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Wave Propagation in a Homogeneous Isotropic Thermoelastic Cylindrical Panel

P. Ponnusamy[1] and R. Selvamani[2]

[1]Department of Mathematics, Govt. Arts College (Autonomous), Coimbatore, Tamil Nadu, India.
[2]Department of Mathematics, Karunya University, Coimbatore, Tamil Nadu, India.
E-mail: selvam1729@gmail.com

Abstract

In this paper the three dimensional wave propagation of a homogenous Isotropic thermo elastic cylindrical panel is investigated in the context of the linear theory of thermo elasticity. Three displacement potential functions are introduced to uncouple the equations of motion. The frequency equations are obtained using the boundary conditions. A modified Bessel functions with complex argument is directly used to analyze the frequency equations and are studied numerically for the material Zinc. The computed non-dimensional frequencies are plotted in the form of dispersion curves with the support of MATLAB.

Keywords: isotropic cylindrical panel, thermo elasticity, modified Bessel function.

Introduction

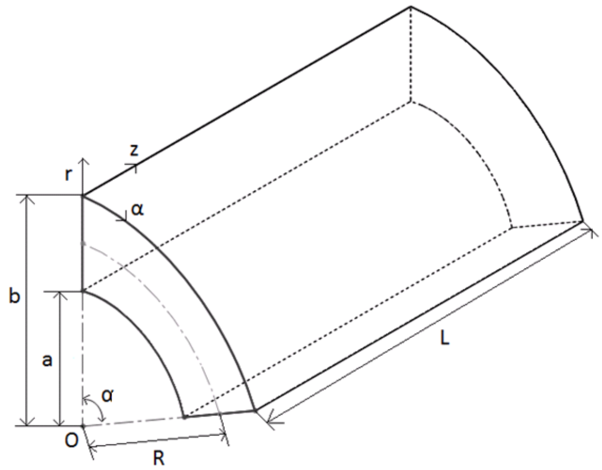
The analysis of thermally induced vibration of cylindrical panel is common place in the design of structures, atomic reactors, steam turbines, supersonic aircraft, and other devices operating at elevated temperature. In the field of nondestructive evaluation, laser-generated waves have attracted great attention owing to their potential application to noncontact and nondestructive evaluation of sheet materials. The high velocities of modern aircraft give rise to aerodynamic heating, which produces intense thermal stresses, reducing the strength of the aircraft structure. In the nuclear field, the extremely high temperatures and temperature gradients originating inside nuclear reactors influence their design and operations. Moreover, it is well recognized that the investigation of the thermal effects on elastic wave propagation has bearing on many seismological application. This study may be used in applications involving nondestructive testing (NDT), qualitative nondestructive evaluation (QNDE) of large diameter pipes and health monitoring of other ailing infrastructures in addition to check and verify the validity of FEM and BEM for such problems.

The static analysis cannot predict the behavior of the material due to the thermal stresses changes very rapidly. Therefore in case of suddenly applied loading, thermal deformation and the role of inertia getting more important. This thermo elastic stress response being significant leads to the propagation of thermo elastic stress waves in solids. The theory of thermo elasticity is well established by Nowacki [1]. Lord and Shulman [2] and Green and Lindsay [3] modified the Fourier law and constitutive relations, so as to get hyperbolic equation for heat conduction by taking into account the time needed for acceleration of heat flow and relaxation of stresses. A special feature of the Green–Lindsay model is that it does not violate the classical Fourier's heat conduction law. Vibration of functionally graded multilayered orthotropic cylindrical panel under thermo mechanical load was analyzed by X.Wang et.al [4]. Hallam and Ollerton [5] investigated the thermal stresses and deflections that occurred in a composite cylinder due to a uniform rise in temperature, experimentally and theoretically and compared the obtained results by a special application of the frozen stress technique of photoelasticity. Noda [6] have studied the thermal-induced interfacial cracking of magneto electro elastic materials under uniform heat flow. Chen et al [7] analyzed the point temperature solution for a pennay-shapped crack in an infinite transversely isotropic thermo-piezo-elastic medium subjected to a concentrated thermal load applied arbitrarily at the crack surface using the generalized potential theory. Banerjee and Pao [8] investigated the propagation of plane harmonic waves in infinitely extended anisotropic solids by taking into account the thermal relaxation time. Dhaliwal and Sherief [9] extended the generalized thermo elasticity to anisotropic elastic bodies. Chadwick [10] studied the propagation of plane harmonic waves in homogenous anisotropic heat conducting solids. Sharma and Sidhu[11] studied the propagation of plane harmonic thermo elastic wave in homogenous transversely isotropic, cubic crystals and anisotropic materials in the context of generalized thermo elasticity. Sharma[12] investigated the three dimensional vibration analysis of a transversly istropic thermo elastic cylindrical panel. The application of powerful numerical tools like finite element or boundary element methods to these problems is also becoming important. Prevost and Tao [13] carried out an authentic finite element analysis of problems including relaxation effects. Eslami and Vahedi [14] applied the Galerkin finite element to the coupled thermoelasticity problem in beams. Huang and Tauchert [15]derived the analytical solution for cross-ply laminated cylindrical panels with finite length subjected to mechanical and thermal loads using the extended power series method.

