

An Inventory Model for Deteriorating Items with Stock Dependent Demand and Partial Backlogging

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Abstract

This paper presents an optimization framework to derive optimal replenishment policy for perishable items with stock dependent demand rate. The present work attempts to model the situations where selling rate depends on the stock level in hand and items have time dependent deterioration rate with partial backlogging. Here shortages are allowed and partially backlogged, where backlogged rate is dependent on the duration of the waiting time up to the arrival of next lot.

Keywords: Stock dependent demand rate, Variable deterioration rate, partial backlogging.

Introduction

Classical inventory models developed for constant demand rate can be applied to both manufacturing and sales environment. But many supermarket managers have observed that, for some items, the demand rate is directly related to the amount of inventory displayed. This phenomenon is termed as 'stock dependent demand rate.'

Deterioration is the important factor in many inventory systems. Deterioration is defined as decay or damage such that the items can not be ignore this factor in inventory system. For items such as steel, hardware, glassware and toys the rate of deterioration is so low in which there is little need for considering deterioration in the determination of the economic lot size. However, some items such as blood, fish, strawberry, alcohol, gasoline, radioactive chemical, medicine and food grains deteriorate rapidly overtime. Thus deterioration of physical goods in stock is a very

realistic feature and inventory modelers felt the need to take this factor in to consideration. . EOQ models for deteriorating items with stock dependent demand have also considered by several researchers like Gupta & Vrat [5], Wee [12], Padmanabhan & Vrat [9], Giri & Chakra [4], Chang & Dye [1]. An inventory model with stock dependent demand in response to temporary sale price is proposed by Horng-Jinh Chang and Chung-Yuan Dye [6]. Chun-Tao Chang and J.T. Ting [2] discuss why it is appropriate maximize the profits of minimizing the costs, in an inventory system with an inventory level dependent demand rate. Dye and Ouyang [3] proposed an EOQ model for perishable items under stock dependent selling rate and time dependent partial backlogging and then established the unique optimal solution to the problem when building up inventory is not profitable. Manjusri Basu and Sudipta Sinha [8] proposes to present a general inventory model with due consideration to the factors of time dependent partial backlogging and time dependent deterioration. Sanjay Jain, Mukesh Kumar, and Priya Advani [11] discuss a deterministic inventory model for infinite time-horizon incorporating inventory level-dependent demand rate, deterioration begins after a certain time, partial backlogging and decrease in demand is develop. Ravinder Kumar Arya, S.R. Singh and Shakya [10] present an optimization framework to derive optimal replenishment policy for perishable items with stock dependent demand rate. Horng Jinh Chang and Wen Feng Lin [7] derive a partial backlogging inventory model for non instantaneous deteriorating items with stock-dependent demand rate under inflation over a finite planning horizon.

In this chapter, inventory models have been developed for single deteriorating items with variable deterioration rate and stock dependent demand with variable holding cost. Shortages are allowed and partially backlogged. The backlogged rate is dependent on the duration of the waiting time up to the arrival of next lot. The present work attempts to model the situations where selling rate depends on the current stock level and items have time dependent deterioration rate with partial backlogging.

Notations

C_1 = Setup cost for each replenishment.

C_2 = Inventory carrying or holding cost per unit time.

C_3 = Shortage cost for backlogged items.

C_4 = Deterioration cost per unit time.

C_5 = The unit cost of lost sale.

$R(t)$ = The variable deterioration rate at any time $t, t \geq 0$.

$I(t)$ = The level of inventory at any time $t, t \geq 0$.

$D(t)$ = The demand rate at any time $t, t \geq 0$.

P = The initial inventory after fulfilling backorders.

I_h = Total number of units holding during $(0, t_1)$.

I_d = Total number of deteriorated units during $(0, t_1)$.

I_s = Total number of shortages units during (t_1, T) .

I_l = Total amount of lost sales during (t_1, T) .

K_1 = Total average cost of the system per unit time.

