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Unsteady Flow and Heat Transfer of a Dusty Fluid between Two Parallel Plates

G.S. Roopa, B.J. Gireesha[*] and C.S. Bagewadi

Department of Studies and Research in Mathematics, Kuvempu University,
Shankaraghatta-577 451, Shimoga, Karnataka, India
**Corresponding Author Email: bjgireesu@rediffmail.com*

Abstract

The purpose of the present paper is to study an unsteady laminar flow and heat transfer of an incompressible, conducting viscous dusty fluid between two infinitely non-conducting parallel plates. The flow is influenced by pulsatile pressure gradient, uniform magnetic field which is applied perpendicular to the plates and due to oscillations of the plates. The governing partial differential equations are solved numerically using finite differences. The effect of variation in the magnetic field and number density on the velocity profiles of both fluid and dust phase have been discussed. Further the effect of Prandtl number and Eckert number on the temperature profiles have been depicted graphically.

Keywords: Dusty fluid, pulsatile pressure gradient, heat transfer, velocities of dust and fluid phase, oscillating plates, Prandtl number, Eckert number, Numerical solution.

AMS Subject Classification (2010): 76T15, 80A20.

Introduction

The flow and heat transfer of dusty fluids between the parallel plates has been analyzed by number of authors due to its important applications in the fields of fluidization, petroleum industry, air cool conditioners, and purification of crude oil, electrostatic precipitation, polymer technology and paint spraying.

Prasad and Ramacharyulu [14] have initiated the solution of an unsteady flow of a dusty incompressible fluid between two parallel plates under varying impulsive pressure gradient. Mitra and Bhattacharyya [13] have examined the solution of unsteady hydromagnetic laminar flow of a conducting dusty fluid between two parallel plates started impulsively from rest. Snigdha Saxena and Sharma [15] have studied on unsteady flow of an electrically conducting dusty viscous liquid between two parallel plates. Unsteady hydromagnetic flows of a dusty viscous fluid between two oscillating plates were investigated by Debnath and Ghosh [12]. Ganguly and Lahiri [9] have determined the oscillatory motion of dusty viscous incompressible fluid between two parallel plates.

Datta and Dalal [7] have obtained the solution for an unsteady flow and heat transfer of a dusty viscous incompressible fluid in a channel. Heat transfer in unsteady MHD oscillatory flow was analyzed by Anjali Devi and Jothimani [2]. Attia and Kotb [3] have investigated the solution of MHD flow between two parallel plates with heat transfer. N.C.Ghosh, B.C.Ghosh and Debnath [10] have obtained the solution of hydromagnetic flow of a dusty visco-elastic fluid between two infinite parallel plates. Bodosa and Borkakati [6] have discussed the MHD Coutte flow with heat transfer between two horizontal plates in the presence of a uniform transverse magnetic field.

Attia [4, 5] has obtained the numerical solution of an unsteady MHD Coutte flow and heat transfer of dusty fluid between parallel plates by varying some variable physical properties. A note on an unsteady flow of viscous fluid due to an oscillating plane wall was analyzed by Erdogan [8]. Gireesha, Bagewadi and Prasannakumara [11] have investigated the flow of an unsteady dusty fluid through rectangular channel under varying pulsatile pressure gradient, in frenet frame field system. Sreeharireddy, Nagarajan and Sivaiah [16] have studied the MHD flow of a dusty viscous conducting liquid between two parallel plates. Ajadi [17] has obtained an analytical solution of unsteady oscillatory particulate visco-elastic fluid between two walls.

The present investigation deals with the study of an unsteady laminar flow and heat transfer of viscous dusty fluid between two oscillating and also moving parallel plates, maintained at two constant but different temperatures. The flow analysis is carried out in two different cases. In both the case (i) and (ii), the flow is due to pulsatile pressure gradient and the uniform magnetic field. The dust particles are assumed to be spherical in shape and uniformly distributed throughout the fluid. The numerical solutions for the velocities and temperature profiles are determined using finite difference method. Further the effect of the magnetic field, number density on velocity and Prandtl number, Eckert number on temperature profiles for both the fluid and dust particles is discussed.