In this paper, the three dimensional wave propagation in a homogeneous isotropic thermo elastic cylindrical panel is discussed using the linear three-dimensional theory of elasticity. The frequency equations are obtained using the boundary conditions. A modified Bessel functions with complex argument is directly used to analyze the frequency equations by fixing the length to mean radius ratio and are studied numerically for the material Zinc. The computed non-dimensional frequencies are plotted in the form of dispersion curves.

The Governing equations

Consider a cylindrical panel as shown in Fig.1 of length L having inner and outer radius a and b with thickness h. The angle subtended by the cylindrical panel, which is known as center angle, is denoted by α. The deformation of the cylindrical panel in the direction r, θ, z are defined by u, v and w. The cylindrical panel is assumed to be homogenous, isotropic and linearly elastic with Young's modulus E, poisson ratio ν and density ρ in an undisturbed state.



In cylindrical coordinate the three dimensional stress equation of motion, strain displacement relation and heat conduction in the absence of body force for a linearly elastic medium are:

$$\sigma_{rr,r} + r^{-1}\sigma_{r\theta,\theta} + \sigma_{rz,z} + r^{-1}\left(\sigma_{rr} - \sigma_{\theta\theta}\right) = \rho u_{,tt}$$

$$\sigma_{r\theta,r} + r^{-1}\sigma_{\theta\theta,\theta} + \sigma_{,rzz} + \sigma_{\theta z,z} + 2r^{-1}\sigma_{r\theta} = \rho v_{,tt}$$

$$\sigma_{rz,r} + r^{-1}\sigma_{\theta z,\theta} + \sigma_{zz,z} + r^{-1}\sigma_{r\theta} = \rho w_{,tt}$$

$$\kappa(T_{,rr} + r^{-1}T_{,r} + r^{-2}T_{,\theta\theta} + T_{,zz}) = \rho C v T_{,t} + \beta T_0(u_{,rt} + r^{-1}(u_{,t} + v_{,\theta t}) + w_{,tz}) \tag{1}$$

where ρ is the mass density, c_v is the specific heat capacity, $\kappa = K/\rho c_v$ is the diffusity, K is the thermal conductivity, T_0 is the reference temperature.

$$\sigma_{rr} = \lambda(e_{rr} + e_{\theta\theta} + e_{zz}) + 2\mu e_{rr} - \beta(T)$$
$$\sigma_{\theta\theta} = \lambda(e_{rr} + e_{\theta\theta} + e_{zz}) + 2\mu e_{\theta\theta} - \beta(T)$$
$$\sigma_{zz} = \lambda(e_{rr} + e_{\theta\theta} + e_{zz}) + 2\mu e_{zz} - \beta(T) \tag{2}$$

where e_{ij} are the strain components, β is the thermal stress coefficients, T is the temperature, t is the time, λ and μ are Lame' constants. The strain e_{ij} are related to the displacements are given by

$$\sigma_{r\theta} = \mu\gamma_{r\theta} \ \ \sigma_{rz} = \mu\gamma_{rz} \ \ \sigma_{\theta z} = \mu\gamma_{\theta z} \ \ e_{rr} = \frac{\partial u}{\partial r} \ \ e_{\theta\theta} = \frac{u}{r} + \frac{1}{r}\frac{\partial v}{\partial \theta} \tag{3}$$

$$e_{zz} = \frac{\partial w}{\partial z} \ \ \gamma_{r\theta} = \frac{\partial v}{\partial r} - \frac{v}{r} + \frac{1}{r}\frac{\partial u}{\partial \theta} \ \ \gamma_{rz} = \frac{\partial w}{\partial r} + \frac{\partial u}{\partial z} \ \ \gamma_{z\theta} = \frac{\partial v}{\partial z} + \frac{1}{r}\frac{\partial w}{\partial \theta} \tag{4}$$