Assumptions

- The replenishment occurs instantaneously at an infinite rate.
- No replenishment or repair of deteriorated items is made during a given cycle.
- A single item is considered over the prescribed period T units of time which is subject to variable deterioration rate.
- The time dependent demand rate $D(t) = a + bI(t) + ct$, where $a > 0$, and $0 < b < 1$.
- Unsatisfied demand is backlogged at a rate $\exp(-qt)$, where t is the time up to next replenishment and q is positive constant.
- Lead time is zero.
- The backlogged rate is dependent on the duration of the waiting time up to the arrival of next lot.
- Shortages are allowed and partially backlogged.
- T is the planning horizon and t_1 is the time at which shortage starts.
- Shortages are allowed and partially backlogged
- Inventory models have been developed for single deteriorating items with variable deterioration rate.

Model

When the variable deterioration rate $R(t) = Rt$, $t > 0$. The differential equations governing the inventory level $I(t)$ an any time t during the cycle $(0, T)$ are such as

$$\frac{dI(t)}{dt} + R(t)I(t) = -D(t), \quad 0 \leq t \leq t_1 \quad \dots \dots (1)$$

$$\frac{dI(t)}{dt} = -D(t)e^{-qt} \quad t_1 \leq t \leq T \quad \dots \dots (2)$$

with the conditions, $I(t) = P$ when $t = 0$ (3)

and $I(t) = 0$ when $t = t_1$ (4)

Substituting the value of $D(t)$ and $R(t)$ in the equation (1) and (2)

$$\frac{dI(t)}{dt} + (Rt + b)I(t) = -(a + ct), \quad 0 \leq t \leq t_1 \quad \dots \dots (5)$$

$$\frac{dI(t)}{dt} = -(a + ct)e^{-qt}, \quad t_1 \leq t \leq T \quad \dots \dots (6)$$

The solution of equation (5) is given by

$$I(t)e^{\left(\frac{Rt^2}{2}+bt\right)} = \int -(a + ct)e^{\left(\frac{Rt^2}{2}+bt\right)}dt + A_1$$

Where A_1 is constant of integration

$$I(t) = -\left[at + \frac{aRt^3}{6} + \frac{abt^2}{2} + \frac{ct^2}{2} + \frac{cRt^4}{8} + \frac{bct^3}{3}\right]e^{-\left(\frac{Rt^2}{2}+bt\right)} + A_1e^{-\left(\frac{Rt^2}{2}+bt\right)} \dots (7)$$

But $I(t) = P$, when $t = 0$. Therefore from equation (7)

$$I(t) = -\left[at + \frac{aRt^3}{6} + \frac{abt^2}{2} + \frac{ct^2}{2} + \frac{cRt^4}{8} + \frac{bct^3}{3}\right]e^{-\left(\frac{Rt^2}{2}+bt\right)} + Pe^{-\left(\frac{Rt^2}{2}+bt\right)} \dots (8)$$

But $I(t) = 0$ when $t = t_1$, therefore

$$P = \left[at_1 + \frac{aRt_1^3}{6} + \frac{abt_1^2}{2} + \frac{ct_1^2}{2} + \frac{cRt_1^4}{8} + \frac{bct_1^3}{3}\right] \dots (9)$$

Substitute the value of P from equation (9) in equation (8), We have

$$I(t) = \left[a(t_1 - t) + (aR + 2bc)\left(\frac{t_1^3}{6} - \frac{t^3}{6}\right) + (ab + c)\left(\frac{t_1^2}{2} - \frac{t^2}{2}\right) + (cR)\left(\frac{t_1^4}{8} - \frac{t^4}{8}\right)\right]e^{-\left(\frac{Rt^2}{2}+bt\right)}, \quad 0 \leq t \leq t_1 \quad (10)$$

The solution of equation (6) is given by

$$\frac{dI(t)}{dt} = -(a + ct)e^{-qt}, \quad t_1 \leq t \leq T$$

Where A_2 is the constant of integration

$$I(t) = \frac{ae^{-qt}}{q} + \frac{cte^{-qt}}{q} + \frac{ce^{-qt}}{q^2} + A_2 \dots (11)$$