Equations of Motion

The governing equations of motion and energy for two phases are given by

For fluid phase

$$\nabla\cdot\vec{u}=0, \text{ (Continuity)} \tag{2.1}$$

$$\frac{\partial\vec{u}}{\partial t}+(\vec{u}\cdot\nabla)\vec{u}=-\frac{1}{\rho}\nabla p+\nu\nabla^2\vec{u}+\frac{KN}{\rho}(\vec{v}-\vec{u})-\frac{1}{\rho}\left(\vec{J}\times\vec{B}\right),$$

$$\text{(Linear Momentum)} \tag{2.2}$$

$$\rho\left\{\frac{\partial E}{\partial t}+(\vec{u}\cdot\nabla\mathrm{E})\right\}=Q+(\vec{v}-\vec{u})\cdot F+k\nabla\cdot(\nabla\mathrm{T}), \text{ (Energy)} \tag{2.3}$$

For dust phase:

$$\nabla \cdot \vec{v} = 0, \text{ (Continuity)} \tag{2.4}$$

$$\frac{\partial \vec{v}}{\partial t} + (\vec{v} \cdot \nabla)\vec{v} = \frac{K}{m}(\vec{u} - \vec{v}), \text{ (Linear Momentum)} \tag{2.5}$$

$$N\left\{\frac{\partial E_p}{\partial t} + (\vec{v} \cdot \nabla \mathrm{E_p})\right\} = -Q, \text{ (Energy)} \tag{2.6}$$

where $E = c_p T,\ E_p = c_m T_p, Q = N c_p (T_p - T)/\tau_T$ is the thermal interaction between fluid and dust particle phase, $F = N(\vec{v} - \vec{u})/\tau_v$ is the velocity interaction force between the fluid and dust particle phase, $\tau_v = m/6\pi a\mu = m/K$ is the velocity relaxation time of the dust particles, $\tau_T = mc_p/4\pi ak$ is the thermal relaxation time of the dust particles, $k\nabla \cdot (\nabla \mathrm{T})$ is the rate of heat added to the fluid by conduction in unit volume, $\vec{u}, \rho, p, \nu, T, c_p$ & k are respectively, the velocity vector, density, pressure, kinematic viscosity, temperature, specific heat and thermal conductivity of the fluid, $\vec{v}$, N, T_p, c_m & m are respectively, the velocity vector, number density, temperature, specific heat and mass concentration of dust particles, K the Stoke's resistance co-efficient (for spherical particles of radius a is $6\pi a\mu$), t is the time and $\vec{J}$ and $\vec{B}$ given by Maxwell's equations and Ohm's law, namely,

$$\nabla \times \vec{H} = 4\pi \vec{J}, \nabla \times \vec{B} = 0, \nabla \times \vec{E} = 0\ , \vec{J} = \sigma\left[\vec{E} + \vec{u} \times \vec{B}\right].$$

Here $\vec{H}$ - magnetic field, $\vec{J}$ - current density, $\vec{B}$ - magnetic Flux, $\vec{E}$ - electric field and σ - the electrical conductivity of the fluid.

It is assumed that the effect of induced magnetic fields produced by the motion of the electrically conducting gas is negligible and no external electric field is applied. With those assumptions the magnetic field $\vec{J} \times \vec{B}$ of the body force in (2.2) reduces simply to $-\sigma B_0^2 \vec{u}$ where B_0 - the intensity of the imposed transverse magnetic field.

Formulation of the Problem

Consider an unsteady laminar flow and heat transfer of an incompressible, viscous dusty fluid between two parallel plates separated by a distance h. Throughout the flow analysis the following assumptions are made,

- Two plates are electrically non-conducting and kept at constant temperatures T_0 for the lower plate and T_1 for the upper plate with $T_1 > T_0$.
- The flow is due to the influence of the pulsatile pressure gradient, uniform magnetic field and oscillations of the plates.
- A pressure gradient is applied in the x-direction and a uniform magnetic field is applied in positive y-direction.
- The dust particles are assumed to be spherical in shape and uniformly distributed throughout the fluid.
- Initially the conducting fluid and non-conducting dust particles are assumed to be at rest.

The number density of the dust particles is taken as a constant.

Under these assumptions the geometry of the flow configuration is shown as in the figure-1.