Where u, v, w are displacements along radial, circumferential and axial directions respectively. $\sigma_{rr}, \sigma_{\theta\theta}, \sigma_{zz}$ are the normal stress components and $\sigma_{r\theta}, \sigma_{\theta z}, \sigma_{zr}$ are the shear stress components, $e_{rr}, e_{\theta\theta}, e_{zz}$ are normal strain components and $e_{r\theta}, e_{\theta z}, e_{zr}$ are shear strain components.

Substituting the equation (3) and equation (2) in equation(1),gives the following three displacement equations of motion:

$$(\lambda+2\mu)\left(u_{,rr}+r^{-1}u_{,r}-r^{-2}u\right)+\mu r^{-2}u_{,\theta\theta}+\mu u_{,zz}+r^{-1}(\lambda+\mu)v_{,r\theta}-r^{-2}(\lambda+3\mu)v_{,\theta}+(\lambda+\mu)w_{,rz}-\beta\left(T_{,r}\right)=\rho u_{,tt}$$

$$\mu\left(v_{,rr}+r^{-1}v_{,r}-r^{-2}v\right)+r^{-2}(\lambda+2\mu)v_{,\theta\theta}+\mu v_{,zz}+r^{-2}(\lambda+3\mu)u_{,\theta}+r^{-1}(\lambda+\mu)u_{,r\theta}+r^{-1}(\lambda+\mu)w_{,\theta z}-\beta\left(T_{,\theta}\right)=\rho v_{,tt}$$

$$(\lambda+2\mu)w_{,zz}+\mu\left(w_{,rr}+r^{-1}w_{,r}+r^{-2}w_{,\theta\theta}\right)+(\lambda+\mu)u_{,rz}+r^{-1}(\lambda+\mu)v_{,\theta z}+r^{-1}(\lambda+\mu)u_{,z}-\beta\left(T_{,z}\right)=\rho w_{,tt}$$

$$\rho c_v\kappa\left(T_{,rr}+r^{-1}T_{,r}+r^{-2}T_{,\theta\theta}+T_{,zz}\right)=\rho c_v T_{,t}+\beta T_0\left[u_{,tr}+r^{-1}(u_{,t}+v_{,t\theta})+w_{,tz}\right] \tag{5}$$

To solve equation (5),we take

$$u = \frac{1}{r}\psi_{,\theta} - \phi_{,r} \ \ v = -\frac{1}{r}\phi_{,\theta} - \psi_{,\varpi} \ \ w = -\chi_{,z}$$

Using Eqs (5) in Eqs (1), we find that ϕ, χ, T satisfies the equations.

$$((\lambda+2\mu)\nabla^2{}_1 + \mu\frac{\partial^2}{\partial z^2} - \rho\frac{\partial^2}{\partial t^2})\phi - (\lambda+\mu)\frac{\partial^2\chi}{\partial z^2} = \beta\ (T) \tag{6a}$$

$$(\mu\nabla_1{}^2 + (\lambda+2\mu)\frac{\partial^2}{\partial z^2} - \rho\frac{\partial^2}{\partial t^2})\chi - (\lambda+\mu)\nabla_1{}^2\phi = \beta(T) \tag{6b}$$

$$(\nabla_1{}^2 + \frac{\partial^2}{\partial z^2} - \frac{\rho}{\mu}\frac{\partial^2}{\partial t^2})\psi = 0 \tag{6c}$$

$$\nabla_1{}^2 T + \frac{\partial^2 T}{\partial z^2} - \frac{1}{k}\frac{\partial T}{\partial t} + \frac{\beta T_0(i\omega)}{\rho C_V K}(\nabla_1{}^2\phi + \frac{\partial^2\chi}{\partial z^2}) = 0 \tag{6d}$$

Equation (6c) in ψ gives a purely transverse wave, which is not affected by temperature. This wave is polarized in planes perpendicular to the z-axis. We assume that the disturbance is time harmonic through the factor $e^{i\omega t}$.We can write the three displacement functions and the temperature change as:

Solution to the problem

The equation (6) is coupled partial differential equations of the three displacement components. To uncouple equation(6),we can write three displacement functions which satisfies the simply supported boundary conditions followed by Sharma [12]

$$\psi(r,\theta,z,t)=\overline{\psi}(r)\sin(m\pi z)\cos(n\pi\theta/\alpha)e^{i\omega t}$$

$$\phi(r,\theta,z,t)=\overline{\phi}(r)\sin(m\pi z)\sin(n\pi\theta/\alpha)e^{i\omega t} \tag{7}$$

$$\chi(r,\theta,z,t)=\overline{\chi}(r)\sin(m\pi z)\sin(n\pi\theta/\alpha)e^{i\omega t}$$

$$T(r,\theta,z,t)=\overline{T}(r,\theta,z,t)\sin(m\pi z)\sin(n\pi\theta/\alpha)e^{i\omega t}$$

Where m is the circumferential mode and n is the axial mode, ω is the angular frequency of the cylindrical panel motion. By introducing the dimensionless quantities:

$$r^{'}=\frac{r}{R}\ z^{'}=\frac{z}{L}\ \overline{T}=\frac{T}{T_0}\ \delta=\frac{n\pi}{\alpha}\ t_L=\frac{m\pi R}{L}\ \overline{\lambda}=\frac{\lambda}{\mu}\ \in_4=\frac{1}{2+\overline{\lambda}}$$

$$C_1^{\,2}=\frac{\lambda+2\mu}{\rho}\ \Omega^2=\frac{\omega^2R^2}{C_1^2} \tag{8}$$

After substituting equation (8) in (7),we obtain the following system of equations:

$$(\nabla_2^2+k_1^{\,2})\overline{\psi}=0 \tag{9a}$$

$$(\nabla_2^2+g_1)\overline{\phi}+g_2\overline{\chi}-g_4\overline{T}=0 \tag{9b}$$

$$(\nabla_2^2+g_3)\overline{\chi}+(1+\overline{\lambda})\nabla_2^2\overline{\phi}+(2+\overline{\lambda})g_4\overline{T}=0 \tag{9c}$$

$$(\nabla_2^{\,2}-t_L^{\,2}+\in_2\Omega^2-i\in_3)\overline{T}+i\in_1\Omega\nabla_2^{\,2}\overline{\phi}-i\in_1\Omega t_L^{\,2}\overline{\chi}=0 \tag{9d}$$

where

$$\nabla_2^2=\frac{\partial^2}{\partial r^2}\frac{1}{r}\frac{\partial}{\partial r}-\frac{\delta^2}{r^2},\ \in_1=\frac{T_0R\beta^2}{\rho^2C_VC_1K}\ \in_2=\frac{C_1^{\,2}}{C_VK}\ \in_3=\frac{C_1R}{K}$$

$$g_1=(2+\overline{\lambda})(t_L^2-\Omega^2)\ g_2=\in_4(1+\overline{\lambda})t_L^{\,2}$$

$$g_3=(\Omega^2-\in_4t_L^{\,2})\ g_4=\frac{\beta T_0R^2}{\lambda+2\mu}\ g_5=\in_1\Omega$$

C_1 wave velocity of the cylindrical panel. A non-trivial solution of the algebraic equations systems (9) exist only when the determinant of equatins (9) is equal to zero.

$$\begin{vmatrix} (\nabla_2^{\ 2}+g_1) & -g_2 & g_4 \\ (1+\overline{\lambda})\nabla_2^{\ 2} & (\nabla_2^{\ 2}+g_3) & (2+\overline{\lambda})g_4 \\ ig_5\nabla_2^{\ 2} & -ig_5 t_L^{\ 2} & (\nabla_2^{\ 2}-t_L^{\ 2}+\in_2\Omega^2-i\Omega\in_3) \end{vmatrix}(\overline{\phi},\overline{\chi},\overline{T})=0 \tag{10}$$

Equation (10), on simplification reduces to the following differential equation:

$$\nabla_2^6+A\nabla_2^4+B\nabla_2^2+C=0 \tag{11}$$

Where,

$$A=-g_1+g_2(1+\overline{\lambda})+g_3-g_4g_5it_L^{\ 2}+\in_2\Omega^2-i\in_3\Omega$$

$$B=-g_1g_3-g_1g_4g_5i-g_2g_4g_5i(2+\overline{\lambda})+t_l^2(g_1-g_2-g_3)+g_4g_5it_L^{\ 2}+g_2\Omega^2\in_2(1+\overline{\lambda})-g_2i\in_3\Omega(1+\overline{\lambda})$$
$$-g_2t_L^{\ 2}\overline{\lambda}+g_3\Omega(\Omega\in_2-i\in_3)+g_1\Omega(i\in_3-\Omega\in_2)$$

$$C=g_1g_3(t_L^{\ 2}+i\in_3\Omega-\in_2\Omega^2)+ig_3g_4g_5t_L^{\ 2}(2+\overline{\lambda})$$

