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Manpower Model for a Two Grade System with Correlated Inter-Decision Times and Univariate Policy of Recruitment

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Abstract

In this paper, an organization having two grades following univariate policy of recruitment wherein depletion of manpower occurs at every decision epoch is analyzed. In order to keep going the system with usual performance it is assumed that the transfer of persons is permitted to the grade which has greater loss. The inter-decision times are exchangeable and constantly correlated exponential random variables. By considering the threshold of the organization as the sum of threshold levels of the two grades, the expectation and the variance of time to recruitment are obtained.

Keywords: Manpower planning, Shock models, Univariate recruitment policy, Mean and Variance of time to recruitment.

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Introduction

Frequent wastage or exit of personnel is common in many administrative and production oriented organization. Whenever the organizations announces revised policies regarding sales target, revision of wages, incentives and perquisites, the exodus is possible. Reduction in the total strength of marketing personnel adversely affects the sales turnover in the organization. As the recruitment involves several costs, it is usual that the organization has the natural reluctance to go in for frequent recruitments. Once the total amount of wastage crosses a certain threshold level, the organization reaches an uneconomic status which otherwise be called the breakdown point and recruitment is done at this point of time. The time to attain the breakdown point is an important characteristic for the management of the organization. Many models using different kinds of wastages could be seen in Barthlomew[1] and Barthlomew and Frobes[2]. Senthamizhselvi and Srinivasan[3] have obtained the mean time to recruitment for a two grade manpower organization under different conditions. Sathiyamoorthi and Parthasarathy [4] have found out the mean and variance of time to recruitment for a two grade organization with transfer of persons between grades. They have assumed that the number of policy decisions announced in two grades are governed by the same renewal process and the threshold level of the organization is the maximum of the thresholds of two grades. Suresh Kumar et al. [5] have analyzed the same system by assuming the threshold level of the organization as the sum of the thresholds of two grades. Esther Clara and Srinivasan[6] have obtained the results for a discrete analog of the research work by Suresh Kumar et al. [5].The present paper extends that result of Suresh Kumar et al. [5] involving combined thresholds when inter-decision times are exchangeable and constantly correlated exponential random variables using the univariate policy of recruitment.

Model description

An organization having two grades I(II) in which decisions are taken atrandom epochs and at every decision making epochs, a random number of persons quit the organization. There is an associated loss of manhours to the organization if a person quits. Let $X_{i\,1}$ ($X_{i\,2}$) be independent and identically distributed random variable denoting manpower depletion in grade I, (grade II) due to i^{th} decision,$i\geq 1$. It is assumed to follow exponential distribution with parameter α_1 (α_2). Its probability density function g(.) (h(.)) and cumulative distribution function is G(.) (H(.)).The mobility of persons from one grade to the other is permitted. Each grade has its own threshold level Y_1 (Y_2) following exponential distribution with parameters θ_1 (θ_2).The loss of manhours is independent and independent of their respective thresholds. Further, Y_1 and Y_2 are independent random variables. The inter-decision times are assumed to form asequence of exchangeable and constantly correlated exponential random variables with common distribution F (.), mean a and correlation R. Let $F_k(.)$ be the k fold convolution of F.The recruitment is done, whenever the cumulative loss of manhours exceeds the combined threshold $Y_1 + Y_2$. Let W be a continuous random variable representing the time to the breakdown of the organization. Its probability density function is l(.) and cumulative distribution function is L(.).

Expected time to recruitment and variance of time to recruitment

As in Suresh kumar et.al [5], the time to recruitment is

$$P(W>t) = 1 - \left(\frac{\theta_1}{\theta_1-\theta_2}\right)[1-q^*(\theta_2)]\sum_{k=1}^{\infty} F_k(t)\,[q^*(\theta_2)]^{(k-1)} + \left(\frac{\theta_2}{\theta_1-\theta_2}\right)[1-q^*(\theta_1)]\sum_{k=1}^{\infty} F_k(t)\,[q^*(\theta_1)]^{(k-1)} \quad (1)$$

And its distribution function is

$$L(t) = 1 - P(W>t) = \left(\frac{\theta_1}{\theta_1-\theta_2}\right)[1-q^*(\theta_2)]\sum_{k=1}^{\infty} F_k(t)\,[q^*(\theta_2)]^{(k-1)} - \left(\frac{\theta_2}{\theta_1-\theta_2}\right)[1-q^*(\theta_1)]\sum_{k=1}^{\infty} F_k(t)\,[q^*(\theta_1)]^{(k-1)} \quad (2)$$