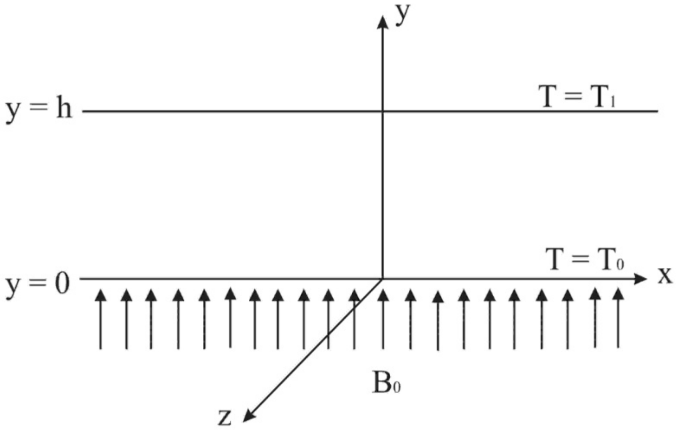

Figure 1: Geometry of the flow configuration.

For the above described flow the velocities of both fluid and dust particles are given by

$\vec{u} = u(y,t)\hat{\imath}\,, \vec{v} = v(y,t)\hat{\imath}\,.$

Solution of the Problem

The governing equations from (2.1) to (2.6) can be decomposed as,

For fluid phase:

$$\frac{\partial u}{\partial t} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \nu\frac{\partial^2 u}{\partial y^2} + \frac{KN}{\rho}(v-u) - \frac{\sigma H_0^2}{\rho}u, \tag{4.1}$$

$$\rho c_p \frac{\partial T}{\partial t} = k\frac{\partial^2 T}{\partial y^2} + \frac{Nc_p}{\tau_T}\left(T_p - T\right) - \frac{KN}{m}(v-u)^2, \tag{4.2}$$

For dust phase:

$$\frac{\partial v}{\partial t} = \frac{K}{m}(u-v), \tag{4.3}$$

$$c_m\frac{\partial T_p}{\partial t} = -\frac{c_p}{\tau_T}\left(T_p - T\right), \tag{4.4}$$

where $u(y,t)$ and $v(y,t)$ denote the velocity of the fluid and of the dust phase respectively.

The initial and boundary conditions on the velocity fields are taken as

Case (i)

Initial conditions: $u = 0, v = 0$ at $t \leq 0$ for all y,

Boundary conditions: $u = a_1 e^{i\omega_1 t} + a_2 e^{-i\omega_1 t}$ at $y = 0$ for $t > 0$, (4.5)
$u = b_1 e^{i\omega_2 t} + b_2 e^{-i\omega_2 t}$ at $y = h$.

where a_1, a_2, b_1, b_2 are complex constants, such that u becomes real on the plates.

Case (ii)
Initial conditions: $u = 0, v = 0$ at $t \leq 0$ for all y,
Boundary conditions: $u = U_1$ at $y = 0$ for $t > 0$, (4.6)
$u = V_1$ at $y = h$.

Here the lower and upper plates start moving with uniform velocities U_1 and V_1 respectively.

The common initial and boundary conditions of the temperature fields for the above two cases are
Initial conditions: $T = T_p = T_0$ at $t \leq 0$,
Boundary conditions: $T = T_p = T_0$ at $y = 0$ for $t > 0$, (4.7)
$T = T_p = T_1$ at $y = h$.

It is assumed that the pulsatile pressure gradient is influenced on the flow and it is of the form,

$$-\frac{1}{\rho}\frac{\partial p}{\partial x} = A\left[1 + \varepsilon e^{i\alpha t}\right] \tag{4.8}$$

where ε is a small quantity, A and α are constants.

Let us consider the following non-dimensional flow variables

$$\bar{u} = \frac{uh}{\nu}, \bar{v} = \frac{vh}{\nu}, \bar{t} = \frac{\nu t}{h^2}, \bar{x} = \frac{x}{h}, \bar{y} = \frac{y}{h},$$
$$\bar{p} = \frac{ph^2}{\rho\nu^2}, \theta = \frac{T - T_0}{T_1 - T_0}, \theta_p = \frac{T_p - T_0}{T_1 - T_0}, \tag{4.9}$$

where θ and θ_p − dimensionless fluid and dust phase temperatures.