The solution of equation (11) are

$$\overline{\phi}=\sum_{i=1}^{3}A_iJ_\delta(\alpha_i r)\phi(r)\sin(m\pi z)\sin(n\pi\theta/\alpha)e^{i\omega t}$$

$$\overline{\chi}(r)=\sum_{i=1}^{3}A_id_iJ_\delta(\alpha_i r)\chi(r)\sin(m\pi z)\sin(n\pi\theta/\alpha)e^{i\omega t}$$

$$\overline{T}(r)=\sum_{i=1}^{3}A_ie_iJ_\delta(\alpha_i r)T(r)\sin(m\pi z)\sin(n\pi\theta/\alpha)e^{i\omega t}$$

$$\overline{\psi}(r)=A_4J_\delta(k_1 r)\psi(r)\sin(m\pi z)\cos(n\pi\theta/\alpha)e^{i\omega t}$$

Here, $(\alpha_i r)^2$ are the non-zero roots of the algebraic equation

$$(\alpha_i r)^6-A(\alpha_i r)^4+B(\alpha_i r)^2-C=0$$

The arbitrary constant d_i and e_i is obtained from

$$d_i=\left[\frac{(1+\overline{\lambda})\delta_i^2-(2+\overline{\lambda})\delta_i^2-g_1}{g_2(2+\overline{\lambda})-\delta_i^2-g_3}\right]$$

$$e_i = \left(\frac{\lambda + 2\overline{\mu}}{\beta T_0 R^2}\right)\left[\frac{\varepsilon_4 \delta_i^2 + \left(\varepsilon_4\left(g_1 + g_3\right) + \varepsilon_4(1+\overline{\lambda})g_2\right)\delta_i^2 + \varepsilon_4 g_1 g_3 + \delta_i^2) - g_1 g_3}{\varepsilon_4 g_3 + \varepsilon_4 \delta_i^2 - g_2}\right]$$

Eq. (9a) is a Bessel equation with its possible solutions are

$$\overline{\psi} = \begin{cases} A_3 J_\delta(k_1 r) + B_3 Y_\delta(k_1 r), k_1^2 > 0 \\ A_3 r^\delta + B_3 r^{-\delta}, k_1^2 = 0 \\ A_3 I_\delta(k_1' r) + B_3 K_\delta(k_1' r), k_1^2 < 0 \end{cases} \tag{12}$$

Where $k_1'^2 = -k_1^2$,and, J_δ and Y_δ are Bessel functions of the first and second kinds respectively while, I_δ and k_δ are modified Bessel functions of first and second kinds respectively. A_3 and B_3 are two arbitrary constants. Generally $k_1^2 \neq 0$,so that the situation $k_1^2 \neq 0$ is will not be discussed in the following. For convenience,we consider the case of $k_1^2 > 0$,and the derivation for the case of $k_1^2 < 0$ is similar.

The solution of equation (9a) is

$$\overline{\psi}(r) = A_4 J_\delta(k_1 r)\psi(r)\sin(m\pi z)\cos(n\pi\theta/\alpha)e^{iwt}$$

Where $k_1^2 = (2+\overline{\lambda})\Omega^2 - t_L{}^2$

Boundary condition and frequency equation

In this section we shall derive the secular equation for the three dimensional vibrations cylindrical panel subjected to traction free boundary conditions at the upper and lower surfaces at

$$r = a, b$$

$$u_r = \left(-\overline{\phi}' - \frac{\delta\overline{\psi}'}{r}\right)\sin(m\pi z)\sin(\delta\theta)e^{i\omega t}$$

$$u_\theta = \left(-\overline{\psi}' - \frac{\delta\overline{\phi}'}{r}\right)\sin(m\pi z)\cos(\delta\theta)e^{i\omega t}$$