But $I(t) = 0$, when $t = t_1$, therefore

$$I(t) = \frac{a}{q} [(e^{-qt} - e^{-qt_1}) + (te^{-qt} - t_1e^{-qt_1})] + \frac{c}{q^2} (e^{-qt} - e^{-qt_1}), \quad t_1 \leq t \leq T \tag{12}$$

total number of units holding during (0, t₁) is given by

$$I_h = \int_0^{t_1} C_2 I(t) dt$$

$$I_h = C_2 \left\{ \frac{at_1^2}{2} + \frac{1}{8} (aR + 2bc)t_1^4 + \frac{1}{3} (ab + c)t_1^3 + \frac{1}{10} cRt_1^5 - \frac{1}{24} aRt_1^4 - \frac{1}{72} (aR + 2bc)Rt_1^6 - \frac{1}{30} (ab + c)Rt_1^5 - \frac{1}{84} cR^2t_1^7 - \frac{1}{6} abt_1^3 - \frac{1}{20} (aR + 2bc)Rt_1^5 - \frac{1}{8} (ab + c)bt_1^4 - \frac{1}{24} cRbt_1^6 \right\} \dots \dots \dots (13)$$

Total amount of deteriorated units during the period (0, t₁) is given by

$$I_d = \int_0^{t_1} R(t)I(t) dt$$

$$I_d = R \left\{ \frac{at_1^3}{6} + \frac{1}{20} (aR + 2bc)t_1^5 + \frac{1}{8} (ab + c)t_1^4 + \frac{1}{24} cRt_1^6 - \frac{1}{40} aRt_1^5 - \frac{1}{112} (aR + 2bc)Rt_1^7 - \frac{1}{48} (ab + c)Rt_1^6 - \frac{1}{128} cR^2t_1^8 - \frac{1}{12} at_1^4 - \frac{1}{36} (aR + 2bc)t_1^6 - \frac{1}{15} (ab + c)bt_1^5 - \frac{1}{42} cRbt_1^7 \right\} \dots \dots (14)$$

Total number of shortage units during the period (t₁, T) is given by

$$I_s = - \int_{t_1}^T I(t) dt$$

$$I_s = \frac{1}{q^3} \left\{ (e^{-qT} - e^{-qt_1}) + q \left[a(e^{-qT} - e^{-qt_1}) + c \left(T + \frac{1}{q} \right) e^{-qT} - c \left(t_1 + \frac{1}{q} \right) e^{-qt_1} + (T - t_1)e^{-qt_1} \right] + q^2 [a(T - t_1)e^{-qt_1} + c(T - t_1)t_1e^{-qt_1}] \right\}, \quad t_1 \leq t \leq T \dots \dots \dots (15)$$

and total amount of lost sales during the period (t₁, T) is given by

$$I_L = \int_{t_1}^T (1 - e^{-qt})a dt$$

$$I_L = a \left\{ (T - t_1) + \frac{(e^{-qT} - e^{-qt_1})}{q} \right\}, \quad t_1 \leq T$$

..... (16)

Hence, the total average cost of the system per unit time is given by

$$K_1 = \frac{1}{T} [C_1 + C_2 I_h + C_3 I_d + C_4 I_s + C_5 I_L] = \frac{S_1}{T}$$

$$K_1 = \frac{1}{T} \left[C_1 + C_2 \left\{ \frac{at_1^2}{2} + \frac{1}{8}(aR + 2bc)t_1^4 + \frac{1}{3}(ab + c)t_1^3 + \frac{1}{10}cRt_1^5 \right. \right.$$

$$\left. - \frac{1}{24}aRt_1^4 - \frac{1}{72}(aR + 2bc)Rt_1^6 - \frac{1}{30}(ab + c)Rt_1^5 - \frac{1}{84}cR^2t_1^7 \right.$$

$$\left. - \frac{1}{6}abt_1^3 - \frac{1}{20}(aR + 2bc)Rt_1^5 - \frac{1}{8}(ab + c)bt_1^4 - \frac{1}{24}cRbt_1^6 \right\}$$

$$+ C_3 R \left\{ \frac{at_1^3}{6} + \frac{1}{20}(aR + 2bc)t_1^5 + \frac{1}{8}(ab + c)t_1^4 + \frac{1}{24}cRt_1^6 \right.$$

$$\left. - \frac{1}{40}aRt_1^5 - \frac{1}{112}(aR + 2bc)Rt_1^7 - \frac{1}{48}(ab + c)Rt_1^6 - \frac{1}{128}cR^2t_1^8 \right.$$