Using the above non-dimensional variables in the equations (4.1)-(4.4), then one can get the following non-dimensionalized form of the equations (on dropping the bars) as follows

$$\frac{\partial u}{\partial t} = \frac{\partial p}{\partial x} + \frac{\partial^2 u}{\partial y^2} + \frac{f}{\tau_v}(v - u) - M^2 u, \tag{4.10}$$
$$\frac{\partial u}{\partial t} = \frac{1}{\tau_v}(u - v), \tag{4.11}$$
$$\frac{\partial \theta}{\partial t} = \frac{1}{Pr}\frac{\partial^2 \theta}{\partial y^2} + \frac{l_1}{\tau_T}\left(\theta_p - \theta\right) + \frac{l_1 N}{\tau_v} Ec (v - u)^2, \tag{4.12}$$
$$\frac{\partial \theta_p}{\partial t} = l_2\left(\theta - \theta_p\right), \tag{4.13}$$

where $f = \frac{Nm}{\rho}$ the mass concentration of dust particles, $\tau_v = \frac{m\nu}{Kh^2}$ relaxation time of particles, $M = B_0 h\sqrt{\frac{\sigma}{\mu}}$ Hartmann number, $l_1 = \frac{h^2}{\mu}, l_2 = \frac{c_p h^2}{c_m \tau_T}, Pr = \frac{\mu c_p}{k}$ Prandtl number, $Ec = \frac{\nu^2}{h^2 c_p (T_1 - T_0)}$ Eckert number.

The non-dimensional form of initial and boundary conditions on the velocity fields are

For Case (i)

Initial conditions: $u = 0, v = 0$ for $t \leq 0$,
Boundary conditions: $u = a_1 e^{i\sigma_1 t} + a_2 e^{-i\sigma_1 t}$ at $y = 0$, (4.14)
$u = b_1 e^{i\sigma_2 t} + b_2 e^{-i\sigma_2 t}$ at $y = 1$.

where $(\sigma_1, \sigma_2) = \frac{h^2}{\nu}(\omega_1, \omega_2)$ and $(\overline{a_1}, \overline{a_2}, \overline{b_1}, \overline{b_2}) = \frac{h^2}{\nu}(a_1, a_2, b_1, b_2)$ are non-dimensional flow parameters.

For Case (ii)

Initial conditions: $u = 0, v = 0$ for $t \leq 0$,
Boundary conditions: $u = U_2$ at $y = 0$, (4.15)
$u = V_2$ at $y = 1$.

where $U_2 = \frac{h}{\nu} U_1$ and $V_2 = \frac{h}{\nu} V_1$.

In the same way, the dimensionless form of initial and boundary conditions on the temperature fields are
Initial conditions: $\theta = \theta_p = 0$ for $t \leq 0$,
Boundary conditions: $\theta = \theta_p = 0$ at $y = 0$, (4.16)
$\theta = \theta_p = 1$ at $y = 1$.

The non-dimensional form of pressure gradient as given by

$$\frac{\partial p}{\partial x} = A\left[1 + \varepsilon e^{i\beta t}\right], \tag{4.17}$$

where $\beta = \frac{\alpha h^2}{\nu}$.

Numerical Solution

The system of partial differential equations from (4.10) to (4.13) under the initial and boundary conditions for case (i) and (ii) are solved numerically using finite difference method with the help of MATLAB software. In the absence of pressure gradient, the equations (4.10) and (4.11) under the initial and boundary conditions (4.14) are solved analytically by Lokenath Debnath and Ghosh [12] and also they have consider the special cases like (a) hydromagnetic fluid flow and (b) Non oscillatory two phase hydromagnetic flow. Further, using the initial and boundary conditions (4.15), the analytical solutions are obtained by Mitra and Bhattacharyya [13]. The accuracy of this method is verified by comparing our obtained numerical solutions with the analytical solutions for both the cases, which are given in the table 1. From this table, we can observed that a very good agreement between the results. Further table 2 shows the variations of temperature of the fluid phase for different values of the parameters Prandtl number, Eckert number and number density at $y = 0.2$. Analyzing this table reveals that the increasing the value of the Prandtl number and number density increases the temperature of the fluid phase $\theta(y,t)$. The table 2 also shows that temperature between the plates increases gradually with increase of Ec in both case (i) and (ii).