$$u_z = \overline{\chi} t_L \cos(m\pi z)\sin(\delta\theta)e^{i\omega t}$$

$$\bar{\sigma}_{rr}=\left[\left(2+\bar{\lambda}\right)\delta\left(\frac{\bar{\psi}'}{r}-\frac{\bar{\psi}}{r^2}\right)+\left(2+\bar{\lambda}\right)\left(\frac{1}{r}\bar{\varphi}'+(\alpha_i^2-\frac{\delta^2}{r^2}\bar{\varphi})\right)+\bar{\lambda}\left(\frac{\delta}{r^2}\bar{\psi}-\frac{1}{r}\bar{\phi}'-\frac{\delta^2}{r^2}\bar{\varphi}-\frac{\delta}{r}\bar{\psi}'-t_L^2\bar{\chi}\right)\right]$$

$$\sin(m\pi)z\cos(\delta\theta)e^{i\omega t}$$

$$\bar{\sigma}_{r\theta}=2\left(\frac{1}{r}\bar{\psi}+(\alpha_i^2-\frac{\delta^2}{r^2})\bar{\psi}-\frac{2\delta}{r}\bar{\varphi}'+\frac{2\delta}{r^2}\bar{\varphi}+\frac{\bar{\psi}'}{r}-\frac{\delta^2}{r^2}\bar{\psi}\right)\sin(m\pi)z\cos(\delta\theta)e^{i\omega t}$$

$$\bar{\sigma}_{rz}=2t_L\left(-\bar{\varphi}'-\frac{\delta}{r}\bar{\psi}+\bar{\chi}'\right)\cos(m\pi)z\sin(\delta\theta)e^{i\omega t}$$

Where prime denotes the differentiation with respect to r $\bar{u}_i=u_i/R,(i=r,\theta,z)$ are three non– dimensional displacements and $\bar{\sigma}_{rr}=\sigma_{rr}/\mu,\bar{\sigma}_{r\theta}=\sigma_{r\theta}/\mu,\bar{\sigma}_{rz}=\sigma_{rz}/\mu$ are three non-dimensional stresses. For the purpose of comparison,we first consider the uncoupled free vibration of isotropic cylindrical panel. In this case both convex and concave surface of the panel are traction free

$$\sigma_{rr}=\sigma_{r\theta}=\sigma_{rz}=0,T_{,r}=0(r=a,b) \quad (13)$$

$$\left|E_{ij}\right|=0 \qquad i,j=1,2,...8 \quad (14)$$

$$E_{11}=(2+\bar{\lambda})\left((\delta J_\delta(\alpha_1t_1)/t_1^2-\frac{\alpha_1}{t_1}J_{\delta+1}(\alpha_1t_1))-((\alpha_1t_1)^2R^2-\delta^2)J_\delta(\alpha_1t_1)/t_1^2\right)$$

$$+\bar{\lambda}\left(\delta(\delta-1)J_\delta(\alpha_1t_1)/t_1^2-\frac{\alpha_1}{t_1}J_{\delta+1}(\alpha_1t_1)\right)+\bar{\lambda}d_1t_L^2J_\delta(\alpha_1t_1)-\beta T_0R^2e_1\bar{\lambda}$$

$$E_{13}=(2+\bar{\lambda})\left((\delta J_\delta(\alpha_2t_1)/t_1^2-\frac{\alpha_2}{t_2}J_{\delta+1}(\alpha_2t_1))-((\alpha_2t_1)^2R^2-\delta^2)J_\delta(\alpha_2t_1)/t_1^2\right)$$

$$+\bar{\lambda}\left(\delta(\delta-1)J_\delta(\alpha_2t_1)/t_1^2-\frac{\alpha_2}{t_1}J_{\delta+1}(\alpha_2t_1)\right)+\bar{\lambda}d_2t_L^2J_\delta(\alpha_2t_1)-\beta T_0R^2e_2\bar{\lambda}$$

$$E_{15}=(2+\bar{\lambda})\left((\delta J_\delta(\alpha_3t_1)/t_1^2-\frac{\alpha_2}{t_2}J_{\delta+1}(\alpha_3t_1))-((\alpha_3t_1)^2R^2-\delta^2)J_\delta(\alpha_3t_1)/t_1^2\right)$$

$$+\bar{\lambda}\left(\delta(\delta-1)J_\delta(\alpha_3t_1)/t_1^2-\frac{\alpha_2}{t_1}J_{\delta+1}(\alpha_3t_1)\right)+\bar{\lambda}d_2t_L^2J_\delta(\alpha_3t_1)-\beta T_0R^2e_3\bar{\lambda}$$