Taking Laplace-Stieltjes transform on both sides, we get

$$L^*(s) = \left(\frac{\theta_1}{\theta_1-\theta_2}\right)[1-q^*(\theta_2)]\sum_{k=1}^{\infty} F_k^*(s)\,[q^*(\theta_2)]^{(k-1)} - \left(\frac{\theta_2}{\theta_1-\theta_2}\right)[1-q^*(\theta_1)]\sum_{k=1}^{\infty} F_k^*(s)\,[q^*(\theta_1)]^{(k-1)} \quad (3)$$

As in [7], we get

$$F_k^*(s) = \frac{(1-R)\,(1+cs)^{1-k}}{(1-R)\,(1+cs)+kRcs} \quad (4)$$

Using (4) in (3) we get

$$E(W) = -\left(\frac{d}{ds}L^*(s)\right)_{s=0} = \left(\frac{a}{\theta_1-\theta_2}\right)\left\{\frac{\theta_1}{[1-q^*(\theta_2)]} - \frac{\theta_2}{[1-q^*(\theta_1)]}\right\} \quad (5)$$

$$E(W^2) = \left(\frac{d^2}{ds^2}L^*(s)\right)_{s=0} = \left(\frac{2a^2}{\theta_1-\theta_2}\right)\left[\frac{\theta_1(1+R^2q^*(\theta_2))}{[1-q^*(\theta_2)]^2} - \frac{\theta_2(1+R^2q^*(\theta_1))}{[1-q^*(\theta_1)]^2}\right] \quad (6)$$

Density function of $X_{i1} + X_{i2}$ is given by

$$q(x) = \int_0^x g(u)\,h(x-u)\,du = \frac{\alpha_1\,\alpha_2\,(e^{-\alpha_2 x} - e^{-\alpha_1 x})}{\alpha_1-\alpha_2}\ .$$

Then

$$q^*(\theta_1) = \frac{\alpha_1\,\alpha_2}{(\theta_1+\alpha_1)\,(\theta_1+\alpha_2)} \text{ and } q^*(\theta_2) = \frac{\alpha_1\,\alpha_2}{(\theta_2+\alpha_1)\,(\theta_2+\alpha_2)} \quad (7)$$

Using (7) in (5) and (6) and on simplification , we have the mean time to recruitment

$$E(W) = \left(\frac{a}{\theta_1 - \theta_2}\right)\left[\frac{\theta_1 (\theta_2 + \alpha_1)(\theta_2 + \alpha_2)}{[\theta_2^2 + \theta_2 \ (\alpha_1 + \alpha_2)]} - \frac{\theta_2 (\theta_1 + \alpha_1)(\theta_1 + \alpha_2)}{[\theta_1^2 + \theta_1 \ (\alpha_1 + \alpha_2)]}\right] \quad (8)$$

And

$$E(W^2) = \left(\frac{\theta_1}{\theta_1 - \theta_2}\right)\frac{2a^2 (\theta_2 + \alpha_1)(\theta_2 + \alpha_2)[(\theta_2 + \alpha_1)(\theta_2 + \alpha_2) + R^2 \alpha_1 \alpha_2]}{[\theta_2^2 + \theta_2 (\alpha_1 + \alpha_2)]^2} - \left(\frac{\theta_2}{\theta_1 - \theta_2}\right)\frac{2a^2 (\theta_1 + \alpha_1)(\theta_1 + \alpha_2)[(\theta_1 + \alpha_1)(\theta_1 + \alpha_2) + R^2 \alpha_1 \alpha_2]}{[\theta_1^2 + \theta_1 (\alpha_1 + \alpha_2)]^2} \quad (9)$$

Using equations (8) and (9) the variance of time to recruitment can be obtained.

Numerical Illustration

The values of the expected time to recruitment and its variance are calculated and presented in the Table by varying a the mean of inter-decision times. Table shows that the expected time and variance of time to recruitment increase for the increase of the mean of inter-decision times.

Table Effect of a on performance measures (θ_1 = 0.2, θ_2 = 0.1, α_1 = 2, α_2 = 1, R = 0.5)

a	Expected time to recruitment	Variance of time to recruitment
0.1	1.0778	0.7857
0.2	2.1556	3.1426
0.3	3.2335	7.0709
0.4	4.3113	12.5705
0.5	5.3891	19.0414
0.6	6.4669	28.2836
0.7	7.5448	38.4971
0.8	8.6226	50.2819

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