Table 1: Comparison of analytical and numerical solutions of fluid velocity $u(y,t)$ for case (i) and (ii).

y	Case(i)		Case(ii)	
	Analytical Solution [12]	Numerical Solution	Analytical Solution [13]	Numerical Solution
0.0	1.9601	1.9601	1.0000	1.0000
0.2	1.2949	1.2395	0.6024	0.5791
0.4	1.1195	1.0543	0.4290	0.4051
0.6	1.4035	1.3289	0.4290	0.4051
0.8	2.2494	2.1661	0.6024	0.5791
1.0	3.9203	3.9203	1.0000	1.0000

Results and Discussion

We have considered an unsteady flow and heat transfer of a dusty fluid between two oscillating and moving non-conducting parallel plates. The numerical solutions are obtained for the governing equations from (4.10) to (4.13) for both the cases. A comprehensive parametric study is performed to show the effects of the Hartmann number M, Number density of the dust particles N, Prandtl number Pr and Eckert number Ec on the velocity and temperature profiles. The variation of velocity and temperature profiles for the fluid and dust particles for case (i) and (ii) are plotted in graphs 2 to 17, which shows parabolic nature. Further one can observe that if the dust is very fine, relaxation time of dust particle decreases and ultimately as $\tau_v \to 0$, the velocities of fluid and dust particles will be the same. Here the result of u and v for case (i) and (ii) describes the velocity of fluid and dust phase respectively for the general case, if $\omega_1 = \omega_2 = \omega$ and if $U_1 = 1, V_1 = 2$. The graphs are drawn for the values $\omega = 1, \varepsilon = 0.2, m = 1, N = 10, \tau_v = 1$, $\tau_T = 0.5$, $c_p = c_m = 0.2$ and $t = 0.2$.

Figures 2 to 5 shows the effect of the strength of the magnetic field M (Hartmann number) on velocity profiles. It is evident from that, as we increase the strength of the magnetic field, it has an appreciable effect on the velocities of fluid and dust particles, i.e., increase in magnetic field shows the decrease of velocity for case (i) and (ii).

In both cases, the velocity distribution of both fluid and dust phase against the vertical distance y for different values of Number density N are shown in the figures 6, 7, 8 and 9. Here one can observe the decreasing of velocity while the number density of the dust particles is increasing. This result is very relevant to the traffic problem when we consider the vehicles as dust particles.

The Figures 10 to 13 are obtained by plotting the temperature distribution for both fluid and dust phase against variable y for different values of Prandtl number Pr i.e., $Pr = 0.72, 1.0, 2.0$ and 3.0 for case (i) and (ii). Form these graphs it is clear that the temperature distribution between the plates increases gradually with the increase of Pr. However the values of temperature increases towards the plate $y > 0$ and decreases towards the plate $y < 0$.

The temperature distribution for both the phases versus y for different values of the Eckert number, are drawn in the figures 14 to 17. The graphs indicates that the temperature profiles for both fluid and dust particles increases with increase in Eckert number.

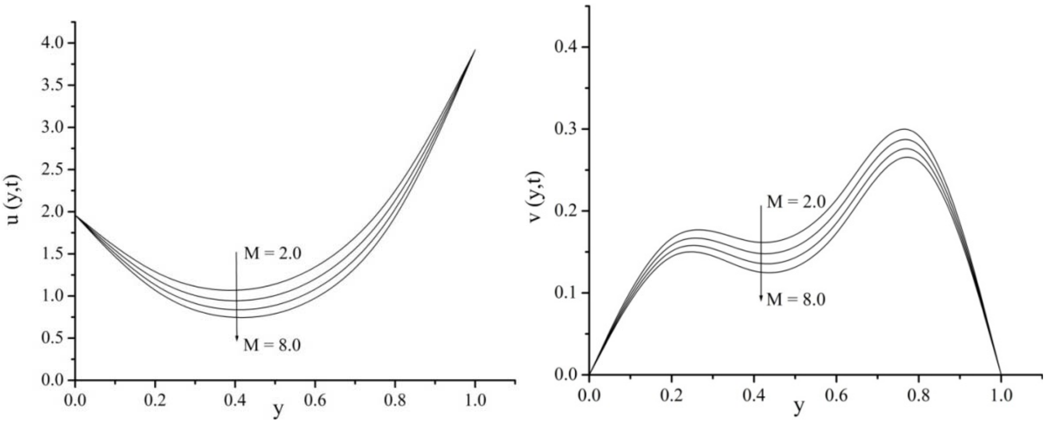

Figure 2, 3: Influence of Magnetic field on the velocity distributions for fluid and dust phase (case (i)).