$$\left. - \frac{1}{12}at_1^4 - \frac{1}{36}(aR + 2bc)t_1^6 - \frac{1}{15}(ab + c)bt_1^5 - \frac{1}{42}cRbt_1^7 \right\}$$

$$+ C_4 \frac{1}{q^3} \left\{ (e^{-qT} - e^{-qt_1}) \right.$$

$$+ q \left[a(e^{-qT} - e^{-qt_1}) + c \left(T + \frac{1}{q} \right) e^{-qT} - c \left(t_1 + \frac{1}{q} \right) e^{-qt_1} \right.$$

$$\left. + (T - t_1)e^{-qt_1} \right] + q^2 [a(T - t_1)e^{-qt_1} + c(T - t_1)t_1e^{-qt_1}] \left. \right\}$$

$$+ C_5 a \left\{ (T - t_1) + \frac{(e^{-qT} - e^{-qt_1})}{q} \right\} \left. \right]$$

$$= \frac{S_1}{T} \quad \text{..... (17)}$$

Approximation Solution Procedure

To minimize total average cost per unit time, the optimal values of t_1 and T can be obtained by solving the following equations simultaneously

$$\frac{\partial K_1}{\partial t_1} = 0 \tag{18}$$

and $\frac{\partial K_1}{\partial T} = 0 \tag{19}$

provided, they satisfy the following conditions

$$\frac{\partial^2 K_1}{\partial t_1} > 0, \quad \frac{\partial^2 K_1}{\partial T^2} > 0 \quad \dots(20)$$

$$\text{and} \quad \left(\frac{\partial^2 K_1}{\partial t_1^2} \right) \left(\frac{\partial^2 K_1}{\partial T^2} \right) - \left(\frac{\partial^2 K_1}{\partial t_1 \partial T} \right)^2 > 0 \quad \dots(21)$$

The equations (18) and (19) are equivalent to the following respectively

$$\begin{aligned} C_2 & \left[\left\{ \left(1 - \frac{1}{6} R t_1^2 - \frac{1}{2} b t_1 \right) \left(a t_1 + \frac{1}{2} (aR + 2bc) t_1^3 + (ab + c) t_1^2 + \frac{1}{2} c R t_1^4 \right) \right\} \right. \\ & + C_2 R \left\{ \left(\frac{1}{2} - \frac{1}{8} R t_1^2 - \frac{1}{3} t_1 \right) \left(a t_1^2 + \frac{1}{2} (aR + 2bc) t_1^4 + (ab + c) t_1^3 \right. \right. \\ & \left. \left. + \frac{1}{2} c R t_1^5 \right) \right\} \\ & + C_4 \left\{ \left(\frac{1 + c + c(qt_1 - 2)}{q} + a \right) t_1 e^{-qt_1} \right. \\ & \left. - \left(\frac{1 + c(qt_1 - 1)}{q} + a \right) T e^{-qt_1} \right\} + C_5 (e^{-qt_1} - 1) \Big] \\ & = 0 \quad \dots \dots \dots (22) \end{aligned}$$

And

$$\begin{aligned} S_1 + \frac{TC_4}{q} \left\{ \left(\frac{1}{q} + a + cT \right) e^{-qT} - \left(\frac{1}{q} + a + ct_1 \right) e^{-qt_1} \right\} \\ + TC_5 a (e^{-qT} - 1) = 0 \quad \dots \dots \dots (23) \end{aligned}$$

The numerical solution of these equations can be obtained by using some suitable computational numerical method.

Conclusion

We have discussed an optimization framework to derive optimal replenishment policy for perishable items with stock dependent demand rate. The demand rate is assumed to be a function of on hand inventory level and the inventory deteriorates per unit of time with variable deterioration rate. Shortages are allowed and partially backlogged with a variable rate, which depends on the duration of waiting time up to the arrival of next lot. The present work attempts to model the situations where selling rate depends on the current stock level and items have time dependent deterioration rate with partial backlogging. The model can be applied to determine optimal inventory policy in situations such as super market bakeries, stationery stores, and fancy items which may exhibit the characteristics modeled here.

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