$$E_{17}=(2+\bar{\lambda})\left((\frac{k_1\delta}{t_1}J_{\delta+1}(k_1t_1)-\delta(\delta-1)J_\delta(k_1t_1)/t_1^2\right)+\bar{\lambda}\left(\delta(\delta-1)J_\delta(k_1t_1)/t_1^2-\frac{k_1\delta}{t_1}J_{\delta+1}(k_1t_1)\right)$$

$$E_{21}=2\delta\left((\alpha_1/t_1)J_{\delta+1}(\alpha_1t_1)-\delta(\delta-1)J_\delta(\alpha_1t_1)\right)$$

$$E_{23}=2\delta\left((\alpha_2/t_1)J_{\delta+1}(\alpha_2t_1)-\delta(\delta-1)J_\delta(\alpha_2t_1)\right)$$

$$E_{25} = 2\delta\left((\alpha_3/t_1)J_{\delta+1}(\alpha_3 t_1) - \delta(\delta-1)J_\delta(\alpha_3 t_1)\right)$$

$$E_{27} = (k_1 t_1)^2 R^2 J_\delta(k_1 t_1) - 2\delta(\delta-1)J_\delta(k_1 t_1)/t_1^{\,2} + k_1/t_1 J_{\delta+1}(k_1 t_1)$$

$$E_{31} = -t_L(1+d_1)\left(\delta/t_1 J_\delta(\alpha_1 t_1) - \alpha_1 J_{\delta+1}(\alpha_1 t_1)\right)$$

$$E_{33} = -t_L(1+d_2)\left(\delta/t_1 J_\delta(\alpha_2 t_1) - \alpha_2 J_{\delta+1}(\alpha_2 t_1)\right)$$

$$E_{35} = -t_L(1+d_3)\left(\delta/t_1 J_\delta(\alpha_3 t_1) - \alpha_2 J_{\delta+1}(\alpha_3 t_1)\right)$$

$$E_{37} = -t_L(\delta/t_1)J_\delta(k_1 t_1)$$

$$E_{41} = e_1[(\delta/t_1)J_\delta(\alpha_1 t_1) - (\alpha_1)J_{\delta+1}(\alpha_1 t_1)]$$

$$E_{43} = e_2[(\delta/t_1)J_\delta(\alpha_2 t_1) - (\alpha_2)J_{\delta+1}(\alpha_2 t_1)]$$

$$E_{45} = e_3[(\delta/t_1)J_\delta(\alpha_3 t_1) - (\alpha_3)J_{\delta+1}(\alpha_3 t_1)]$$

$$E_{47} = 0$$

In which $t_1 = a/R = 1 - t^*/2$, $t_2 = b/R = 1 + t^*/2$ and $t^* = b - a/R$ is the thickness - to-mean radius ratio of the panel. Obviously $E_{ij}\left(j = 2,4,6,8\right)$ can obtained by just replacing modified Bessel function of the first kind in $E_{ij}\left(i = 1,3,5,7\right)$ with the ones of the second kind, respectively, while $E_{ij}\left(i = 5,6,7,8\right)$ can be obtained by just replacing t_1 in $E_{ij}\left(i = 1,2,3,4\right)$ with t_2

Numerical results and discussion

The frequency equation (14) is numerically solved for Zinc material. For the purpose of numerical computation we consider the closed circular cylindrical shell with the center angle $\alpha = 2\pi$ and the integer n must be even since the shell vibrates in circumferential full wave. The frequency equation for a closed cylindrical shell can be obtained by setting $\delta = l\left(l = 1,2,3.....\right)$ where l is the circumferential wave number in equations(14). The material properties of a Zinc is

$\rho = 7.14\times 10^3 kgm^{-3}$, T_0=296 K

$\mu = 0.508\times 10^{11} Nm^{-2}$, β=1

$\lambda = 0.385\times 10^{11} Nm^{-2}$ and Poisson ratio ν=0.3. Cv=3.9×10^2 J kg^{-1} deg^{-1}

The roots of the algebraic equation (11) was calculated using a combination of Birge-Vita method and Newton-Raphson method. In the present case simple Birge-Vita method does not work for finding the root of the algebraic equation. After obtaining the roots of the algebraic equation using Birge-Vita method, the roots are corrected for the desired accuracy using the Newton-Raphson method. This combination has overcome the difficulties in finding the roots of the algebraic equations of the governing equations. A dispersion curve is drawn between the non-dimensional length to mean radius ratio versus dimensionless frequency for the different thickness parameters t*= 0.01,0.1 0.25 with the axial wave number $t_L = 1$ and $t_L = 2$ and the circumferential wave number $\delta = 1,2$ is shown in Fig.2 and Fig.3 respectively.