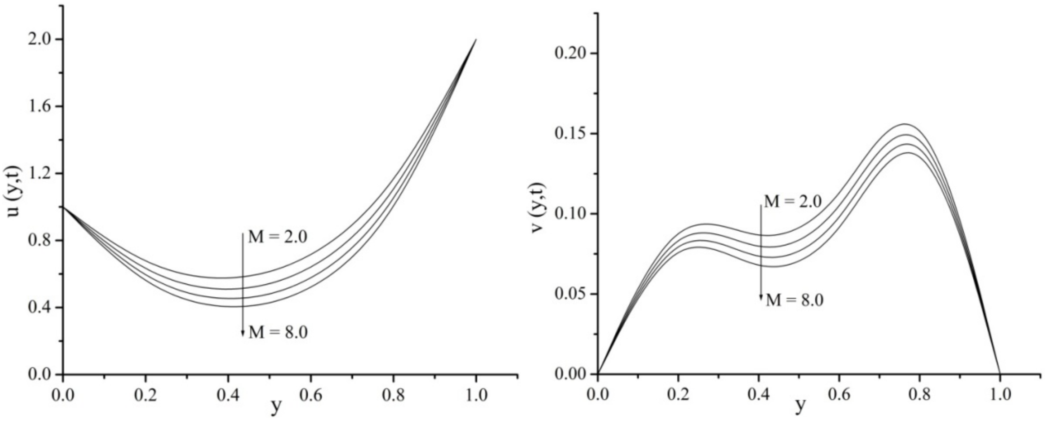

Figure 4, 5: Influence of Magnetic field on the velocity distributions for fluid and dust phase (case (ii)).

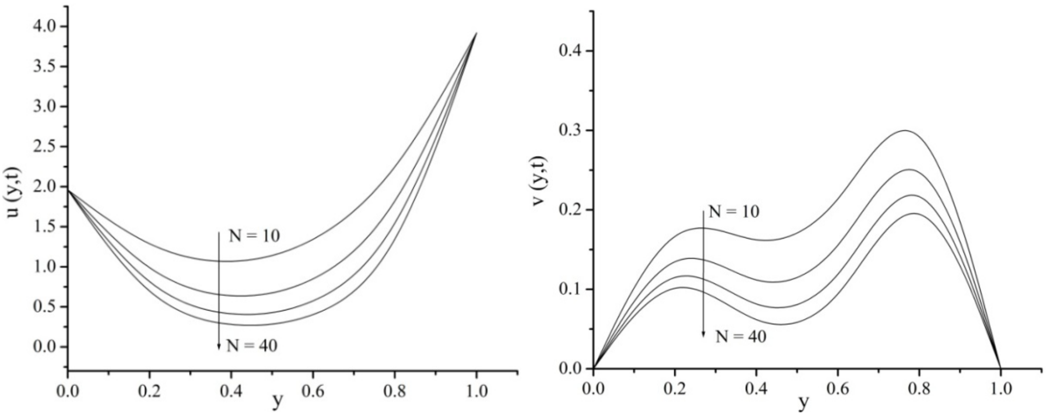

Figure 6, 7: Influence of Number density on the velocity distributions for fluid and dust phase (case (i)).

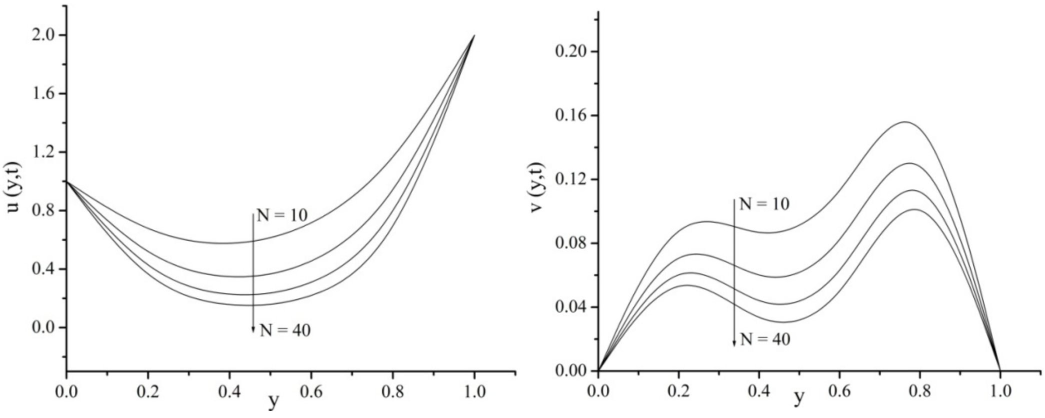

Figure 8, 9: Influence of Number density on the velocity distributions for fluid and dust phase (case (ii)).

