)+1\right]}$$

After simplification, we get,

$\frac{1}{2}\le v\le 1.$

$\left|c_2-vc_1^{\,2}\right|+(1-v)\left|c_1^{\,2}\right|\le 2.$

$$\left|a_3-\mu a_2{}^2\right|+\left[1-\frac{(1-\beta)}{(1-k)}2\mu\frac{\left(2(3\lambda\alpha+\lambda-\alpha)+1\right)-1}{(2\lambda\alpha+\lambda-\alpha+1)^2}\right]\frac{|a_2|^2}{(1-k)^2(2\lambda\alpha+\lambda-\alpha+1)^2}$$

$$\leq\frac{(1-\beta)}{(1-k)\left[2(3\lambda\alpha+\lambda-\alpha)+1\right]}$$

Which prove the result in (3.5)

This completes the proof of the Theorem.

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