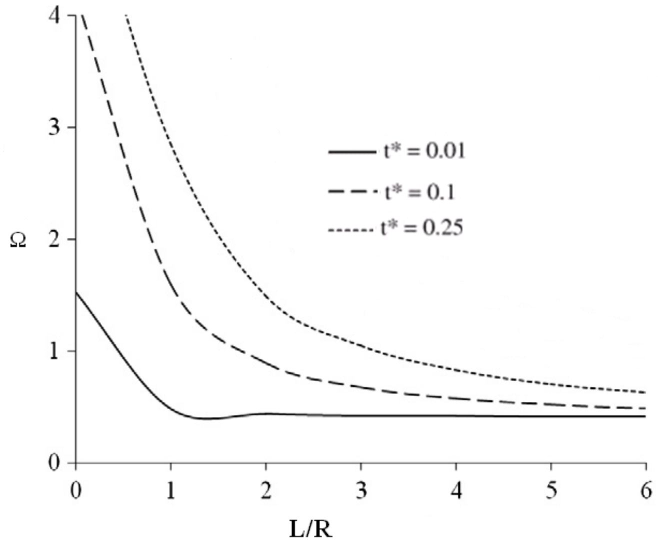


Figure 2: Variation of length to mean radius ratio verses Non-dimensional frequency with different t* for $t_L = 1$ and $\delta = 1$

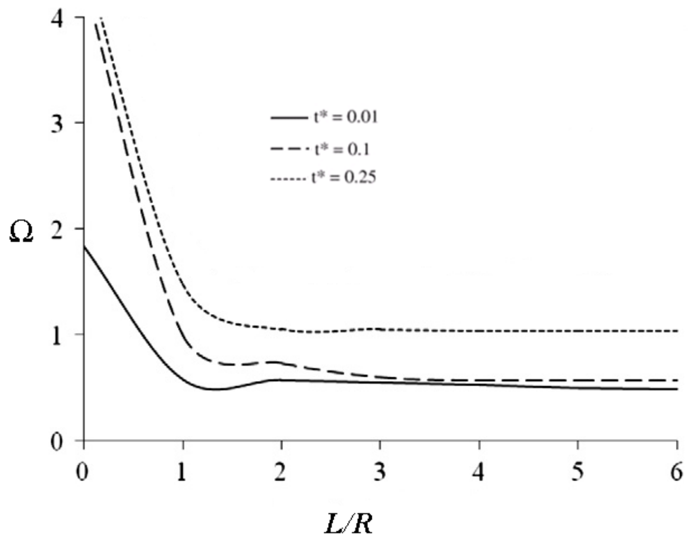


Figure 3: Variation of length to mean radius ratio verses Non-dimensional frequency with different t* for $t_L = 2$ and $\delta = 1$

From the Figs.2 and 3, it is observed that the non-dimensional frequency decreases rapidly to become linear at L/R=3 for both $t_L = 1$ and $t_L = 2$ .when the thickness of the cylindrical panel is increased, the dimensionless frequency is decreases. This is the proper physical behavior of a cylindrical panel with respect to its thickness. The comparison of Fig.2 and Fig.3 shows that the non-dimensional frequency decrease exponentially for L/R< 3 in case of the axial wave number $t_L = 1$ and $t_L = 2$ and the circumferential wave number $\delta = 1$,2 for all value of t*,but the case when L/R >3 the non-dimensional frequency is steady and slow for all values of t*.

Conclusion

The three dimensional vibration analysis of a homogenous isotropic cylindrical panel subjected to simply supported boundary conditions has been considered for this paper. For this problem, the governing equations of three dimensional linear elasticity have been employed and solved by modified Bessel function with complex argument. Comparison of numerical results with those of related publications proves the feasibility and effectiveness of the present method. The effect of the length to mean radius ratio on the natural frequencies of a closed Zinc cylindrical shell is investigated and the results are presented as dispersion curves.

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