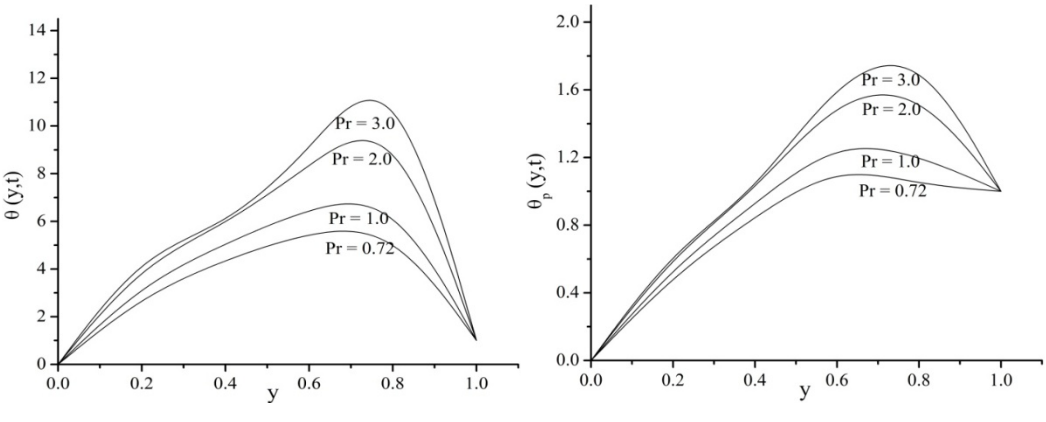

Figure 10, 11: Influence of Prandtl number on the temperature distributions for fluid and dust phase (case (i)).

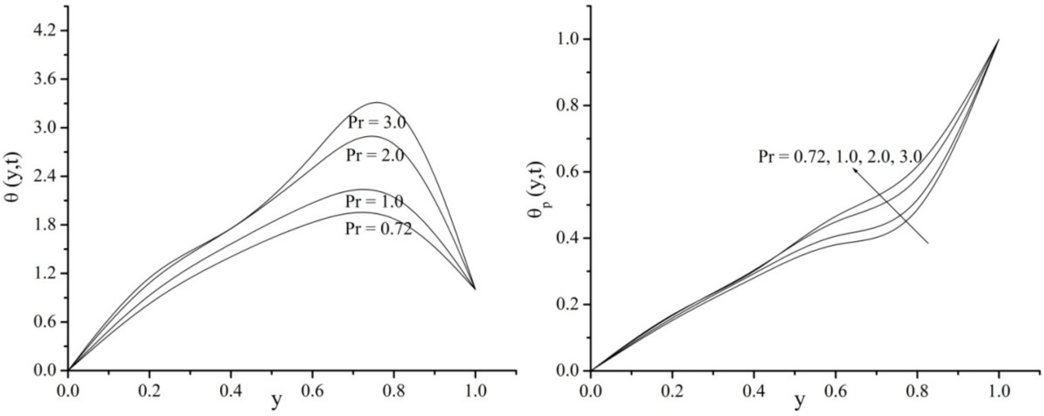

Figure 12, 13: Influence of Prandtl number on the temperature distributions for both fluid and dust phase (case (ii)).

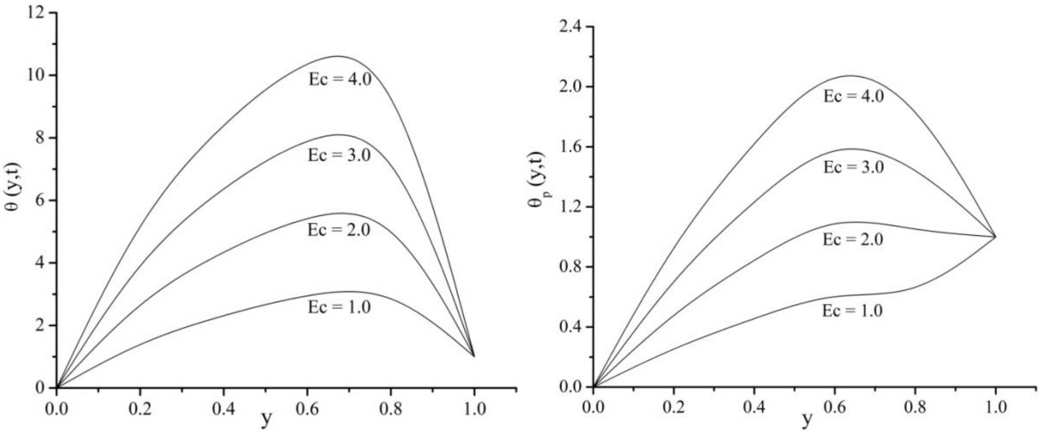

Figure 14, 15: Influence of Eckert number on the temperature distributions for fluid and dust phase (case (i)).

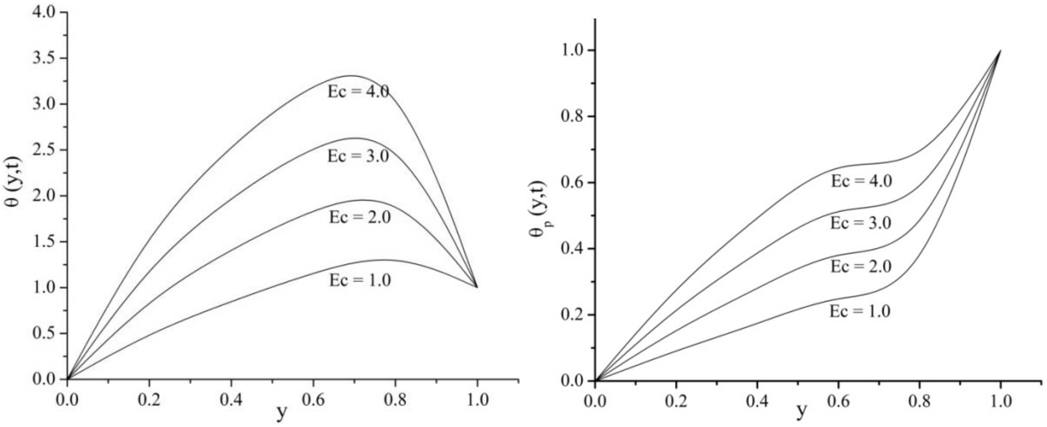

Figure 16, 17: Influence of Eckert number on the temperature distributions for fluid and dust phase (case (ii)).

Conclusions

In this paper, an unsteady laminar flow of an incompressible viscous fluid with uniform distribution of dust particles between two infinitely non-conducting parallel plates with a uniform magnetic field is considered. Here the flow is studied in the following two cases, (i) between two oscillating plates and (ii) impulsive motion of two parallel plates. In both cases, the effect of pulsatile pressure gradient is considered. The governing equations for this investigation were non-dimensionalized and solved numerically using finite difference method with the help of MATLAB software. Table 1 show the accuracy of these numerical solutions was validated for the case (i) and (ii) by a comparison with the analytical solutions reported by [12] and [13]. It can be seen from this table that a very good agreement between the results exists. Table 2 shows the variations of temperature of the fluid phase for different values of the parameters Prandtl number, Eckert number and number density. Further graphical results of velocity profiles for both fluid and dust phase are presented and discussed to show the effects of the Hartmann number and the number density. In addition, the numerical solutions of energy equations were performed graphically to show the influence of the Prandtl number and Eckert number on the temperature profiles, of both phases for the case (i) and (ii). It shows that, decreases the velocities of both fluid and dust phase for the effects of increasing the strength of the magnetic field and number density. And also the temperature distribution between plates increases gradually with the increase of Prandtl number Pr and Eckert number Ec.

Table 2: The variation of temperature of the fluid phase for different values of the parameters Pr, Ec and $N \; at \; y = 0.2$.

Pr	Ec	N	$\theta(y,t)$	
			Case(i)	Case(ii)
0.72	2.0	10	2.8710	0.8858
1.0			3.3713	1.0033
2.0			4.2417	1.1980
0.72	1.0	10	1.5011	0.5086
	2.0		2.8710	0.8858
	3.0		4.2407	1.2630
0.72	2.0	10	2.8710	0.8858
		20	3.6210	1.0821
		30	4.0698	1.1938

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